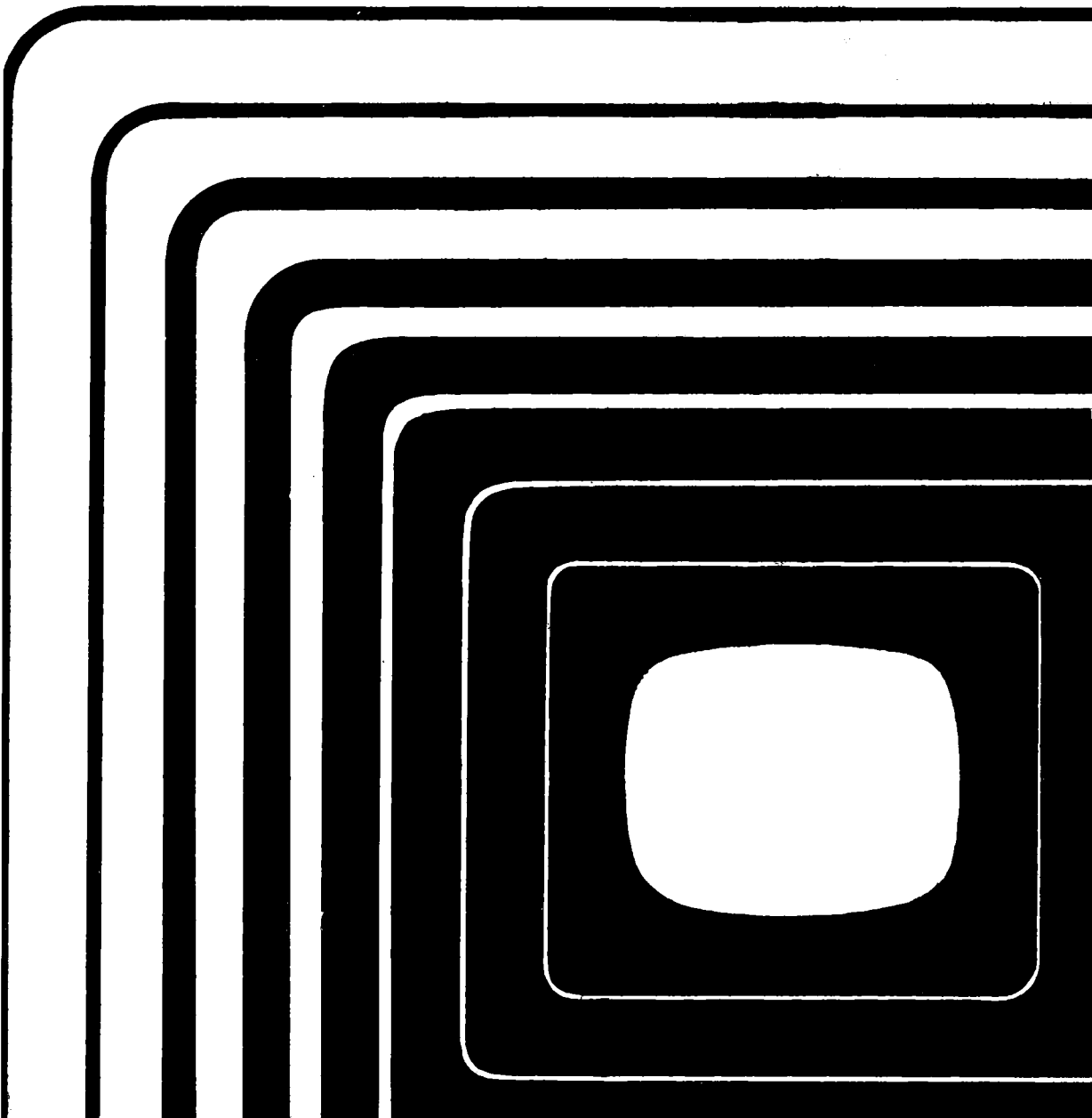


Electron Optics and Electron-beam Devices

A. Zhigarev

Mir Publishers · Moscow



The book covers the fundamentals of electron optics, including geometrical electron optics and electron optics of intensive beams; adequately dealt with are electron guns, deflecting and focusing systems and fluorescent screens utilized in the oscillographic, radar, television and computer cathode-ray tubes.

This textbook is designed for electronic engineering students following the courses on electronics, especially on electron optics and electron-beam devices. It will interest engineers, technicians and all other specialists engaged in research, design and use of cathode-ray tubes and microwave devices as well as those concerned with simulating electron beams and experimental techniques involved.

The author, Cand. Sc. (Tech.), post-graduate and since 1962 Scientific Secretary of the Electronics Committee under the auspices of the USSR Ministry of Higher and Secondary Special Education, has conducted the work of supervisor for more than 25 years at the higher educational establishments, at the same time undertaking scientific research.

Credited to him are more than thirty published scientific works, a monograph on cathode-ray tubes, five school textbooks, a manual and three inventor's certificates for inventions in the field of electronic instrument engineering.



А. А. Жигарев

ЭЛЕКТРОННАЯ
ОПТИКА
И ЭЛЕКТРОННО-
ЛУЧЕВЫЕ
ПРИБОРЫ

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A. Zhigarev

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by N. Utkin

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The Russian Alphabet and Transliteration

А а	а	К к	к	Х х	kh
Б б	b	Л л	l	Ц ц	ts
В в	v	М м	m	Ч ч	ch
Г г	g	Н н	n	Ш ш	sh
Д д	d	О о	o	Щ щ	shch
Е е	e	П п	p	Ъ ъ	”
Ё ё	ë	Р р	r	Ы ы	y
Ж ж	zh	С с	s	Ь ь	’
З з	z	Т т	t	Э э	e
И и	i	У у	u	Ю ю	yu
Й й	y	Ф ф	f	Я я	ya

The Greek Alphabet

Α α	Alpha	Ι ι	Iota	Ρ ρ	Rho
Β β	Beta	Κ κ	Kappa	Σ σ	Sigma
Γ γ	Gamma	Λ λ	Lambda	Τ τ	Tau
Δ δ	Delta	Μ μ	Mu	Υ υ	Upsilon
Ε ε	Epsilon	Ν ν	Nu	Φ φ	Phi
Ζ ζ	Zeta	Ξ ξ	Xi	Χ χ	Chi
Η η	Eta	Ο ο	Omicron	Ψ ψ	Psi
Θ θ	Theta	Π π	Pi	Ω ω	Omega

На английском языке

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Preface

This book, primarily intended for university students studying the “Electron Optics and Electron-Beam Devices” course, presents the fundamentals of electron optics including intensive-beam optics and detailed description of basic components of electron-beam devices as well as the most common types of such devices.

The main emphasis is made on study of the physical processes underlying the operating principle of electron-beam devices. Hence the first chapters dealing with the problems of electron optics occupy a considerable portion of the book. Rapid development of superhigh-frequency systems has raised a problem of design of devices capable of shaping electron beams with perveance up to 10^{-6} A/V^{3/2} and higher. The problems of shaping of such beams and the electron guns for their generation are discussed in Chapters 2 and 3 which may be used as an independent teaching material for those specializing in the superhigh-frequency engineering.

Chapters 4, 5, 6 give detailed description of the components of electron-beam devices such as electron guns, deflection systems and luminescent screens. Chapters 7 through 12 are devoted to specific types of electron-beam devices ranging from a simple oscillograph tube to complex TV camera tubes. It is impossible, of course, to present the total variety of up-to-date electron-beam devices within a scope of a single book, and the author did not intend to get such a goal, keeping in mind that knowledge of the fundamental physical processes taking place in the typical electron-beam devices

will help the reader to understand the design and operation of some newly developed devices which are not described in this book. For the same reason, the book does not include tables with parameters of electron-beam devices, since a text-book should by no means be a substitution of special reference literature but should simply teach a student to use various sources including handbooks, thus assisting in selection of a device with necessary parameters.

All equations in this book are written in the International System (SI). Therefore all values in the design formulas should be presented in the SI system. The only exception is the value of magnetic induction—Gauss, which is widely used in the practice of electromagnetic measurements and is permitted for use as a decimal unit multiple to the basic unit of the SI system—Tesla ($1 \text{ G} = 10^{-4} \text{ T}$).

The list of references includes only those books and papers which are available to the reader

Part One

Fundamentals of Electron Optics

Geometrical Electron Optics

1.1. Principal Concepts of Electron Optics

Electron optics is a branch of physics dealing with the problems of motion of charged particles (electrons and ions) in electric and magnetic fields, forming and focusing flows of charged particles, and producing images by means of electron and ion beams. As it will be shown (Section 1.3), the motion of charged particles in electric and magnetic fields is analogous to the propagation of light rays so that in many cases the known theses of ordinary (light) optics can be used for solving problems of electron optics.

Like the light optics, the electron optics makes use of an electron-optical refractive index and electron lenses, prisms and mirrors. Electron lenses, like optical lenses, are characterized by optical density, position of the principal focal points, principal planes, etc.

Electron optics, born in the 20s of our century has been greatly developed in recent years and now is often regarded as an independent science involving a wide scope of problems associated with the motion of charged particles in an electron-optical medium, i.e. in electric and magnetic fields under vacuum conditions.

Electron optics, like light optics, can be divided into two parts: geometrical electron optics and electron wave optics. The geometrical electron optics studies the problems of motion of charged particles in electric and magnetic fields, the methods of calculation of the paths of electrons and ions, the methods of focusing and formation of images with no allowance for the wave properties of moving particles. The electron wave optics considers a moving electron as a De Broglie wave and studies such purely wave phenomena as diffraction and interference of waves. The De Broglie wavelength $\lambda = \frac{h}{mv}$ (h is Planck's constant, m and v are the mass and velocity of the electron) even for comparatively slow electrons is very low. For example, for an electron having passed through an accelerating field at a potential difference of 100 V the length of the De Broglie wave is equal approximately to 1.25×10^{-10} m, i.e. shorter than 1 nm. Therefore, when considering the physical processes occurring in the majority of electron-beam devices, the wave properties of electrons may not

be taken into account. Only in some special cases, for example, during estimation of the theoretical limit of a resolving power of an electron microscope, it is necessary to employ the methods of wave optics.

Within the scope of geometrical electron optics we can neglect the Heisenberg uncertainty principle, i.e. we may assume that the velocity and coordinates of the electron can be determined simultaneously and with a sufficient accuracy. Indeed, the uncertainty relation may be written in the form

$$\Delta x \Delta v \approx \frac{h}{m_0} \quad (1.1)$$

where x and v are respectively the coordinate and velocity of the particle; h is Planck's constant ($h = 6.6 \times 10^{-34}$ joule $\times$ second); m_0 is the rest mass of the particle.

Substituting into (1.1) the mass of a resting (or moving with a velocity much lower than the light velocity) electron, $m_0 = 9.1 \times 10^{-31}$ kg, we obtain

$$\Delta x \Delta v = 0.73 \times 10^{-3} \text{ m}^2/\text{s}$$

In electron-beam devices the electron velocities are usually higher than 10^6 m/s. Then, taking the possible error in determining the velocity to be equal to 0.1%, the coordinate can be calculated with an accuracy of $0.73 \cdot 10^{-6}$ m which is quite permissible.

When studying the motion of electrons in electron-beam devices in many cases it is possible to use the equations of classical mechanics, i.e. to use the law of conservation of energy

$$\frac{mv^2}{2} = eU \quad (1.2)$$

where m , v and e are the mass, velocity and charge of the electron ($e = 1.6 \times 10^{-19}$ k) U is the potential difference of the electric field passed by the electron (it is assumed that with $U = 0$, $v = 0$, i.e. the electron starts moving in the electric field with no initial velocity)

The expression for the electron velocity is obtained directly from (1.2):

$$v = \sqrt{\frac{2e}{m} U} = 5.93 \times 10^5 \sqrt{U} \text{ [m/s]} \approx 600 \sqrt{U} \text{ km/s} \quad (1.3)$$

where the potential difference U is measured in volts.

However, if expression (1.3) is used for determining the velocity of an electron accelerated by the potential difference in the order of 1 MV, the value for v will exceed the velocity of light and this is in contradiction with the theory of relativity. Therefore, for the electrons accelerated by considerable potential differences (high-speed electrons) the law of conservation of energy should be written

in accordance with the theory of relativity in the form

$$m_0 c^2 \left(\frac{1}{\sqrt{1-v^2/c^2}} - 1 \right) = eU \quad (1.4)$$

where c is the velocity of light ($c \approx 3 \times 10^8$ m/s).

Thus the electron velocity

$$v = 5.93 \times 10^5 \sqrt{U} \frac{\sqrt{10^{-6}U+1}}{2 \times 10^{-6}U+1} \text{ [m/s]} \quad (1.5)$$

(the potential difference U is in volts).

It is evident that with $U \ll 10^6$ V expression (1.5) transforms into (1.3). To evaluate the limits of applicability of the laws of

Table 1.1

U , V	v , m/s	$\beta=v/c$	U , V	v , m/s	$\beta=v/c$
0.1	1.87×10^5	6.62×10^{-4}	10^4	5.80×10^7	1.95×10^{-1}
1.0	5.90×10^5	1.98×10^{-3}	10^5	1.64×10^8	5.90×10^{-1}
10	1.87×10^6	6.20×10^{-3}	10^6	2.84×10^8	0.95
10^2	5.90×10^6	1.98×10^{-2}	10^7	2.99×10^8	0.998
10^3	1.87×10^7	6.20×10^{-2}			

classical mechanics to moving electrons, it is convenient to use special tables (Table 1.1) or graphs (Fig. 1.1).

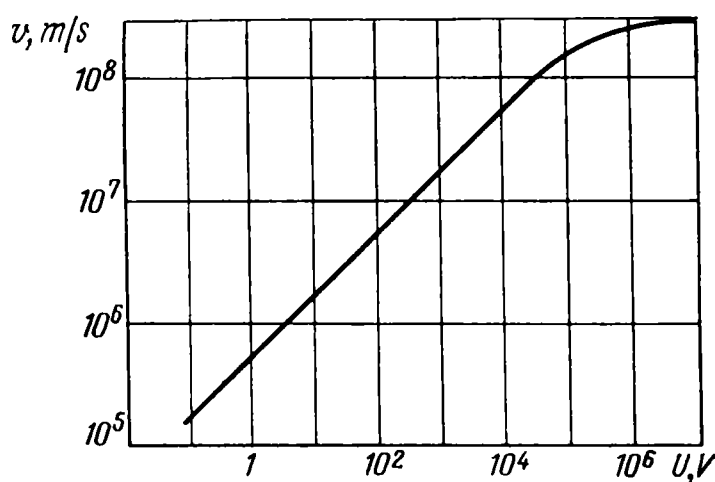


Fig. 1.1. Electron velocity versus potential difference

Figure 1.1 illustrates a logarithmic dependence of the electron velocity on the potential difference. With a low potential difference the dependence of v on $\sqrt{U}$ is linear. At high accelerating potential

differences the linear dependence is perceptibly disturbed and relativistic formulas should be used in such a case. In practice, when using the equations of classical mechanics, an allowable error occurs at accelerating potential differences up to 25-30 kV.

The relativistic equations are used only in certain cases. An example is studying the focusing and deflection of electrons in projection kinescopes, where the accelerating potential differences are as high as 70-80 kV, and in electron microscopes where the value of U is often higher than 100 kV.

In studying the fundamentals of geometrical electron optics we shall use such concepts as "electron flow", "electron pencil" and "electron beam". The *electron flow* is a general term defining a plurality of electrons moving approximately in the same direction. In some cases the flow can be discussed without taking into account the boundary conditions, i.e. being considered as unbounded. The *electron pencil* is a shaped flow with definite external boundaries. As a rule, the pencil is long, i.e. extends along one coordinate axis rather than along the two other axes. Hence we may speak of a configuration of a pencil, calculate the boundary trajectories and focal point of it.

The *electron beam* is an electron pencil so thin that its cross section may be considered negligible, while its trace on a screen is a point. A real electron pencil may be regarded as a bundle of electron beams. Often, by an electron beam is meant a narrow focused electron "ray" having a very small cross section area at least at one point. Such pencils, rays, are used in the most of electron-beam devices determining the general name of this class of cathode-ray devices.

In the study of geometrical electron optics the electric and magnetic fields specifying the motion of charged particles are assumed to be completely external, i.e. produced by external electrodes with predetermined potentials, as well as by current-carrying coils or magnets. At the same time, the electron, which is a charged particle, forms its own electric field while a moving charge also produces its own magnetic field. Therefore, strictly speaking, the electric and magnetic fields acting on the electron flow should be regarded as the superposition of the fields produced by the external electrodes and the magnets and the fields produced by the electron flow itself.

A perceptible effect of intrinsic field of electrons starts manifesting itself at comparatively high densities of the space charge in the pencil and at low velocities of electrons (see Section 2.1). A perceptible effect of the intrinsic magnetic field of electron flow starts to appear only at electron velocities close to the velocity of light, i.e. relativistically.

From the viewpoint of space charge and, therefore, the inherent field of the charged particles the electron optics can be divided into two parts: the optics of electrons with no allowance for the influ-

ence of the space charge and the optics of intensive pencils (beams) in which the influence of the space charge imposes certain restrictions on the possibility of focusing.

In the first case the focusing can be effected, high-quality images are obtainable and the methods of calculation based on the analogy of electron and light optics can be applied. In the other case the focusing as such, i.e. convergence of the electron trajectories to one point becomes impossible and satisfactory images cannot be obtained.

The intermediate region, in which a satisfactory image can be obtained, but its quality being well affected by the action of the space charge, is still underdeveloped. In this chapter we will discuss the methods of calculation and study of electron-optical systems without taking into account the influence of the space charge of the pencil electrons, while the optics of intensive beams (for which the effect of the space charge is important) will be studied in Chapter 2

1.2. Motion of Electrons in Electrostatic and Magnetic Fields

Let us consider the motion of electrons in electric and magnetic fields. An electron moving in electric and magnetic fields is acted on by the Lorentz force

$$\mathbf{F} = -e \{ \mathbf{E} + [\mathbf{v}\mathbf{B}] \} \quad (1.6)$$

where $\mathbf{E}$ is the electric field intensity vector; $\mathbf{B}$ is the magnetic induction vector.

In the case of an electric field only, the equation of motion of electrons in the Cartesian coordinates (for nonrelativistic electrons) is as follows

$$\left. \begin{aligned} m\ddot{x} &= -eE_x \\ m\ddot{y} &= -eE_y \\ m\ddot{z} &= -eE_z \end{aligned} \right\} \quad (1.7)$$

where E_x , E_y and E_z are the components of the electric field intensity.

As an example, let us consider motion of electrons in a uniform electric field ($E_y = \text{const}$, $E_x = E_z = 0$) (Fig. 1.2).

Assume that an electron enters the field along the axis OZ with an initial velocity v_{z0} ($v_{x0} = v_{y0} = 0$).

Integrating the system of equations (1.7) will give us:

$$\begin{aligned} v_z &= v_{z0} = \text{const}, \quad z = v_{z0}t \\ v_y &= \frac{e}{m} E_y t, \quad y = \frac{1}{2} \times \frac{e}{m} E_y t^2 \end{aligned}$$

By excluding t from the right-hand equations we obtain the equation describing the electron trajectory

$$y = \frac{1}{2} \times \frac{e}{m} E_y \frac{z^2}{v_z^2} \quad (1.8)$$

Therefore, the path of an electron moving in a uniform electric field is a parabola. The definition of the angle of deflection of the

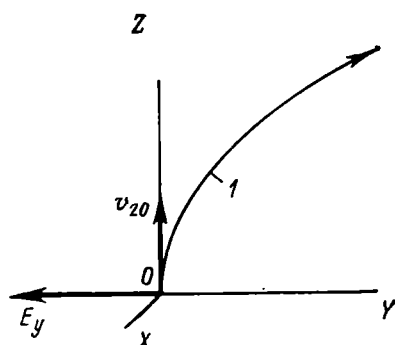


Fig. 1.2. Motion of electron in uniform electric field
1—trajectory

path of the electron from the original direction is of practical interest. By differentiating expression (1.8) with respect to z , we find

$$\frac{dy}{dz} = \tan \alpha = \frac{e}{m} E_y \frac{z}{v_z^2} \quad (1.9)$$

In the case of a magnetic field the equations describing the motion of electrons (in the Cartesian coordinates) assume the form

$$\left. \begin{aligned} m\ddot{x} &= -e(v_y B_z - v_z B_y) \\ m\ddot{y} &= -e(v_z B_x - v_x B_z) \\ m\ddot{z} &= -e(v_x B_y - v_y B_x) \end{aligned} \right\} \quad (1.10)$$

It is clear that the force acting on the electron in a magnetic field is always perpendicular to the direction of the velocity. This leads to a very important conclusion that in a magnetic field the velocity and energy of an electron remain constant, while the direction of its motion varies. Indeed, the work done by the force is equal to the scalar product of the force vector and the velocity vector. In accordance with (1.6) we can write

$$W = \mathbf{F} \mathbf{v} = -e [\mathbf{v} \mathbf{B}] \mathbf{v} = -e [\mathbf{v} \mathbf{v}] \mathbf{B} = 0 \quad (1.11)$$

In the practically interesting case of motion of an electron in a uniform transverse ($\mathbf{v} \perp \mathbf{B}$) magnetic field (Fig. 1.3) the Lorentz force is a centripetal force, and the electron describes a circular

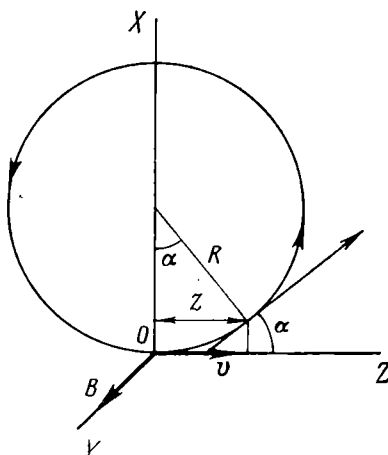


Fig. 1.3. Motion of electron in uniform magnetic field

trajectory whose radius can be determined by the equation

$$\frac{mv^2}{R} = evB \quad (1.12)$$

hence

$$R = \frac{mv}{eB} \quad (1.13)$$

Let us determine the angle of deflection of an electron in a transverse magnetic field (Fig. 1.3)

$$\sin \alpha = \frac{z}{R} \quad (1.14)$$

Substituting the value of R from (1.13) and assuming that $\sin \alpha \approx \tan \alpha$ for small angles, we obtain

$$\tan \alpha = \frac{eBz}{mv} \quad (1.15)$$

Thus, in order to find the paths of the electrons moving in electric and magnetic fields, one must solve the systems of differential equations (1.7) and (1.10). The system (1.7) can be solved if the intensity of the electric field or the potential are given in the form of functions of coordinates

$$U = U(x, y, z), \quad E_x = -\frac{\partial U}{\partial x}, \quad E_y = -\frac{\partial U}{\partial y}, \quad E_z = -\frac{\partial U}{\partial z}$$

In a space free of an electric charge the potential satisfies the Laplace equation

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = \Delta U = 0 \quad (1.16)$$

The solution of the Laplace equation with specified boundary conditions makes it possible to find the potential U as a function of the coordinates and, therefore, the components of the field intensity. The precise analytic solution of the Laplace equation is possible only in a few simple cases. Consequently, approximate and experimental methods of finding the potential distribution are widely used for solving electron-optical problems (see Section 1.4).

To solve system (1.10), it is necessary to know the distribution of magnetic induction $B = B(x, y, z)$. The magnetic field induction is determined by the vector potential A in accordance with the relationship

$$\mathbf{B} = \text{rot } \mathbf{A} \quad (1.17)$$

The vector potential can be calculated in many cases (see Section 1.4), but in the case of nonsymmetric magnetic fields or in the presence of ferromagnets in the field the analytic calculation is difficult so that experimental methods of studying magnetic fields are to be employed.

1.3. Optico-mechanical Analogy.

Electron-optical Refractive Index

The first papers on electron optics relate to the 20s of our century but its premises were known in the middle of the 19s century. More than a hundred years ago English scientist Hamilton noticed an

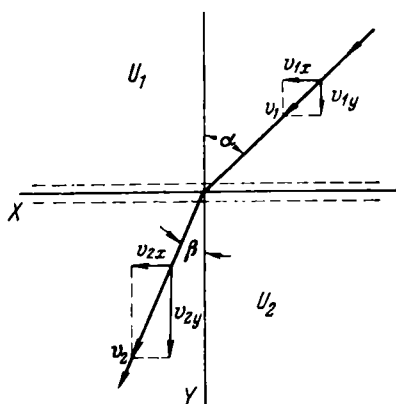


Fig. 1.4. Electron beam refraction

analogy in the propagation of light and the motion of material particles in a force field. This analogy is so significant that when considering the motion of electrons in an electric field in many cases it is convenient to use the equations describing the propagation of light through media with different optical characteristics. Thus,

for example, the optical law of refraction

$$\frac{\sin \alpha}{\sin \beta} = \frac{n_2}{n_1} \quad (1.18)$$

where α and β are the angles formed by the incident and refracted beams with a normal to the boundary of two media having refractive indices n_1 and n_2 also holds for an electron beam passing from the region with a potential U_1 to the region with a potential U_2 .

In fact, when considering the motion of an electron near the interface of two media having different potentials (Fig. 1.4), one can easily see that the velocity component parallel to the interface of the media remains unchanged, while the component perpendicular to this interface varies in magnitude (increases when $U_2 > U_1$).

The equality of the components of velocities v_{1x} and v_{2x} may be written as follows

$$v_1 \sin \alpha = v_2 \sin \beta \quad (1.19)$$

If an electron enters the region U_1 at a zero initial velocity, then according to (1.3)

$$v_1 = \sqrt{\frac{2e}{m} U_1}, \quad v_2 = \sqrt{\frac{2e}{m} U_2} \quad (1.20)$$

Substituting these values of velocities into (1.19) and reducing the charge and mass, we obtain the electron-optical refractive law

$$\frac{\sin \alpha}{\sin \beta} = \frac{\sqrt{U_2}}{\sqrt{U_1}} \quad (1.21)$$

Expression (1.21) coincides with expression (1.18), provided $\sqrt{U}$ plays the role of a refractive index in electron optics.

In a similar way, from the principle of least action known in mechanics and formulated as

$$\int_{t_1}^{t_2} 2W_{\text{kin}} dt = \text{extr} \quad (1.22)$$

while taking into account that $2W_{\text{kin}} dt = mv ds = \sqrt{2emV} ds$, it is possible to obtain an expression similar to the Fermat principle

$$\int_a^b n ds = \int_a^b \sqrt{U} ds = \text{extr} \quad (1.23)$$

i.e. the motion of an electron in an electric field with some potential distribution $U(x, y, z)$ is similar to the propagation of a light ray in media with a refractive index $n = \sqrt{U}$. The difference

between the dimensionalities of n and $\sqrt{U}$ is not important, since during solution of optical problems the equations always include relative refractive indices which are dimensionless values.

The use of the Fermat principle in the electron optics shows that charged particles in electric fields move along the trajectories for which the time spent for displacing a particle between two points is minimum (or maximum), i.e. in the electron optics of all possible trajectories the practical ones are those for which the value of "optical path" $\left(\int n ds \right)$ is extreme.

Thus, considering the equipotential surfaces as refractive surfaces of an optical medium (interfaces between media with different refractive indices), we can find the trajectories of electrons in electric fields by employing the laws of light optics.

A force acting in a magnetic field on an electric charge depends on the magnitude and direction of the velocity of the charged particle. Therefore, in the case of a magnetic field there is no such an analogy to the optics. We may say that from the optical viewpoint a magnetic field is an anisotropic medium as distinct from an isotropic medium (electric field).

The refractive index in a magnetic field is represented by the value

$$n = \frac{e}{m} (As) \quad (1.24)$$

where A is the vector potential and s is the unit vector of a tangent to the trajectory.

Though the analogy between the light and electron optics is sufficiently deep, it is necessary to note some important differences between the propagation of light and the motion of charged particles.

First, the energy of electrons moving in an electric field continuously varies, which is equivalent to the frequency variation of a light ray with its propagation, whereas in a transparent medium the energy of photons of a light ray (therefore, the frequency of oscillations in accordance with the law $W = h\nu$) does not change.

Second, the refractive index in the light optics changes stepwise at the interface between two media with different refractive indices, while in the electron optics the potential and, therefore, the refractive index varies continuously from point to point. In this connection the path of a light ray is usually a broken line consisting of sections of straight lines, while the trajectory of an electron is a smoothly varying curve.

The Laplace equation (1.16) shows that the shape of equipotential surfaces can be definitely determined by giving potential values at some points of space. This shows the third difference between

the light and electron optics. In the light optics the shape of refracting surfaces and the refractive index are not interconnected. In the electron optics the refractive index ($\sqrt{U}$) and the shape of refracting (equipotential) surfaces in most cases cannot be changed independently.

Finally, it should be noted that the refractive index in the electron optics can be varied in a wide range by a simple change in the electrode potential. In the light optics the refractive index of a given optical element is constant and the range of possible values of n is limited (approximately from 1 to 3).

The optico-mechanical analogy has a great influence on the development of the electron optics, since the knowledge of the laws of light optics made it possible to contemplate some ways of designing electron-optical systems. However, the above-mentioned differences in the behaviour of light and electron beams dictate careful application of this analogy.

1.4. Calculations and Experimental Analysis of Electrostatic and Magnetic Fields

As it was shown earlier, analytic calculation of an electric field (solution of the Laplace equation) can be made only in a number of simple cases. However, even when it is possible to obtain an analytic solution, the final expressions are often too awkward and unsuitable for practical application.

As an example, let us present a solution of a Laplace equation for an axisymmetric field created by two cylinders having a radius R

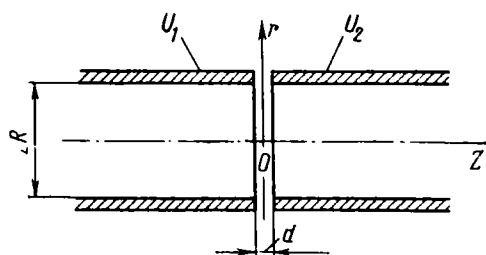


Fig. 1.5 Field calculation of two coaxial cylinders

with a gap $d \ll R$ between the cylinders and with potentials U_1 and U_2 (Fig. 1.5).

In this case the Laplace equation is preferably solved in the cylindrical system of coordinates, using the method of separation of variables. Expressing $U(z, r)$ as the product of two functions, each of which depends only on one variable

$$U(z, r) = N(z) M(r) \quad (1.25)$$

we obtain two equations

$$\left. \begin{aligned} \frac{d^2 N}{dz^2} - k^2 N &= 0 \\ \frac{d^2 M}{dr^2} + \frac{1}{r} \times \frac{dM}{dr} - k^2 M &= 0 \end{aligned} \right\} \quad (1.26)$$

Let us write the solutions of these equations

$$\left. \begin{aligned} N_k(z) &= A_k \sin kz + B_k \cos kz \\ M_k(r) &= C_k I_0(kr) + D_k K_0(kr) \end{aligned} \right\} \quad (1.27)$$

where I_0 is the modified zero-order Bessel function of the first kind; K_0 is the modified zero-order Bessel function of the second kind; A_k , B_k , C_k , D_k are the constant coefficients.

The expression for the potential is found by integrating the product of the obtained solutions through the entire range of change in k

$$U(z, r) = \int_k [A_k(k) \sin kz + B_k(k) \cos kz] \times \\ \times [C_k(k) I_0(kr) + D_k(k) K_0(kr)] dk \quad (1.28)$$

Since the function $K_0(kr)$ becomes infinite with $r = 0$ (on the axis), while the potential cannot be infinitely high, the terms with $K_0(kr)$ should be rejected. The values of $A_k(k)$, $B_k(k)$ and $C_k(k)$ are determined from the boundary conditions. The final expression for the distribution of potential takes the form

$$U(z, r) = \frac{U_1 + U_2}{2} + \frac{U_2 - U_1}{\pi} \int_0^\infty \frac{\sin kz I_0(kr)}{k I_0(kr)} dk \quad (1.29)$$

Sometimes it is sufficient to know only the distribution of potential along the axis (with $r = 0$). In this case, taking the quantity $\xi = z/R$ for an independent variable, we can obtain more simple expression

$$U(\xi) = \frac{U_1 + U_2}{2} + \frac{U_2 - U_1}{\pi} \int_0^\infty \frac{\sin k\xi}{k} \cdot \frac{1}{I_0(k)} dk \quad (1.30)$$

The other term in expression (1.30) is adequately approximated by the hyperbolic tangent. Therefore, the following expression can be used for approximate calculations

$$U(\xi) \approx \frac{U_1 + U_2}{2} + \frac{U_2 - U_1}{\pi} \tanh(1.315\xi) \quad (1.31)$$

The given example shows that even in a comparatively simple case the analytic solution is rather awkward. Therefore, widely used for solving electron-optical problems are approximate methods of calculation and experimental study of electric fields.

Let us consider the method of finite differences based on the replacement of derivatives in the starting equation by small differences.

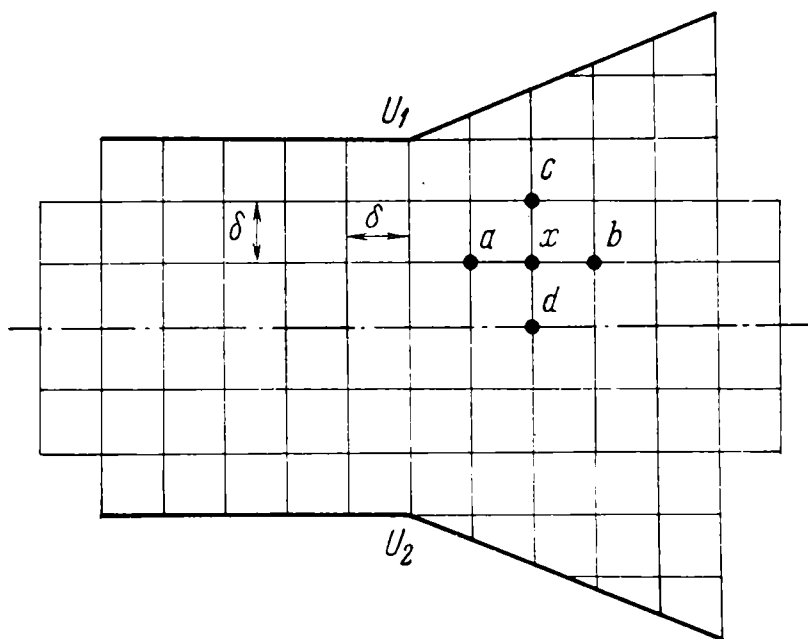


Fig. 1.6. Field calculation by approximation method

Assume that it is necessary to find the potential distribution in a space between two nonparallel plates of a crt deflection system (Fig. 1.6).

Divide the space between the plates into squares with equal sides δ . It is clear that with the potentials U_a , U_b , U_c and U_d known, the value of the potential U_x at the point x equidistant from the points a , b , c and d can be defined as

$$U_x = \frac{U_a + U_b + U_c + U_d}{4} \quad (1.32)$$

Set up the differences

$$\frac{U_a - U_x}{\delta}, \quad \frac{U_x - U_b}{\delta}, \quad \frac{U_c - U_x}{\delta}, \quad \frac{U_x - U_d}{\delta}$$

With adequately low magnitudes of δ we may assume that these differences are approximately equal

$$\left. \frac{\partial U}{\partial x} \right|_{ax}, \quad \left. \frac{\partial U}{\partial x} \right|_{xb}, \quad \left. \frac{\partial U}{\partial y} \right|_{cx}, \quad \left. \frac{\partial U}{\partial y} \right|_{xd}$$

Using repeatedly the same operation, we obtain

$$\left. \begin{aligned} \frac{U_a - U_x}{\delta} - \frac{U_x - U_b}{\delta} &\approx \delta \frac{\partial^2 U}{\partial x^2} \\ \frac{U_c - U_x}{\delta} - \frac{U_x - U_d}{\delta} &\approx \delta \frac{\partial^2 U}{\partial y^2} \end{aligned} \right\} \quad (1.33)$$

Adding equations (1.33) we obtain a Laplace equation in the right-hand part.

$$U_a + U_b + U_c + U_d - 4U_x = \delta^2 \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right) = 0 \quad (1.34)$$

Equation (1.32) stems directly from equation (1.34). The solution of the finite-difference equation can be performed by successive approximation methods (iterative methods).

By assigning boundary values of the electrode potentials, we can tentatively (by eye) plot the values U at the points $a_1, a_2, \dots, b_1, b_2, \dots, c_1, c_2, \dots, d_1, d_2, \dots$ and calculate the potential values at the points $x_1, x_2, \dots$ using the (1.32). After that, taking the found values of U_x as the starting data, we can calculate the potentials at the points a, b, c and d , correct the value of U_x once more, and so on. Using this technique repeatedly, we can gradually approach to the precise values of the potential within the whole area. In many cases the 4th and 5th approximations are slightly different from the preceding one and further correction is unnecessary. None the less, more than ten approximations are sometimes required.

This method can be used for calculation of not only plane but axisymmetric fields as well (see Section 1.5). In the case of an axisymmetric field the value of potential at the points of the plane passing through the system axis ("meridian plane") is determined by the expression

$$U_x = \frac{U_a + U_b + U_c \left(1 - \frac{\delta}{2R}\right) + U_d \left(1 + \frac{\delta}{2R}\right)}{4} \quad (1.35)$$

where R is the distance between the point x and the axis of the system.

After determining the potential values at all the points of the investigated field by the method of successive approximation and connecting the points having equal potentials by continuous curves, we obtain a picture of equipotential lines which characterizes the electron-optical system with sufficient accuracy. The method of successive approximations, although being comparatively simple for obtaining a picture of equipotential lines, becomes time-consuming where it is necessary to change the shape of the electrodes, their mutual position or the potential applied thereto, since in this case all the calculations have to be made anew. Therefore, widely used in the process of designing electron-optical systems is an experi-

mental method of finding the picture of a field on models of a system in an electrolytic tank (bath).

The electrolytic tank method is based on the analogy between an electrostatic field in a vacuum and a current field in a uniformly conductive fluid. The electrostatic field in vacuum is described by the Laplace equation (1.16) with boundary conditions $U = U_1, U_2, \dots, U_n$ on the electrodes. If a model of an electron-optical system with the same potentials $U_1, U_2, \dots, U_n$ on the electrodes is immersed into homogeneous electrically conductive fluid with specific conductivity λ , which does not depend on the coordinates, an electric current flow starts through the fluid. The density of this current is

$$\mathbf{j} = \lambda \mathbf{E} = -\lambda \text{grad } U \quad (1.36)$$

If there is no current sources or leaks in the fluid contained in the interelectrode space, it is obvious that $\text{div } \mathbf{j} = 0$, or

$$\lambda \text{div grad } U = \lambda \Delta U = 0 \quad (1.37)$$

In other words, the current field in an electrolyte is described by the same equation $\Delta U = 0$ (Laplace equation) with the same boundary conditions as the electrostatic field in vacuum. Consequently, the distribution of potentials at all the points of the electrolyte accurately coincides with the distribution of potentials in vacuum. The distribution of potentials in the electrolyte can easily be measured by means of a probe submerged into the electrolyte and connected to a measuring instrument. The measuring instrument should have a very high input resistance (such as a valve voltmeter or an oscilloscope), otherwise the measuring circuit can change the distribution of potentials in the tank. Constant conductivity of the electrolyte is a prerequisite of these measurements. When using a d-c supply to the tank this condition obviously cannot be satisfied due to unavoidable polarization of the electrolyte with resultant change in the ion concentration, and, hence in the conductivity. Therefore, the tank is fed with an alternating current. The frequency of the supply voltage may range from 10 to 1,000 Hz. The lower frequency limit is dictated by the mobility of the ions of the electrolyte. When the frequency is too low, the ions can be displaced to a considerable extent during a half-period with resultant disturbances to the uniformity of conductivity. When the frequency is too high there appear capacitive currents which can distort the field in the tank. For the sake of convenience the tank can be fed with 50 Hz commercial current. The tank can also be fed from a special a-c generator with a frequency of 400-800 Hz.

A diagram of a set-up with an electrolytic tank is shown in Fig. 1.7.

An oscillograph or another sufficiently sensitive a-c instrument can be used as the recorder in the probe circuit. The electrodes are

usually made of iron (technical steel), since oxidation of the iron does not disturb the normal operation of the tank because of high conductivity of iron hydroxide, while the use, for example, of aluminium would result in formation of a non-conducting film of aluminium oxide which can change the distribution of potentials in the tank.

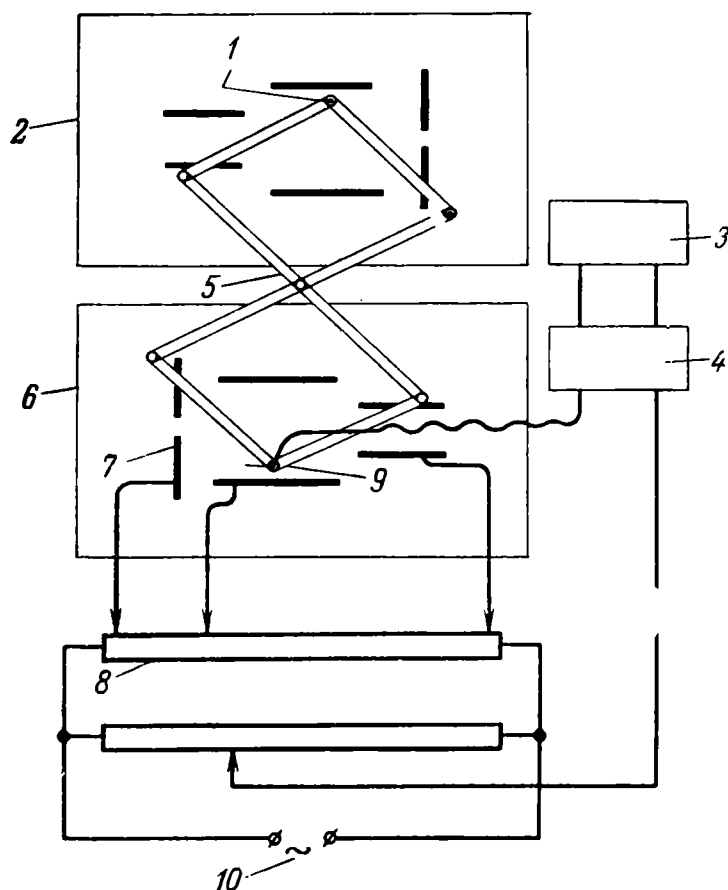


Fig. 1.7. Set-up with electrolytic tank

1—recorder; 2—recording tape; 3—null-indicator; 4—amplifier; 5—pantograph; 8—electrolytic tank; 9—electrode system model; 7—voltage divider; 9—probe; 10—a-c voltage generator

Since equation (1.16) is homogeneous with respect to the potentials, the voltages applied to the electrodes of the model immersed into the tank can be proportionally decreased (or increased) as compared to the potentials of the electrodes of the system being simulated.

As a rule, the supply voltage to the tank is low (1-30 V) for the reasons of safety engineering and in order to avoid perceptible heating of the electrolyte as considerable local overheating can lead to a change in the electrolyte conductivity. Tap water can be used as an

electrolyte because its conductivity. ($\sim 10^{-2}$ S/m) is sufficiently low as compared to that of the electrodes (7×10^6 S/m).

The electrolytic tank is particularly suitable for simulating plane and axisymmetric fields, i.e. the fields in which the potential depends only on two coordinates: $U = U(x, y)$ for a plane field and $U = U(z, r)$ for an axisymmetric field. Such fields have planes of symmetry: $z = \text{const}$ in the plane field and $\psi = \text{const}$ in the axisymmetric field (meridian plane, see Section 1.5). The normal component of intensity of the electric field at any point of these planes is equal to zero. Hence, when the system of the electrodes is cut by a plane of symmetry and the part lying at one side of this plane is rejected, the field of the remaining part of the system does not change. It directly follows from the field intensity component perpendicular to the plane of symmetry being equal to zero that no current will flow through the plane of symmetry when a model of a system of electrodes having such a plane of symmetry is immersed into an electrically conductive fluid. Therefore, during the simulating process we may restrict ourselves to making only one part of the model, lying at one side of the plane of symmetry. During immersing the model into the electrolyte the plane of symmetry must be aligned with the surface of the fluid. Then the other (absent) symmetrical part of the system will be as if reflected in the electrolyte-to-air interface, and the field of the immersed portion of the electrodes will exactly correspond to the field of a complete system.

Such a section method of simulating axisymmetric systems permits use of a thin wedge-shaped layer of electrolyte contained between an inclined non-conducting bottom of a shallow tank and the electrolyte surface (Fig. 1.8).

The possibility of replacement of an axisymmetric system by a "wedge" follows directly from the fact that in an axisymmetric system any plane passing through the axis (meridian plane) is a plane of symmetry. During the modelling any meridian plane can be replaced by a dielectric plane. At the same time making a small part of a model and the use of a shallow tank offer many practical advantages. What is more, in this case the measurement of the potential distribution is effected rather simply by means of a short probe transported over the open surface of the electrolyte.

When simulating plane fields, it is also expedient to use a shallow tank, the electrodes being directly set on the nonconducting bottom of the tank.

Using the above-described electrolytic tank, it is possible to obtain potential values accurate within 1%. Plotting the equipotential lines can be performed automatically, in which case the operation on the tank takes less time. A grid of equipotential surfaces produced by the electrolytic tank method for a system of two cylinders with different potentials is given in Fig. 1.9.

While having material advantages (possibility of simulating almost any electric field, sufficient practical accuracy), the electrolytic tank method possesses some disadvantages, the main of which consists in large overall dimensions of the set-up associated with

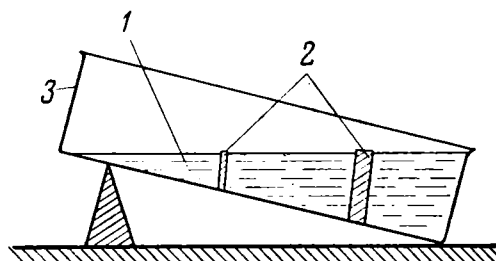


Fig. 1.8. Simulating axisymmetric field in inclined electrolytic tank
1—tank; 2—electrolyte; 3—electrodes

use of great amounts of electrolyte. Therefore, in addition to the electrolytic tank method, some other methods are employed for simulating electric fields.

In simulating plane fields use can be made of the electric field simulating on semiconducting paper, which approximates the electrolytic tank method. Indeed, when simulating a plane (two-dimensional) field in an electrolytic tank, it is possible to use only a thin

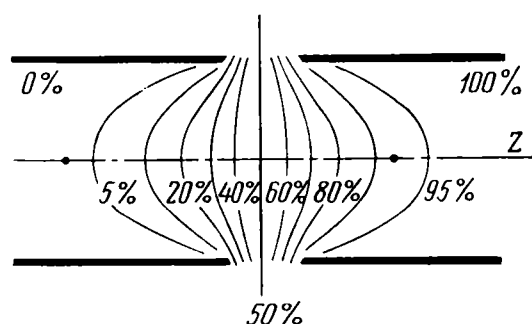


Fig. 1.9. Field of two coaxial cylinders

layer of electrolyte found between the non-conductive bottom of the tank and the surface of the fluid. The potential distribution will apparently not change, if the thin layer of the electrically conductive fluid is replaced by a sheet of an electrically conductive material with electric conductivity much higher than that of the insulating substrate and much lower than that of the metal electrodes.

The electrically conductive sheet is made of special paper containing finely divided carbon (soot or graphite) as a filler covering the paper fibers. Depending on the carbon content the specific resistance of this paper varies within the range of 10^2 to 10^3 Ohm · cm.

When preparing for the simulating a sheet of electrically conductive paper is laid on an insulating plate. The model electrodes are made of sheet metal (copper or brass) and tightly pressed against the paper (or glued to it by electrically conductive adhesive). The electrically conductive adhesive can be prepared of BF-2 glue with addition of soot (approximately 10% by weight). A metallic needle moved over the paper serves as a probe.

In contrast to electrolyte, the electrically-conductive paper features electronic (and not ionic) conduction and there is no polarization in the paper so that it is principally possible to use d-c supply sources. Therefore, the model electrodes are usually fed from a storage battery or a stabilized rectifier. Application of d-c voltage allows highly sensitive d-c measuring instruments to be used as indicators. When measuring the potential at different points of the field simulated on the paper, it is expedient to employ the zero method similar to the electrolytic tank method. In this case a galvanometer with a sensitivity in the order of 10^{-6} A/div serves as a null indicator.

Using high-quality paper and careful conducting of the experiment make it possible to obtain an accuracy sufficient for practical purposes. Thus, for example, when simulating the fields amenable to analytic calculation, the accuracy lies within 1.5-2%.

The simulating of electric fields on the electrically conductive paper is a procedure very simple and convenient. However, the fact that the class of the fields amenable to simulating is limited somewhat restricts the possibilities of this method. Moreover, local heterogeneities of the paper considerably affect the accuracy of simulating.

The simulating of an electric field in an electrolytic tank and on an electrically conductive paper are processes using continuous conducting media. The simulating of electric fields by means of conducting media (solid or liquid) is in fact one of the methods of integration of the Laplace equation which is followed by the potential distribution in an electric field free of space charges. Therefore, any integrator (electrical or mechanical) is suitable for solving the problems of finding a potential distribution in a clear form [for instance, in the form $U = U(x, y, z)$] in a given domain with known boundary conditions.

Electric fields can successfully be analyzed by means of electrical integrators in the form of a grid of resistors. These integrators also serve as devices simulating an electric field, but in contrast to the simulating in an electrolytic tank or on an electrically conductive paper, the continuous conducting media are in this case replaced by a set of discrete elements, i.e. resistors.

The grid of resistors is constructed so that each elementary volume of a continuous conducting medium in the form of a cube or parallelepiped is represented by a structural element of the grid composed

of six resistors whose terminals at one end are connected into a node point (Fig. 1.10).

Assume that an elementary volume of the solid conducting medium with conductivity λ has a form of a parallelepiped with sides

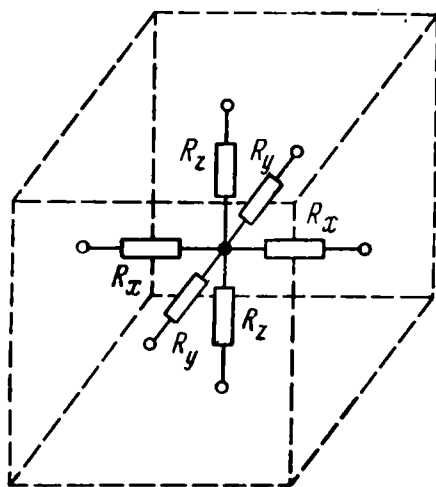


Fig. 1.10. Element of grid of resistors

Δx , Δy , Δz . In this case the conductivities between the opposite sides of the parallelepiped are equal respectively to

$$\Lambda_x = \lambda \frac{\Delta x \Delta y}{\Delta x}, \quad \Lambda_y = \lambda \frac{\Delta x \Delta y}{\Delta y}, \quad \Lambda_z = \lambda \frac{\Delta x \Delta y}{\Delta z} \quad (1.38)$$

Since in the replacement of a continuous conducting medium with resistors each element of the grid of resistances comprises six resistors, the resistance of each of them must be half the resistance between the opposite faces of the elementary parallelepiped, i.e.

$$R_x = \frac{1}{2\Lambda_x}, \quad R_y = \frac{1}{2\Lambda_y}, \quad R_z = \frac{1}{2\Lambda_z} \quad (1.39)$$

Having composed a spatial grid of resistors selected in such a way, we obtain a simulating device equivalent to a continuous medium. When simulating an electric field on a grid of resistors, the node points of the grid corresponding to the electrodes of the system being simulated are interconnected and the voltage of the given electrode is applied to them. The potential distribution is determined by potential values at the node points of the grid located in the interelectrode space. The measurements are usually carried out by the zero method similar to the electrolytic tank method.

There is no need to use a three-dimensional grid of resistors when investigating plane and axisymmetric fields. In these cases a two-dimensional grid is usually employed (Fig. 1.11), which is very

simple and convenient in operation and serves as an analog of a shallow electrolytic bath with a thin layer of electrolyte.

When simulating plane fields, the resistance of all resistors of the grid must be the same. When simulating axisymmetric fields, the resistance of the resistors must vary similarly to the variation of the resistance of the layer of electrolyte in a tank with an inclined bottom. It is obvious that the values of resistances must be inversely proportional to the distance from the straight line taken as the axis of the system being simulated.

As compared to the electrolytic tank, the grid of resistors has certain advantages, such as the absence of electrolyte, the possibility to use direct current and, therefore, more sensitive and accurate measuring instruments, convenience of measurements directly

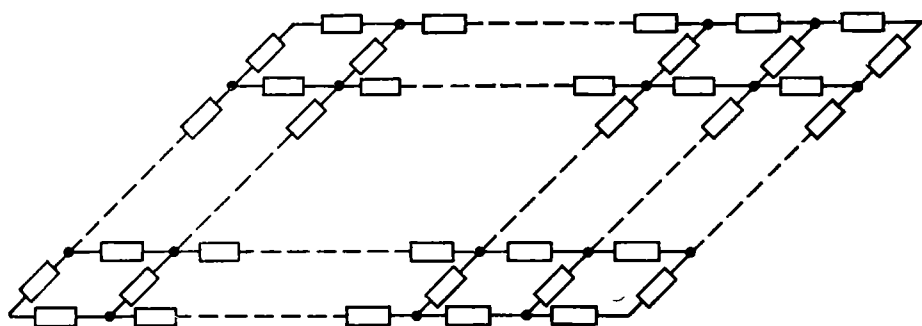


Fig. 1.11. Two-dimensional grid of resistors

at the nodes of the grid. It is clear that the accuracy of this method increases with the number of elements of the grid. It should be noted, however, that the resistors constituting the grid have resistance allowances and an excessive number of resistors may reduce the accuracy of measurements. Furthermore, the higher the number of resistors, the more laborious is the process of potential distribution measurements. A grid consisting of few thousand resistors can practically be used for determining potential distribution with an accuracy of 0.01%.

Still greater opportunities are offered by electronic computers which can be used for calculating any electric fields with very high accuracy.

When solving the problems of electron optics, use is sometimes made of an illustrative but very rough method of simulating an electric field with the aid of an elastic diaphragm. The simulating of the field by this method is based on the fact that the equation describing the shape of the surface of the elastic diaphragm coincides under certain conditions with the Laplace equation and, therefore, the amount of deformation of the diaphragm makes it possible to evaluate the potential distribution in the system being investigated.

As is known, the surface area of a deformed elastic diaphragm is defined by the double integral

$$S = \iint \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} dx dy \quad (1.40)$$

where z is the amount of deformation, i.e. deviation of the diaphragm surface from the position of equilibrium.

Since the area of the surface of the deformed elastic diaphragm is minimal, we have an integral equation $\delta S = 0$, which is reduced to the Euler differential equations. The equation of the deformed diaphragm surface is written as follows

$$\left[1 + \left(\frac{\partial z}{\partial x}\right)^2\right] \times \frac{\partial^2 z}{\partial y^2} + \left[1 + \left(\frac{\partial z}{\partial y}\right)^2\right] \times \frac{\partial^2 z}{\partial x^2} + 2 \frac{\partial^2 z}{\partial x \partial y} \times \frac{\partial z}{\partial x} \times \frac{\partial z}{\partial y} = 0 \quad (1.41)$$

where the derivatives $\partial z/\partial x$ and $\partial z/\partial y$ determine the angle of inclination of the deformed diaphragm surface. Restricting ourselves to small angles of inclination [$(\partial z/\partial x) \ll 1$ and $(\partial z/\partial y) \ll 1$], the square of the first derivatives compared to unity can be neglected and equation (1.41) transforms into the Laplace equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0 \quad (1.42)$$

The simulating of a field by means of an elastic diaphragm is practically carried out as follows. A rubber sheet is stretched over a rigid frame. The frame is installed horizontally. The uniformity of the rubber tension is checked by means of net of identical squares drawn on the sheet before the stretching. The sheet is tensioned so that the net of squares is not distorted, i.e. the squares maintain their geometrical form. The tension should be adequate so that the rubber would not sag under gravity. Then, installed under the rubber (or above it) are pressing members deforming the rubber in conformity with the disposition and potentials of the electrodes of the system being simulated. In this case the scale factor is selected so that the angle of inclination at the places of the greatest potential variations does not exceed 10-15°. After that a mechanical indicator is used for measuring the amount of deformation z and the formula

$$U = kz \quad (1.43)$$

where k is the selected scale factor, is employed to calculate the value of potential $U(x, y)$.

The experience of using the rubber diaphragm shows that careful experiments permit obtaining potential measurements with an accuracy of 2-3% which is sufficient in many practical cases.

The magnetic fields employed in electron-beam devices are most often produced by short circular coils or long solenoids in which an electric current flows. In these cases the calculation of the magnetic field can be performed in rather a simple manner by using the Biot-Savart law. According to the Biot-Savart law, the magnetic induction at the axis of a single plane circular turn having a radius R and passing a current I is determined as follows

$$B(z) = \frac{\mu_0 R I}{2(R^2 + z^2)^{3/2}} \quad (1.44)$$

where z is counted off from the turn plane.

If in (1.44) R and z are taken in metres and I in amperes, the induction B is read in teslas. If R and z are in centimetres, the induction B is read in gaussses which is usually convenient in practical computations.

Expression (1.44) with fairly good approximation can be used for magnetic induction calculation at the axis of a short coil having n turns with a mean radius R_m .

$$B(z) = \frac{\mu_0 R_m n I}{2(R_m^2 + z^2)^{3/2}} \quad (1.45)$$

Produced inside a long solenoid having n turns uniformly arranged over the entire length L is a homogeneous magnetic field whose induction is

$$B = \frac{\mu_0 n I}{L} \quad (1.46)$$

The given formulas make it possible to calculate magnetic fields produced by axisymmetric (circular) coils having no ferromagnetics. If the coils have no axial symmetry and are provided with ferromagnetics (these are often employed for making magnetic optical systems), the field can approximately be calculated by empirical formulas and the magnetic induction can be determined experimentally. Various methods of experimental study of magnetic fields are employed in the electron optics. The method of a ballistic galvanometer based on the Hall effect has gained wide recognition the last years. A diagram of the apparatus with a ballistic galvanometer is illustrated in Fig. 1.12.

A small measuring coil MC is placed into a magnetic field, the plane of the coil being perpendicular to the lines of force of the field. The coil terminals are connected to a ballistic galvanometer BG . If the coil has n turns and its cross section is equal to S , the magnetic flux of the coil

$$\Psi = nSB \quad (1.47)$$

By switching on or reversing the direction of the current in the solenoid winding, the magnetic flux is varied by nSB or $2nSB$. As

this happens an electromotive force is induced in the measuring coil. The result is that a current pulse flows through the ballistic galvanometer. As is known the quantity of electricity flowing in the coil is proportional to the variation in the magnetic flux produced by this coil, while the deflection of the galvanometer pointer is, in its turn, proportional to the quantity of electricity flowing in the coil.

Thus, knowing the geometrical dimensions, the number of turns and the galvanometer constant, we can calibrate the dial of the measuring instrument directly in units of magnetic induction. When employing sufficiently small measuring coils (described are coils 0.4 mm

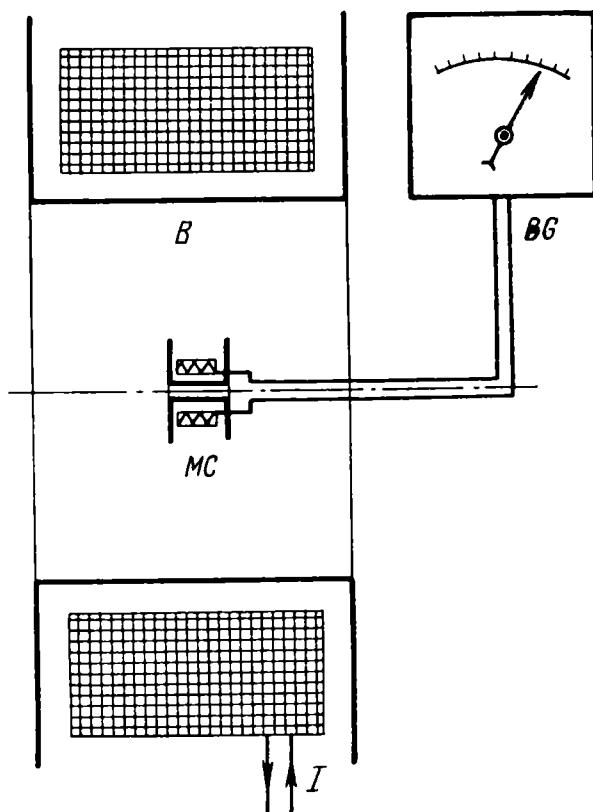


Fig. 1.12. System with ballistic galvanometer

in diameter and 0.3 mm long containing 100 turns of copper wire 0.025 mm in diameter), we can determine magnetic induction with an accuracy of 1%. The disadvantages of the ballistic galvanometer method are high consumption of labour (impossibility of continuous measurement with displacement of the measuring coil) and difficulties associated with the necessity of orientation of the plane of the turns of the measuring coil normal to the lines of force of the field whose direction is not known beforehand.

Continuous measurement of magnetic induction can be made by the method of a rotating coil moving in a magnetic field. A small

The condition of axial symmetry in cylindrical coordinates can be written as follows

$$U(z, r, \psi) \equiv U(z, r, 0) \quad (1.50)$$

i.e. the values z and r uniquely define the value U regardless of the angle of turn [$U = U(z, r)$].

Usually, the potential distribution along the axis of a system ($r = 0$) can be calculated and measured most simply. Therefore,

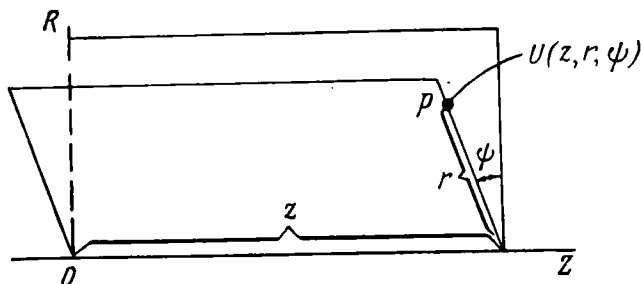


Fig. 1.14. Cylindrical coordinates

it is convenient to represent the potential distribution at some area near the system axis through the value of the known potential on the axis itself.

In a field free from a space charge the potential satisfies the Laplace equation which in the cylindrical coordinate system is written as follows

$$\frac{\partial^2 U}{\partial z^2} + \frac{1}{r} \times \frac{\partial U}{\partial r} + \frac{\partial^2 U}{\partial r^2} + \frac{1}{r^2} \times \frac{\partial^2 U}{\partial \psi^2} = 0 \quad (1.51)$$

In an axisymmetric field [$U = U(r, z) \neq f(\psi)$] the last term of the equation (1.51) vanishes and the equation assumes the form

$$\frac{\partial^2 U}{\partial z^2} + \frac{1}{r} \times \frac{\partial U}{\partial r} + \frac{\partial^2 U}{\partial r^2} = 0 \quad (1.52)$$

Let us find the solution of the equation (1.52) in the form of a series by the powers of r

$$U(z, r) = U_0(z) + U_2(z) r^2 + U_4(z) r^4 + \dots \quad (1.52a)$$

The series contains only even powers of r because of the following reasons. When the space is cut by a plane passing through the axis, the terms z and r on this plane may be considered as the Cartesian coordinates (Fig. 1.15).

Two points a and b symmetric with respect to the axis OZ are characterized by the same potential, since one of them transforms into another during the rotation of the plane through an angle $\psi = \pi$. At the same time, the point b differs from the point a only in the

sign of the coordinate r which will not change the potential value only when all the powers of r are even.

The first term of the series $U_0(z)$ determines the potential distribution along the axis of the system ($r = 0$). Let series (1.52a) be differentiated twice with respect to z , one time and then twice with respect to r and substituted into equation (1.52) combining the terms with the same powers of r

$$U_0''(z) + 4U_2(z) + [U_2''(z) + 16U_4(z)]r^2 + [U_4''(z) + 36U_6(z)]r^4 + \dots = 0$$

(here the two primes stand for differentiation with respect to z).

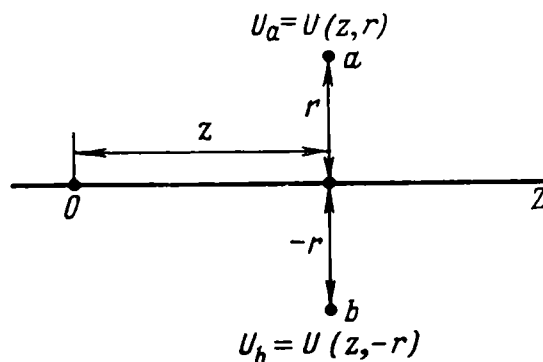


Fig. 1.15. Meridian plane turn

Since this equation is valid for any value of r , it is obvious that the coefficients must be equal to zero at any power of r

$$U_0''(z) + 4U_2(z) = 0, \quad U_2''(z) + 16U_4(z) = 0$$

from which

$$U_2(z) = -\frac{1}{4} U_0''(z), \quad U_4(z) = \frac{1}{64} U_0^{IV}(z)$$

or generally

$$U_{2k} = (-1)^k \frac{U_0^{(2k)}(z)}{2^{2k} (k!)^2}$$

Substituting the values of the coefficients for r^k into the series, we obtain an expression for the potential of an axisymmetric electric field

$$U(z, r) = U_0(z) - \frac{1}{4} U_0''(z) r^2 + \frac{1}{64} U_0^{IV}(z) r^4 - \dots$$

or generally

$$U(z, r) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(k!)^2} U_0^{(2k)}(z) \left(\frac{r}{2}\right)^{2k} \quad (1.53)$$

The last expression allows an axisymmetric electric field to be calculated, provided the potential distribution along the axis $U_0(z)$ is known.

Narrow near-axial electron pencils are often used in cathode-ray tubes. In this case it is no longer necessary to investigate the field at a distance from the system axis, since only the near-axial (paraxial) region of the field affects the formation of the electron beam. Within the scope of paraxial optics, instead of complete decomposition of potential (1.53) we may restrict ourselves to the first two terms of the series with an adequate accuracy, i.e. to discuss the potential distribution near the system axis

$$U(z, r) \approx U_0(z) - \frac{1}{4} U_0''(z) r^2 \quad (1.54)$$

The axisymmetric field in the near-axial region has some interesting features. Let expression (1.54) be differentiated with respect to r

$$\frac{\partial U}{\partial r} = -\frac{1}{2} U_0''(z) r \quad (1.55)$$

From (1.55) it follows that the radial component of the electric field intensity, $E_r = -dU/dr = 1/2 U_0''(z) r$ is directly proportional to r , i.e. rises linearly with distance from the axis. At the same time, on the axis itself, ($r = 0$) the radial component of the intensity vanishes, while the second derivative $U(z, r)$ by r

$$\frac{\partial^2 U}{\partial r^2} = -\frac{1}{2} U_0''(z) \quad (1.56)$$

is not equal to zero on the axis.

Select a certain point with a coordinate $z = z_0$ on the axis and expand the term $U_0(z)$ in the Taylor series near this point:

$$U_0(z) = U_0(z_0 + \Delta z) = U_0(z_0) + U_0'(z_0) \Delta z + \frac{1}{2} U_0''(z_0) (\Delta z)^2 + \dots$$

At a small distance from the axis (in the paraxial area) $U(z, r)$ near the point z_0 may be presented in the form

$$U(z, r) = U_0(z_0) + U_0'(z_0) \Delta z + \frac{1}{2} U_0''(z_0) (\Delta z)^2 - \frac{1}{4} U_0''(z_0) r^2 \quad (1.57)$$

Let us consider the equipotential surface intersecting the axis at the point z_0 . Along this surface $U(z, r) = U_0(z_0) = \text{const.}$ From equation (1.57) we then obtain an equation of the equipotential surface passing through point z_0 .

$$\frac{1}{4} U_0''(z_0) r^2 = \frac{1}{2} U_0''(z_0) (\Delta z)^2 + U_0'(z_0) \Delta z \quad (1.58)$$

Equation (1.58) is a hyperbola equation from which it directly follows that the equipotential surfaces near the axis of an axisymmetric field are hyperboloids of revolution. Thus, any electric field having axial symmetry is hyperbolic near the axis.

Determine the radius of curvature of the equipotential line in the cross section area of the equipotential surface cut by a meridian plane at the hyperbola apex (on the axis). If the equipotential line is presented by an equation $r = r(z)$, then the radius of curvature can be found by the known formula

$$R = \frac{\left[1 + \left(\frac{dr}{dz}\right)^2\right]^{3/2}}{\frac{d^2r}{dz^2}} \quad (1.59)$$

Since the potential is constant along the equipotential line: $U(z, r) = U_0(z) = \text{const}$ and differentiating $U(z, r)$ with respect to z , we obtain

$$\frac{\partial U}{\partial z} + \frac{\partial U}{\partial r} \frac{dr}{dz} = 0$$

from which

$$\frac{dr}{dz} = -\frac{\frac{\partial U}{\partial z}}{\frac{\partial U}{\partial r}} \quad (1.60)$$

The second differentiation (1.59) with respect to z leads to the following expression

$$\frac{d^2r}{dz^2} = \frac{1}{\left(\frac{\partial U}{\partial r}\right)^3} \left[\frac{\partial^2 U}{\partial z^2} \left(\frac{\partial U}{\partial r}\right)^2 - 2 \frac{\partial^2 U}{\partial r \partial z} \times \frac{\partial U}{\partial z} \times \frac{\partial U}{\partial r} + \frac{\partial^2 U}{\partial r^2} \left(\frac{\partial U}{\partial z}\right)^2 \right] \quad (1.61)$$

On the axis, at the point $z = z_0$, $r = 0$ and

$$\frac{\partial U}{\partial r} = 0, \quad \frac{\partial U}{\partial z} = U'_0(z_0) \quad (1.62)$$

$$\frac{\partial^2 U}{\partial r^2} = -\frac{1}{2} U''_0(z_0), \quad \frac{\partial^2 U}{\partial r \partial z} = 0, \quad \frac{\partial^2 U}{\partial z^2} = U''_0(z_0)$$

Substituting the obtained expressions for dr/dz and d^2r/dz^2 with allowance for (1.62) into the equation (1.59), we get the radius of curvature of the equipotential line on the axis

$$R(z_0) = -\frac{2U'_0(z_0)}{U''_0(z_0)} \quad (1.63)$$

Thus, the radius of curvature of the equipotential surface near the axis is uniquely defined by the axial potential distribution $U_0(z)$. From this it follows directly that when using the fields free of a space charge in the electron optics, it is in principle impossible to vary the refractive index (potential distribution) and the shape of refracting (equipotential) surfaces. This constitutes a significant difference between the light and electron optics (see Section 1.3).

When analyzing the fields having axial symmetry the field intensity is often equal to zero at a certain point. As an example, we may point out to the field of a circular diaphragm positioned between two flat electrodes having the same potential which differs from the potential of the diaphragm (Fig. 1.16).

It is clear that the potential rises with the distance from the specific point (z_0) in either direction and drops down with the distance in

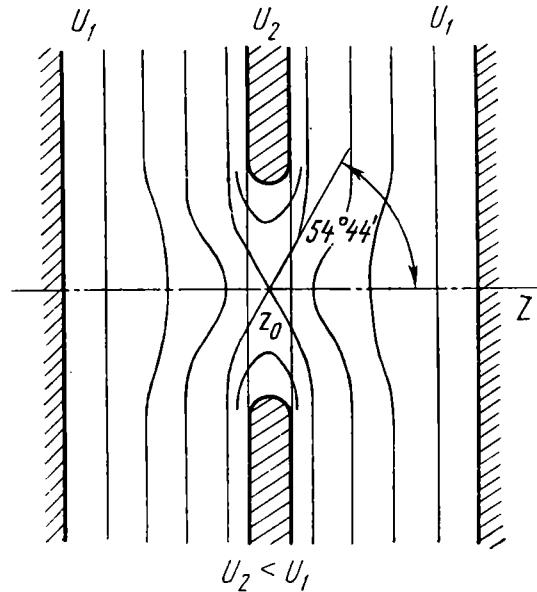


Fig. 1.16. Plane diaphragm field

the radial direction. This specific point is usually called a “*saddle point*” of the field.

At the saddle point the direction of the vector of the field intensity becomes indefinite. Therefore, the derivatives $\partial V/\partial z$ and $\partial V/\partial r$ at this point vanish. Then from expansion (1.58), which is valid for the saddle point as well, it directly follows that $r^2 = \frac{1}{2} (\Delta z)^2$, since $U''_0(z_0) \neq 0$.

Thus, at the saddle point the hyperbolic equipotential lines degenerate into two intersecting straight lines serving as asymptotes of hyperbolas. The equation of these straight lines has the following form

$$r = \pm \sqrt{2} \Delta z \quad (1.64)$$

Consequently, the equipotential surfaces, which are generally hyperboloids of revolutions near the axis of the axisymmetric field, transform at the saddle (specific) point into a cone with an apex angle equal to $2 \arctan \sqrt{2} = 109^\circ 28'$ regardless of the potential distri-

bution at a distance from the specific point. This particularity is often used as a criterion for estimation of the accuracy of approximate calculation or experimental finding of the equipotential surfaces in fields having saddle points. The noticeable difference of the angle of inclination of the equipotential line passing through the saddle point from the value $\arctan \sqrt{2} \approx 55^\circ$ indicates an error in the calculations or during the experiments.

Magnetic fields having axial symmetry are conveniently presented in the form of three components in cylindrical coordinates. Since $\mathbf{B} = \text{rot } \mathbf{A}$, then

$$\left. \begin{aligned} B_z &= (\text{rot } \mathbf{A})_z = \frac{1}{r} \left[\frac{\partial (rA_\psi)}{\partial r} - \frac{\partial A_r}{\partial \psi} \right] \\ B_r &= (\text{rot } \mathbf{A})_r = \frac{1}{r} \left[\frac{\partial A_z}{\partial \psi} - \frac{\partial (rA_\psi)}{\partial z} \right] \\ B_\psi &= (\text{rot } \mathbf{A})_\psi = \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \end{aligned} \right\} \quad (1.65)$$

Due to the axial symmetry $B_\psi = 0$, i.e. $\mathbf{B}$ and $\mathbf{A}$ do not depend on ψ . Thus system (1.65) transforms into the equations

$$\left. \begin{aligned} B_z &= \frac{1}{r} \times \frac{\partial (rA_\psi)}{\partial r} \\ B_r &= -\frac{\partial A_\psi}{\partial z} \\ B_\psi &= 0 \end{aligned} \right\} \quad (1.66)$$

From the last equation it follows that in an axisymmetric field

$$\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} = 0 \quad (1.67)$$

Without disturbing the generality, we may assume that in an axisymmetric field $A_r = A_z = 0$, i.e. the vector potential has only one (azimuth) component $A_\psi \neq 0$. This conclusion also follows from the Biot — Savart law written in the vector form, since the current producing the axisymmetric field flows through a circular turn, i.e. has an azimuthal component only.

In a space free of currents $\text{rot } \mathbf{B} = 0$ and, hence

$$(\text{rot } \mathbf{B})_\psi = \frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r} = 0 \quad (1.68)$$

Substituting the expression B_z and B_r into (1.68) from (1.66), we obtain an equation for the vector potential similar to the Laplace equation for the electric potential

$$\frac{\partial^2 A_\psi}{\partial z^2} + \frac{\partial^2 A_\psi}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial A_\psi}{\partial r} - \frac{A_\psi}{r^2} = 0 \quad (1.69)$$

For an axisymmetric field the induction B is an even function of r and, consequently, the vector potential A (connected with the value B through the operation of differentiation) must be an odd function of r . Therefore, expand the vector potential in a series by the odd powers of r (as in the axisymmetric field A has only one component, it is sufficient to consider the expansion only for A_ψ)

$$A_\psi(z, r) = f_1(z)r + f_3(z)r^3 + f_5(z)r^5 + \dots \quad (1.70)$$

Let series (1.70) be differentiated two times with respect to z and one time and then two times with respect to r and be substituted into the equation (1.69) combining the terms with similar powers of r

$$[f_1''(z) + 8f_3(z)]r + [f_3''(z) + 24f_5(z)]r^3 + \dots = 0 \quad (1.71)$$

This equation must be valid for any r , what is possible when the coefficients for r , r^3 , r^5 ... vanish. By equating these coefficients to zero, we can determine the functions f_1 , f_3 , f_5 etc.

$$\left. \begin{aligned} f_3(z) &= \frac{f_1''(z)}{8} \\ f_5(z) &= \frac{f_3''(z)}{24} \\ &\dots \dots \dots \end{aligned} \right\} \quad (1.72)$$

To determine $f_1(z)$, substitute $A_\psi(z, r)$ from (1.70) into the first equation (1.66) and assume that $r = 0$. Then

$$B_z(z, 0) = 2f_1(z)$$

from which

$$f_1(z) = \frac{B_z(z, 0)}{2} = \frac{1}{2} B_0(z) \quad (1.73)$$

i.e. the value of f_1 is half the value of the magnetic induction on the field axis.

Substituting the values of f_1 , f_3 , ... into series (1.70), we find the expression for the vector potential of the axisymmetric field

$$A_\psi(z, r) = \frac{1}{2} B_0(z)r - \frac{1}{16} B_0''(z)r^3 + \frac{1}{384} B_0^{IV}(z)r^5 - \dots \quad (1.74)$$

or

$$A_\psi(z, r) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(k+1)} B_0^{(2k)}(z) \left(\frac{r}{2}\right)^{2k+1} \quad (1.75)$$

The obtained equations allow the vector potential to be calculated at any point of an axisymmetric field provided that the magnetic induction on the axis is known. As mentioned in Section 1.4, determination of the value of magnetic induction is a simple operation when dealing with systems having no ferromagnetic elements.

For circular coils with a screen (ferromagnetic envelope) frequently used in practice the distribution of magnetic induction along the coil axis is well approximated by the expression

$$B_0(z) = B_{\max} e^{-\frac{z^2}{1.44a^2}} \quad (1.76)$$

where $B_{\max}$ is the maximum value of the magnetic induction, while a is half the "semiwidth" of the field (Fig. 1.17).

Formula (1.76) is satisfactory only when the ferromagnetic core is not saturated. In the case of saturation the field on the axis of the screened coil can be calculated approximately by using the formula

$$B_0(z) = \frac{B_{\max}}{1 + \left(\frac{z}{a}\right)^2} \quad (1.77)$$

or be determined experimentally by any of the above-described methods.

1.6. Electron Trajectories in Axisymmetric Fields

Compose an equation of motion of an electron in an axisymmetric electric field. As before, make use of cylindrical coordinates. Assume that the electron velocity is much less than the velocity of light, i.e. the relativistic mass variation be neglected. Assume that the electron starts moving at a zero initial velocity from the point with a potential $U = 0$ near the axis of the system. Since the axisymmetric field has no azimuthal component, the electron starting to move in a certain plane passing through the axis (meridian plane) will further move along a trajectory lying in this plane, i.e. the trajectory will be a plane curve. Owing to the axial symmetry it is sufficient to study the motion of electrons in one of the meridian planes, while all the other trajectories are obtained from rotation of this plane about the axis.

First consider the near-axial symmetry equation, which is more convenient and simple for analysis. Remaining within the framework of paraxial optics, we may approximately consider that the electron velocity at any point of this area is determined according to (1.3) by the expression

$$v \approx v_z = \frac{dz}{dt} = \sqrt{\frac{2e}{m}} \sqrt{U_0(z)} \quad (1.78)$$

where $U_0(z)$ is the value of the potential on the axis of the system.

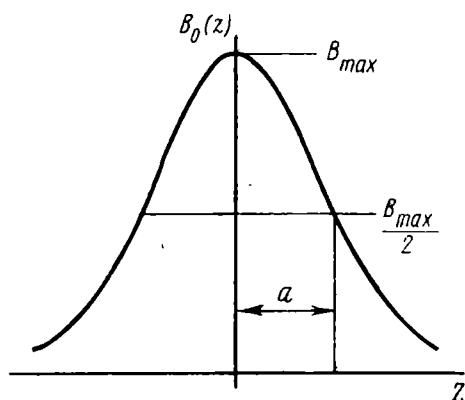


Fig. 1.17. Bell-shaped magnetic field

In the region of a nonuniform axisymmetric field the electron is acted on by the radial force

$$F_r = -eE_r \quad (1.79)$$

The radial component of electric field intensity E_r in the near-axial area is determined by expression (1.55). In this case the equation of motion in the radial direction is written in the form

$$m \frac{d^2 r}{dt^2} = -\frac{1}{2} e U_0''(z) r \quad (1.80)$$

To obtain a trajectory equation of the form $r = r(z)$, it is necessary to exclude t from (1.80). For this purpose use will be made of a differential operator

$$\frac{d}{dt} = \frac{dz}{dt} \times \frac{d}{dz}$$

Applying this operator to (1.80) two times and using (1.78), we get

$$\frac{d}{dz} \left(\sqrt{U_0(z)} \frac{dr}{dz} \right) = \frac{1}{4 \sqrt{U_0(z)}} U_0''(z) r \quad (1.81)$$

or after the differentiation and minor transformations

$$\frac{d^2 r}{dz^2} + \frac{U_0'(z)}{2U_0(z)} \times \frac{dr}{dz} + \frac{U_0''(z)}{4U_0(z)} r = 0 \quad (1.82)$$

Equation (1.82) is fundamental for the near-axial electron optics. The analysis of this second-order homogeneous differential equation allows us to arrive at the following important conclusions:

(a) The equation does not include the charge and mass of the electron. Therefore, the paths of any charged particles (electrons, positive and negative ions) moving in an electrostatic field with non-relativistic velocities coincide. The difference will be only in their transit time [see equation (1.80)];

(b) Since these paths lie in a single plane and do not depend on the charge and mass, they are reversible, i.e. when a charged particle is sent in the reverse direction (at a velocity determined by the starting point potential), it will follow the same path as during the forward flight through the given area of the field;

(c) The equation is homogeneous with respect to the potential. Therefore, an equal increase (or decrease) in the U at all the points of the field does not change the trajectory. This indicates that the supply voltage may be unstabilized when using electrostatic lenses, provided that all the elements of the electron-optical system are connected to a common supply source. From the homogeneity of the equation with respect to U a possibility follows that to analyze electron-optical systems on models with proportionally decreased or increased voltages, which is particularly important for simulating in

the electrolytic bath where, as shown above, the supply voltage may not exceed a few dozens of volts;

(d) The equation is homogeneous with respect to r . This makes it possible to analyze the trajectories on models made to a larger or smaller scale, in which case the trajectories remain geometrically similar. The homogeneity with respect to r is widely used when simulating trajectories by means of trajectory tracers and also in graphoanalytic construction of trajectories, since greatly enlarged models enable the accuracy of the construction to be materially improved.

Finally, the analysis of equation (1.82) shows that the resulting effect of a nonhomogeneous axisymmetric electric field on a pencil of charged particles (in the paraxial region) is similar to the action of an optical lens on a light pencil passing through it. Let us substantiate this fundamental assumption.

The general solution of the equation (1.82) $r = r(z)$ can be presented in the form of a sum of two particular solutions

$$r(z) = c_1 r_1(z) + c_2 r_2(z) \quad (1.83)$$

where c_1 and c_2 are the constants determined by the initial conditions; $r_1(z)$ and $r_2(z)$ are two particular linearly independent solutions.

Let us take particular solutions $r_1(z)$ and $r_2(z)$ so that $r_1(z_a) = 0$, $r_1(z_b) = 0$ (here z_a and z_b are the coordinates of the planes of the object and its image) and $r_2(z_a) = 1$. The second particular solution is undoubtedly possible. The first particular solution, which vanishes twice, is possible, if the second derivative of the potential with respect to z is positive in the nonhomogeneous region of the field ($U''_0 \neq 0$). Indeed, consideration of the expression (1.80) shows that the sign of the second derivative $U''_0(z)$ defines the direction of the force acting on the charged particle. When $U''_0(z) > 0$ the force is directed towards the axis and, therefore, the trajectory, once having intersected the axis, will approach the axis again under the effect of the radial force and intersect this axis at another point. Thus the first particular solution is possible with $U''_0 > 0$, i.e. in the case of *converging* lenses which are of a great interest for the electron-beam optics.

With the above selection of the particular solutions the general solution $r(z)$ uniquely determines the position of the points of the image (in the plane z_b) by the position of the points of the object (in the plane z_a).

In the plane of the object $r(z_a) = c_2$, i.e. c_2 is none other than the initial distance between the electron and the axis. In the plane of the image $r(z_b) = c_2 r_2(z_b)$, i.e. all the particles coming from the point of the object at the distance c_2 from the axis will again gather to a single point in the plane of the image. This result was obtained without imposing any conditions on the direction of the velocities

of particles (it had been only assumed that the trajectories do not come beyond the paraxial region).

The image scale (linear amplification) is

$$M = \frac{c_2 r_2(z_b)}{c_2} = r_2(z_b)$$

i.e. it is constant and independent of the initial conditions.

The given analysis shows that any nonhomogeneous axisymmetric electric field in the paraxial region is an electron lens. This important conclusion can also be drawn on the basis of the fact that any axisymmetric electric field in the near-axial region is hyperbolic [see (1.58)]. As is known in the light optics a refractory surface in the form of a hyperboloid of revolution has the properties of a lens.

The assumption $U''_0(z) > 0$ used during the analysis of equation (1.82) does not reduce the generality of the consideration, since the field is a *dispersing* lens in the case $U''_0(z) < 0$. The fields having properties of *dispersing* lenses are employed comparatively rarely, as they cannot create *real* images that can be seen on a luminescent screen or fixed on a photographic paper.

In complex optical systems certain regions of nonhomogeneous axisymmetric electric fields can, of course, serve as dispersing lenses, but, as it will be shown below, in many practically interesting cases the collecting action of the field is predominant and the presence of dispersing regions only reduces the total (positive) optical power of the system.

We considered the equation of motion of low-velocity electrons ($v \ll c$). If we deal with high-speed electrons (the electron velocity v is commensurable with the velocity of light), then the equation of motion is derived, proceeding from the general expression

$$\frac{d}{dt}(mv) = -eE \quad (1.84)$$

and m and v are determined on the basis of the law of conservation of energy written in the relativistic form [see (1.4)]. If that is the case, the equation of the path of an electron moving at any velocity (up to the velocity of light) assumes the form

$$\frac{d^2 r}{dz^2} + \frac{U'_0}{2U_0} \times \frac{1 + \frac{eU_0}{m_0 c^2}}{1 + \frac{eU_0}{2m_0 c^2}} \times \frac{dr}{dz} + \frac{U''_0}{4U_0} \times \frac{1 + \frac{eU_0}{m_0 c^2}}{1 + \frac{eU_0}{2m_0 c^2}} r = 0 \quad (1.85)$$

Comparing this equation with (1.82), it is readily seen that with $v \ll c$ ($eU_0 \ll m_0 c^2$) equation (1.85) transforms into (1.82), and at the relativistic velocities there appear additional multipliers in the fundamental equation

$$\left(1 + \frac{eU_0}{m_0 c^2}\right) / \left(1 + \frac{eU_0}{2m_0 c^2}\right)$$

and the equation becomes nonhomogeneous with respect to U_0 . However, the homogeneity with respect to r is still preserved, and, therefore, all the conclusions regarding the focusing and formation of an image are valid in the relativistic case.

It is of interest to note that in the limiting relativistic case, when $v \approx c$ ($eU_0 \gg m_0c^2$), the fundamental equation again becomes homogeneous with respect to U_0 , and varied are only the numerical coefficients at the second and third terms

$$\frac{d^2r}{dz^2} + \frac{U'_0}{U_0} \times \frac{dr}{dz} + \frac{U''_0}{2U_0} r = 0 \quad (1.86)$$

This can be physically explained by the fact that at low velocities an increase in the energy of the particle in an accelerating field takes place only at the expense of an increase in the velocity. In the limiting relativistic case it occurs only at the expense of an increase of the mass, whereas in the intermediate area the energy varies solely due to the variation of the velocity and mass.

Now let us consider the motion of an electron in a nonhomogeneous axisymmetric magnetic field. Let the equation of motion of a charged particle in a magnetic field be written in the general form

$$ma = -e[vB] \quad (1.87)$$

In order to present this equation in the cylindrical system of coordinates, let us use the known relation for the transfer from the Cartesian coordinates to the cylindrical ones

$$\left. \begin{aligned} x &= r \cos \psi \\ y &= r \sin \psi \\ z &= z \end{aligned} \right\} \quad (1.88)$$

The force components are respectively represented in the cylindrical coordinates by the following expressions

$$\left. \begin{aligned} F_z &= m\ddot{z} \\ F_r &= m\ddot{x} \cos \psi + m\ddot{y} \sin \psi \\ F_\psi &= m\ddot{x} \sin \psi + m\ddot{y} \cos \psi \end{aligned} \right\} \quad (1.89)$$

Let expression (1.88) be differentiated twice and substituted into (1.89). In this case

$$\left. \begin{aligned} F_r &= m(\ddot{r} - r\dot{\psi}^2) \\ F_\psi &= \frac{m}{r} \times \frac{d}{dt}(r^2\dot{\psi}) \end{aligned} \right\} \quad (1.90)$$

Expressing the components $\mathbf{v}$ and $\mathbf{B}$ in cylindrical coordinates, we obtain the following equations of motion

$$\left. \begin{aligned} \ddot{m}z &= -e (\dot{r}B_\psi - r\dot{\psi}B_r) \\ m (\ddot{r} - r\dot{\psi}^2) &= -e (r\dot{\psi}B_z - \dot{z}B_\psi) \\ \frac{m}{r} \times \frac{d}{dt} (r^2\dot{\psi}) &= -e (\dot{z}B_r - \dot{r}B_z) \end{aligned} \right\} \quad (1.91)$$

In an axisymmetric field $B_\psi = 0$ and the components B_z and B_r can be expressed through the vector potential according to (1.66) and substituted into (1.91)

$$\left. \begin{aligned} \ddot{m}z &= -er\dot{\psi} \frac{\partial A_\psi}{\partial z} \\ m (\ddot{r} - r\dot{\psi}^2) &= -e\dot{\psi} \frac{\partial (rA_\psi)}{\partial r} \\ \frac{m}{r} \times \frac{d}{dt} (r^2\dot{\psi}) &= e\dot{z} \frac{\partial A_\psi}{\partial z} + e\frac{\dot{r}}{r} \times \frac{\partial (rA_\psi)}{\partial r} = \frac{e}{r} \times \frac{d}{dt} (rA_\psi) \end{aligned} \right\} \quad (1.92)$$

The third equation (1.91) can be integrated with respect to time

$$\dot{\psi} = \frac{e}{m} \times \frac{A_\psi}{r} \quad (1.93)$$

The constant term of integration in (1.93) is equal to zero, since with $A_\psi = 0$ (outside the field) assuming that the electron enters the field without an azimuth component of the velocity, $\psi = 0$.

If we again restrict ourselves to the near-axial area of the field, in expansion (1.74) the terms with r in powers higher than 2 can be rejected, i.e. we can assume that $A_\psi = \frac{1}{2} B_0(z) r$. In this case

$$\dot{\psi} = \frac{1}{2} \times \frac{e}{m} B_0(z) \quad (1.94)$$

i.e. the azimuthal velocity is independent of the initial position of the electron and is uniquely determined by the axial component of the magnetic induction.

Substituting the value of $\dot{\psi}$ from (1.94) into the second equation (1.92) and making necessary transforms, we obtain the equation of motion in the form

$$\ddot{m}z = -\frac{e^2}{4m} B_0^2 r \quad (1.95)$$

Using twice the differential operator $\frac{d}{dt} = \frac{dz}{dt} \times \frac{d}{dz}$ and noting that $\frac{dz}{dt} = v_z = \sqrt{\frac{2e}{m} U_0}$ (for the near-axial area), where U_0 is the

potential at the axis of the system, we obtain the equation of the trajectory

$$\frac{d^2r}{dz^2} = -\frac{e}{8mU_0} B_0^2 r \quad (1.96)$$

Equation (1.96) is a trajectory equation in the projection onto the meridian plane. However, as the electron in a magnetic field also acquires an azimuthal velocity, to describe the motion completely it is necessary to calculate the angle of turn of the trajectory or the angle of rotation of the meridian plane about the axis in which the trajectory described by equation (1.96) lies. To this end, transform equation (1.94) using again the differential operator

$$\frac{d\psi}{dz} = \frac{1}{v_z} \times \frac{d\psi}{dt} = \sqrt{\frac{e}{8mU_0}} B_0 \quad (1.97)$$

Thus, the motion of an electron in an axisymmetric magnetic field in the near-axial area (when $v \ll c$) is described by the system of equations

$$\left. \begin{aligned} \frac{d^2r}{dz^2} &= -\frac{e}{8mU_0} B_0^2 r \\ \frac{d\psi}{dz} &= \sqrt{\frac{e}{8mU_0}} B_0 \end{aligned} \right\} \quad (1.98)$$

Analysis of equations (1.98) shows that, first, the trajectories in the magnetic field are not flat but have an appearance of spatial coils with a variable radius. Second, the equations include e and m , i.e. particles with different charges and masses describe different trajectories in the magnetic field. Third, the equations are nonhomogeneous with respect to B_0 and U_0 , therefore, when changing B_0 k times U_0 must be changed k^2 times in simulating the trajectories in magnetic fields. Finally, it should be noted that in magnetic fields the trajectories are irreversible because the angle of turn depends on the direction of motion of the particles.

The value U_0 in equations (1.98) should be regarded as an amount of electron energy rather than the real value of the electric potential at the axis. However, since the electron is usually accelerated in an electric field before entering a magnetic field, the value of U_0 is numerically equal to the accelerating potential difference, i.e. coincides with the value of the potential at the axis (it is assumed that the electron starts moving at a zero initial velocity).

Since the first equation (1.98) is homogeneous with respect to r , then after making an analysis similar to that for the electric field, we may draw a conclusion that a nonhomogeneous axisymmetric magnetic field in the near-axial area is an electron lens. The only difference lies in the fact that in contrast to the electrostatic field, the image formed by the magnetic lens is turned through a certain angle determined by the second equation (1.98).

Equations (1.98) are valid for low-velocity electrons. In the case of relativistic velocities these equations must be changed in accordance with the general expression for the law of conservation of energy (1.4).

In the presence of electric and magnetic axisymmetric fields within a certain area (the case of "superimposing" fields), the equations of motion of the electrons can be obtained by adding the component forces acting on the electron from the sides of electric and magnetic fields

$$\left. \begin{aligned} \frac{d^2 r}{dz^2} + \frac{U'_0}{2U_0} \times \frac{dr}{dz} + \left(\frac{U''_0}{4U_0} + \frac{e}{8mU_0} B_0^2 \right) r &= 0 \\ \frac{d\psi}{dz} - \sqrt{\frac{e}{8mU_0}} B_0 &= 0 \end{aligned} \right\} \quad (1.99)$$

In this case, the possibility to obtain an image remains, but the image turn angle inherent in the magnetic field remains either.

The above equations of motion are valid for narrow near-axial pencils (paraxial optics). It is the narrow beam, which may be considered paraxial with good approximation, that is employed in many

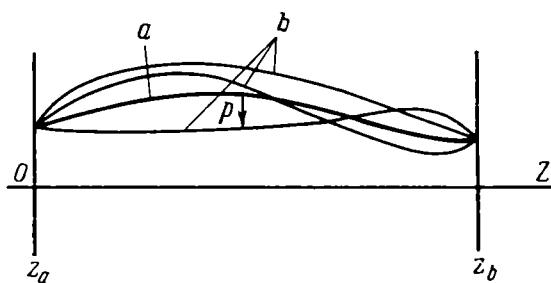


Fig. 1.18. Main (a) and adjacent (b) trajectories

types of electron-beam devices. However, in some devices, such as electron-optical image converters and in TV camera tubes with image transfer, use is made of wide beams that cannot be considered as paraxial even to very rough approximation. In the general case the paths of electrons in wide beams can be found by setting equations of motion similar to those given above by using not one or two terms in expansions (1.53) and (1.74), but the terms of higher orders. It is natural that the solution of such complex nonhomogeneous equations is associated with considerable difficulties, and the obtained results will scarcely be suitable for practical calculations because of their awkwardness.

There is another way to solve the problem of focusing wide beams. Assume that electrons emerge from a point in the plane z_a (Fig. 1.18) at different angles to this plane. An electric or magnetic field symmetric with respect to OZ adjoins the plane z_a . Let us study the path of an electron moving normally to the plane z_a . This electron descri-

bes a curve in the field and reaches some point of the plane z_b . Let the given electron trajectory be called main, while the trajectories of the electrons emerging from the same point of the plane z_a but not normally be referred to as adjacent trajectories. Next, find the conditions under which the adjacent trajectories in the plane z_b meet at the point of intersection of this plane with the main trajectory. Thus, the problem is reduced to finding adjacent trajectories which can be regarded as paraxial with respect to the main trajectory serving as an axis for a family of adjacent trajectories. In the general case this axis can be curvilinear.

Should the main trajectory be known, the position of an adjacent trajectory for any value of z can be uniquely determined by the value and direction of the section lying in a plane perpendicular to the axis (with the given value of z) connecting the main trajectory to the adjacent one. This section can be presented as a vector $[p(z)]$ expressed in a complex form through variables $p(z)$ and $\varphi(z)$

$$p(z) = p(z) e^{j\varphi(z)} \quad (1.100)$$

If the adjacent trajectories are paraxial with respect to the main trajectory, they can be described by the known equations of paraxial optics

$$\left. \begin{aligned} \frac{d^2 p}{dz^2} + \frac{U'}{2U} \times \frac{dp}{dz} + \left(\frac{U''}{4U} + \frac{e}{8mU} B^2 \right) p &= 0 \\ \frac{d\varphi}{dz} - \sqrt{\frac{e}{8mU}} B &= 0 \end{aligned} \right\} \quad (1.101)$$

where U and B are the values of electric potential and magnetic induction along the main trajectory.

In order that the adjacent trajectories coming from one point in the plane z_a be collected at one point in the plane z_b , there must be solutions of equations (1.101) under the boundary conditions $p(z_a) = 0$ and $p(z_b) = 0$. If that is the case an image of the object (plane z_a) is produced in the plane z_b , each point of the object being reproduced by the image point determined by the main trajectory coming from the given point of the object. However, we cannot speak of geometric similarity of the image and the object on the basis of focusing the adjacent trajectories into a point, since this similarity is defined by the main trajectories, which in the case of wide beams may not satisfy the equations of paraxial optics.

The analytic calculation of the main trajectories is often difficult and such trajectories are estimated approximately or plotted by means of graphoanalytic methods (see Section 1.7). Therefore, the problem of focusing wide beams falls into two stages: (1) finding the family of main trajectories for which the geometric similarity of the image and object is valid; (2) finding the conditions under which

the adjacent trajectories intersect the main trajectory in the plane of the image, i.e. be focused in a point in the plane z_b .

A particular case of the considered problem is focusing wide beams of electrons by a longitudinal homogeneous magnetic field (Fig. 1.19) used in some electron-beam devices.

With $B = B_z = \text{const}$ and $U = \text{const}$ the main trajectories coincide with the lines of force of the magnetic field within the whole area of propagation of the pencil while the adjacent trajectories are "wound" on the main ones forming a number of knots and bundles.

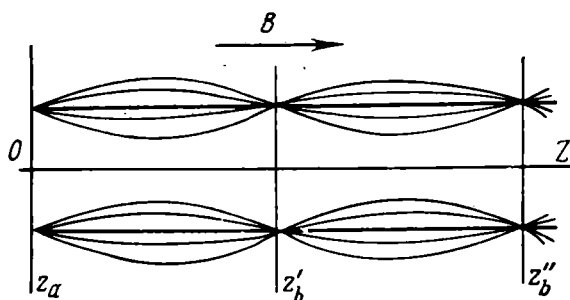


Fig. 1.19. Focusing wide electron beam by uniform magnetic field

In the projection onto a plane normal to the axis OZ the electrons moving along the adjacent trajectories describe circles whose radius is determined from (1.13)

$$R = \frac{mv_r}{eB_z} \quad (1.102)$$

where v_r is the velocity component normal to the axis.

Hence, the adjacent trajectories are represented by helical lines wound on cylinders having a radius defined by expression (1.102), all the cylinders for the given family of adjacent trajectories having a common generating line, i.e. the main trajectory. One can easily see that it is possible to obtain an image and focus it in a homogeneous magnetic field. Indeed, the period of revolution of the electron about the main trajectory (the time of passing one turn of the spiral)

$$T = \frac{2\pi R}{v_r} = \frac{2\pi m}{eB_z} \quad (1.103)$$

is independent either of the velocity component v_r or the radius of the spiral and is uniquely determined by the magnitude of magnetic induction, i.e. all the electrons describe one turn at one and the same time. If the velocity component along the OZ axis is about the same for all the electrons (which in practice is often accomplished by accelerating the pencil electrons before they enter a magnetic field with the aid of a longitudinal electric field), then the electrons cover the same distance along the axis ($z = v_z T$) during one turn along the spiral and meet again at the main trajectory.

Placing the receiver of electrons at a distance $v_z T$, $2v_z T$, etc. from the object, we will obtain a direct (not reversed) image of the object, which is neither enlarged nor reduced in size. Thus, a longitudinal homogeneous magnetic field "transfers" the image from one plane to another in a scale of 1 in 1, i.e. strictly speaking is not an electron

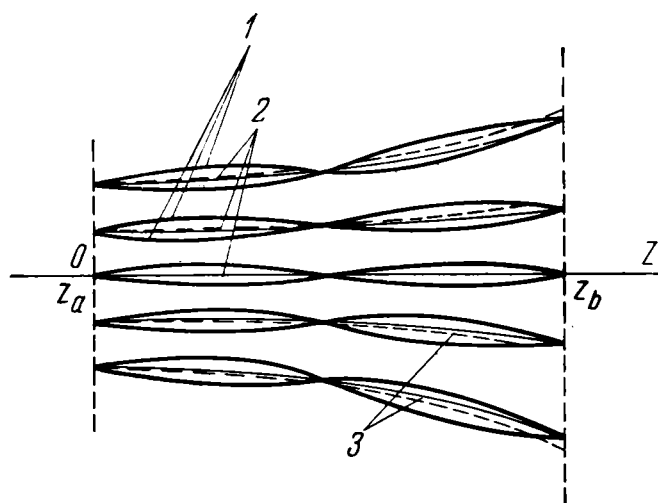


Fig. 1.20. Focusing wide electron beam by nonuniform magnetic field
1—adjacent trajectories; 2—main trajectories; 3—lines of force

lens (cannot collect a parallel beam into a point and form enlarged or reduced images).

In the case of nonuniform magnetic field diminishing along the axis the main trajectories follow diverging lines of force (Fig. 1.20), i.e. such a field will be a diverging lens for the main trajectories. At the same time, the adjacent trajectories "wound" on the main ones can be focused in a point in the plane z_b . If that is the case, we obtain an enlarged image of the object, since the scale of the image exceeds unity due to the divergence of the main trajectories.

The above-discussed system of focusing used in image-transfer television camera tubes has no analog in light optics, since physical diverging lenses cannot in principle produce a real image.

1.7. Methods of Finding Electron Trajectories

The equations of motion of electrons in electric fields discussed in the preceding section can be solved analytically only in rare occasions. Therefore, in practice the paths of the electrons moving in electric fields are usually found by means of approximate methods of solutions of the fundamental equations or by plotting the trajectories grapho-analytically.

If the potential distribution along the axis of an axisymmetric electric field is given in an analytic form or if the experimental curve

of distribution of the axial potential can be approximated to a sufficient accuracy by a suitable analytic function, the solution of the fundamental equation of paraxial optics (1.82) can be found by the method of successive approximations.

Assume that in a certain plane perpendicular to the axis intersecting the axis at a point z_a there are specified the following initial conditions: the distance between the trajectory and the axis $r(z_a) = r_a$ and the inclination of a tangent to the trajectory $(dr/dz)_{z_a} = r'_a$. Using the fundamental equation of the form (1.81) we obtain

$$\frac{d}{dz} \left(\sqrt{U_0} \frac{dr}{dz} \right) = -\frac{1}{4} \times \frac{U_0''}{\sqrt{U_0}} r$$

Next, this equation is integrated in the range from z_a to a certain value of z

$$\sqrt{U_0} r'(z) - \sqrt{U_a} r'_a = -\frac{1}{4} \int_{z_a}^z \frac{U_0''}{\sqrt{U_0}} r(z) dz$$

where $U_a = U_0(z_a)$. Now the obtained equation is solved with respect to $r'(z)$

$$r'(z) = \frac{\sqrt{U_a}}{\sqrt{U_0}} r'_a - \frac{1}{4 \sqrt{U_0}} \int_{z_a}^z \frac{U_0''}{\sqrt{U_0}} r(z) dz$$

The second integration of this equation gives an expression for the sought function $r(z)$

$$r(z) = r_a + \sqrt{U_a} r'_a \int_{z_a}^z \frac{dz}{\sqrt{U_0}} - \frac{1}{4} \int_{z_a}^z \frac{1}{\sqrt{U_0}} \left[\int_{z_a}^z \frac{U_0''}{\sqrt{U_0}} r(z) dz \right] dz \quad (1.104)$$

In equation (1.104) the sought function $r(z)$ is found both in the right-hand and left-hand parts. From the theory of differential equations it is known that, when inserting some approximate value $r_0(z)$ into the right-hand part of the equation [in our case (1.104)], its solution $r_1(z)$ produces better approximation to the real value $r(z)$.

Substituting $r_1(z)$ again into the right-hand part of equation (1.104) and making the integration, we obtain another approximation $r_2(z)$, which is still closer to the real value $r(z)$. Repeating this operation several times we can obtain a dependence $r_n(z)$ that approximates the precise solution of equation (1.82) to a great degree of accuracy. This procedure constitutes the method of successive approximations. With a good choice of the zero approximation $r_0(z)$ it is sufficient to carry out the integration 3 to 4 times to obtain the equation of the trajectory with an accuracy suitable for practical purposes. Good results are achieved when the first two terms of equation (1.104) are taken as zero approximation, i.e. $r_0(z)$ is presented

in the form

$$r_0(z) = r_a + \sqrt{U_a} r'_a \int_{z_a}^z \frac{dz}{\sqrt{U_0}} \quad (1.105)$$

Figure 1.21 illustrates the paths of electrons in an electric field formed by two coaxial cylinders of the same radius and with different potentials calculated by the method of successive approximations.

As seen from this figure, the third approximation $r_3(z)$ and the fourth approximation $r_4(z)$ practically coincide, and further calculations are unnecessary.

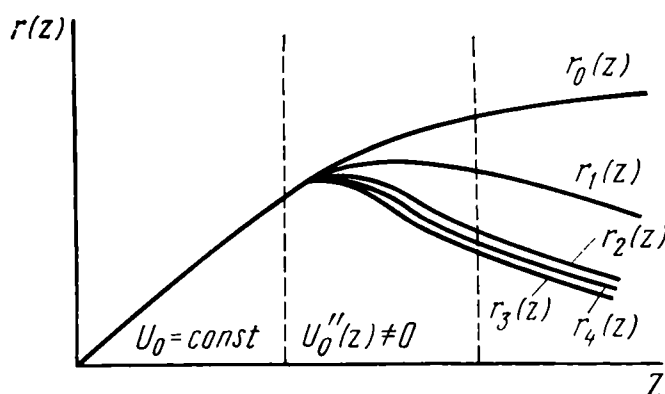


Fig. 1.21. Constructing electron trajectories by approximation method

Where the axial potential distribution found experimentally or roughly calculated is not amenable to approximation by the analytic function, use can be made of approximate solution of the fundamental equation, which is called the polygon method or the method of linear sections of axial potential. In using this method, the area of the axisymmetric electric field, in which it is necessary to find the trajectory of a paraxial electron, is subdivided into a number of sections within each of which the axial potential distribution is represented by a linear function z , i.e. the field intensity is constant inside every section and changes stepwise with changing to another section. In other words, a smooth curve of distribution of the axial potential is replaced by a broken line consisting of sections of straight lines (Fig. 1.22).

With such representation inside each section $U_0(z) = kz$, $U'_0(z) = \text{const}$, $U''_0(z) = 0$. The $U''_0(z)$ becomes infinite at the section junctions.

Let us denote the coordinates of the section junctions through $z_1, z_2, z_3, \dots, z_n$, the values of the axial potential at the section junctions through $U_1, U_2, U_3, \dots, U_n$, the values of the first deriva-

tives dU_0/dz within the sections through $U'_1, U'_2, U'_3, \dots, U'_n$ and the departure of the trajectories from the axis at the section junctions through $r_1, r_2, \dots, r_n$. Since $U''_0 = 0$ within each section, equation (1.82) for the regions enclosed between the planes with coordinates $(z_1, z_2), (z_2, z_3), \dots, (z_{n-1}, z_n)$ assumes the form

$$r'' + \frac{1}{2} \frac{U'_0}{U_0} r' = 0 \quad (1.106)$$

where the dashes stand for the differentiation with respect to z . Rewrite equation (1.106) in the form

$$\frac{r''}{r'} = -\frac{1}{2} \frac{U'_0}{U_0} \quad (1.107)$$

and integrate it with respect to z within the section from z_1 to a certain

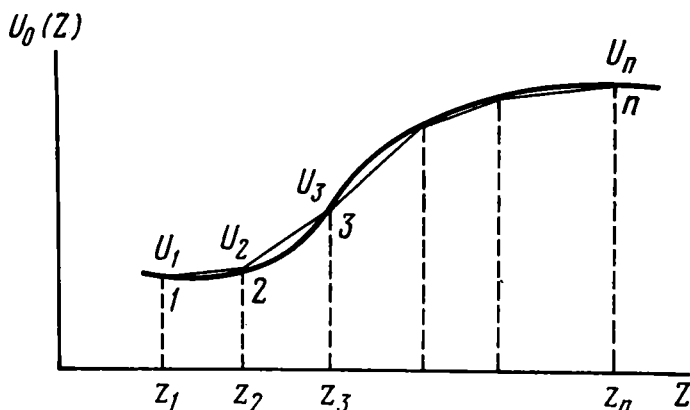


Fig. 1.22. Constructing electron trajectories by line sections

value of z lying between z_1 and z_2

$$\int_{z_1}^z \frac{r''}{r'} dz = -\frac{1}{2} \int_{z_1}^z \frac{U'_0}{U_0} dz \quad (1.108)$$

It is clear that the integrands may be conveniently presented by logarithms, i.e. the (1.108) may be transformed into the equation

$$\int_{z_1}^z d(\ln r') = -\frac{1}{2} \int_{z_1}^z d(\ln U_0) \quad (1.109)$$

which after the integration is written as

$$\ln \left(\frac{r'}{r'_1} \right) = -\frac{1}{2} \ln \left(\frac{U_0}{U_1} \right) \quad (1.110)$$

where $r'_1 = (dr/dz)_{z_1}$ (to the right from the plane having coordinate z_1).

From (1.110) it follows directly that within the section in question

$$r' = r'_1 \sqrt{\frac{U_1}{U_0}} = \frac{c_1}{\sqrt{U_0(z)}} \quad (1.111)$$

Here $c_1 = r'_1 \sqrt{U_1} = \text{const}$ is a constant value defined by the initial conditions such as the inclination of the tangent to the trajectory and the value of the potential at the point z_1 .

Integrating equation (1.111) within the range of the section (from z_1 to z) leads to an expression for $r(z)$ inside the section under consideration

$$r(z) = r_1 + \frac{2c_1 (\sqrt{U_0(z)} - \sqrt{U_1})}{U'_1} \quad (1.112)$$

where $U'_1 = \frac{U_0(z) - U_1}{z - z_1}$

At the right-hand junction of the section (with $z = z_2$)

$$r_2 = r_1 + \frac{2r'_1 \sqrt{U_1} (\sqrt{U_2} - \sqrt{U_1}) (z_2 - z_1)}{U_2 - U_1} \quad (1.113)$$

$$r'_{2l} = r'_1 \sqrt{\frac{U_1}{U_2}}$$

(The index "l" indicates an angle of inclination of the trajectory to the left from the junction between the first and second sections.)

Expressions (1.113) make it possible to construct a length of a paraxial electron trajectory inside the section limited by the planes perpendicular to the axis, within the range from z_1 to z_2 .

The value of z_2 calculated at the end of the first section is used as the starting value during the calculation of the trajectory in the second section. However, the value of r'_{2l} calculated at the end of the first section cannot be regarded as starting for calculation of the trajectory in the second section, as U'_0 is broken at the junction of the sections and the trajectory has a discontinuity, i.e. $r'_{2r} \neq r'_{2l}$ (the index "r" indicates an inclination of the trajectory to the right from the junction between the sections).

The calculation of the angle of inclination of the trajectory at the beginning of the second section r'_{2r} is based on the fact that in the vicinity of the point z_2 the values $U_0(z)$ and $U'_0(z)$ remain finite, while the value U''_0 becomes infinite. Owing to this we may neglect the value $(U'_0/2U_0) r'$ close to the section junction as compared with $(U''_0/4U_0) r$ in the equation (1.82) i.e. the (1.82) may be presented in the form

$$r'' = -\frac{1}{4} \times \frac{U_0}{U_0} \quad (1.114)$$

We may also assume that in the region in question $U_0(z) = U_0(z_2) = \text{const}$. In this case the integration of equation (1.114)

in the range between the points z_{2l} and z_{2r} lying close to z_2 gives

$$r'_{2r} = r'_{2l} - \frac{r_2}{4U_2} (U'_2 - U'_1) \quad (1.115)$$

where $U_2 = \frac{U_3 - U_1}{z_3 - z_1}$.

Having determined the r_2 and r'_{2r} from (1.113) and (1.115) and taking them as the starting data for the left-hand junction of the second section, the trajectory in the second section is found, r_3 and r'_{3r} are calculated and taken as the starting data for the third section, etc. Thus there can be found the whole electron trajectory in the near-axial area of an axisymmetric electric field. In most cases it is

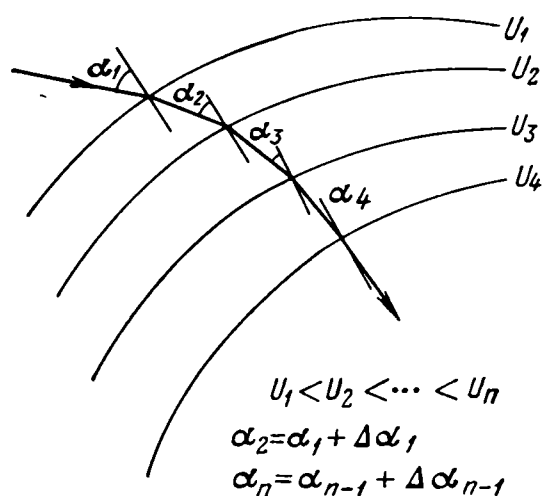


Fig. 1.23. Constructing electron trajectories by broken line

not necessary to determine the value $r(z)$ at the intermediate points z lying inside the selected sections. One can find only $r_1, r_2, \dots, r_n$ and connect the obtained points by a smooth curve which coincides with the sought trajectory with an accuracy sufficient for practical purposes. The obtained curve will be the closer to the real trajectory, the greater the number of the sections dividing the area of the field under consideration or, in other words, the closer the broken line approximating the axial potential distribution to the

real curve $U_0(z)$. However, with a great number of sections the calculations are labour consuming, while the gain in accuracy is insignificant.

If the potential distribution is given in the form of equipotential lines obtained, for example, by means of simulating in the electrolytic bath, the trajectories of the electrons can be found by grapho-analytic methods.

Of many proposed grapho-analytic methods for constructing electron trajectories the following two methods enjoy the widest application: the method of a plane capacitor ("broken line method") and the method of radii of a curvature ("method of circles").

In the first case an electric field produced by a system of equipotential lines is regarded as a number of regions with constant potentials (Fig. 1.23), the potential at the junction of the region (which is taken as plane for a small distance) being changed in a stepwise manner. In this case the trajectory is shown by a broken line and the law

of refraction (1.21) is used for determining the angle of turn of the trajectory at the region junctions.

Assuming that the potential difference ΔU between the adjacent regions is small, we can determine the angle ($\Delta\alpha$) of turn of the trajectory. On the basis of (1.21) we can write that

$$\frac{\sin \alpha}{\sin (\alpha + \Delta \alpha)} = \frac{\sqrt{U + \Delta U}}{\sqrt{U}} \quad (1.116)$$

The small potential difference ΔU results in a low value of $\Delta\alpha$; in this case we may assume that $\sin \Delta\alpha \approx \Delta\alpha$, $\cos \Delta\alpha \approx 1$ and tran-

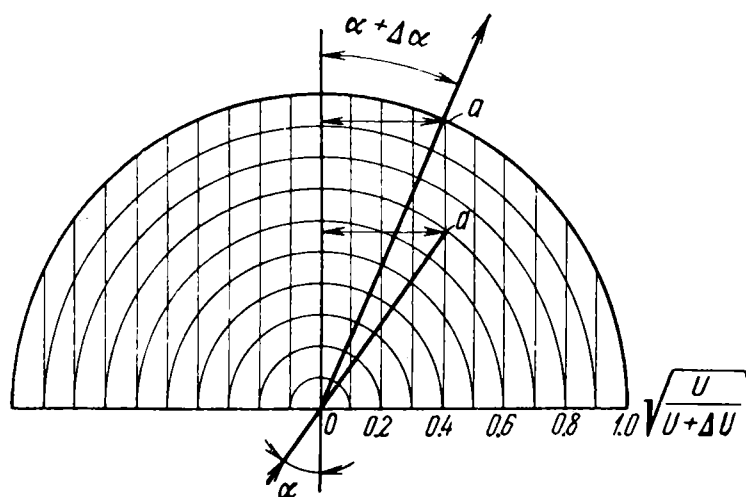


Fig. 1.24. "Protractor" for constructing trajectories

sform expression (1.116) in the following manner

$$\sin \alpha \sqrt{\frac{U}{U + \Delta U}} = \sin \alpha + \cos \alpha \Delta \alpha$$

from which

$$\Delta \alpha = \left(\sqrt{\frac{U}{U + \Delta U}} - 1 \right) \tan \alpha \quad (1.117)$$

Determining in succession the variations of the angle $\Delta\alpha$ by the flying angle α of the particle in the region of the field, the whole trajectory is constructed. The broken-line method allows the trajectory to be constructed with an accuracy of 3-5 %, however, it is time-consuming. The time consumption can be considerably reduced by using a simple contrivance in the form of a special "protractor" of a transparent material (Fig. 1.24).

The relations $\sqrt{\frac{U}{U + \Delta U}}$ are plotted along the diameter, and circular arcs and straight lines perpendicular to the diameter are drawn at equal intervals. The trajectories are constructed as follows. The

"protractor" is positioned so that its base diameter is a tangent to the equipotential U at the point of entrance of the particle. A straight line is drawn in the given prescribed starting direction α until it intersects with the circle corresponding to the found relation $\sqrt{\frac{U}{U+\Delta U}}$, where $U + \Delta U$ is the potential of the next equipotential. A perpendicular is erected from the point obtained to the tangent to the equipotential until it intersects with the circle passing through the mark $\sqrt{\frac{U}{U+\Delta U}} = 1$. The obtained point is connected by a straight line with the centre of the "protractor". This straight line is the section of trajectory between the selected equipotentials. The validity of this assertion can easily be proved by expressing the section a in Fig. 1.24 through the angles α and $\alpha + \Delta\alpha$

$$a = \sin(\alpha + \Delta\alpha), \quad a = \sqrt{\frac{U}{U+\Delta U}} \sin \alpha$$

Equating the right-hand terms of the obtained expressions, gives us expression (1.116).

The use of a "protractor" allows the time required for plotting the trajectories to be considerably reduced. However, the error is still significant mainly due to geometrical errors when laying the "protractor" along the tangent to the equipotential, particularly, when the latter is very curved.

The method of curvature radii is based on the equality of the force acting on a particle in an electric field to the centripetal effective during the movement of the particle along a curved line with an instantaneous radius of curvature r

$$\frac{mv^2}{r} = eE_n$$

where E_n is a component of the electric field intensity normal to the trajectory.

Expressing the particle velocity through the passed potential difference U , we obtain the following relation for the instantaneous radius of curvature of the trajectory

$$r = \frac{2U}{E_n} \quad (1.118)$$

Determining the values U and E_n at certain points of the field, a trajectory can be constructed in the form of a number of conjugate circular arcs. The method of curvature radii makes it possible to construct trajectories with a better accuracy than in the case of the plane-capacitor method. However, this method takes much time either and is practically inconvenient when r is of a high value. The relative error of this method is 2-3 %.

In many cases the approximate and grapho-analytic methods of finding the trajectories of electrons in electric fields provide the required practical accuracy, but they take much time and call for skillful operators. The equations of motion of electrons can be solved with any degree of accuracy by means of electronic computers, yet in this case the computation of trajectories in complex electric fields requires a lot of machine time.

Automatic devices known as trajectory tracers have lately found wide applications for rapid and accurate calculation of the paths of electrons in electric fields having plane or axial symmetry. By their principle of operation these devices should be regarded as special analog computers solving the equations of motion of charged particles in an electric field.

Two types of trajectory tracers are used in practice: the tracers solving the equations of motion in the Cartesian coordinates and those solving the equations in the "natural" coordinates (in the projection to the tangent and main normals to the trajectory). In both types of tracers the electric field is simulated in an electrolytic bath, although it is possible to equip the trajectory tracers with electrically conductive paper or a flexible diaphragm as a field simulating device.

The trajectory tracers operating in the Cartesian coordinates automatically solve the equations of motion of an electron in a plane electric field $[E(x, y)]$ or in an axisymmetric field $[E(z, r)]$. The equation of motion of the electron in the plane field is written in the form [see (1.7)]

$$\left. \begin{aligned} \ddot{x} &= -\frac{e}{m} E_x \\ \ddot{y} &= -\frac{e}{m} E_y \end{aligned} \right\} \quad (1.119)$$

The study of the trajectories in axisymmetric fields is based on an assumption that in the starting point the electron has no azimuthal component of the velocity ($v^\psi = 0$). In this case the electron trajectory lies in the meridian plane, since in the axisymmetric field $E_\psi = 0$ and, therefore, there is no force that would drive the electron from this plane. Therefore, the coordinates of the meridian plane (z, r) might be considered as Cartesian and the equation of motion of the electron in the meridian plane can be written similarly to system (1.119)

$$\left. \begin{aligned} \ddot{z} &= -\frac{e}{m} E_z \\ \ddot{r} &= -\frac{e}{m} E_r \end{aligned} \right\} \quad (1.120)$$

The solution of the system of equations (1.119) or (1.120) is effected by means of commercial-type electronic integrators. It is clear that the above-considered system of equations can be solved by means

of four integrators performing in succession the operations of finding the components of the electron velocity (v_x , v_y) and then the coordinates (x , y)

$$\begin{aligned} v_x &= -\frac{e}{m} \int_0^t E_x dt, \quad x = \int_0^t v_x dt \\ v_y &= -\frac{e}{m} \int_0^t E_y dt, \quad y = \int_0^t v_y dt \end{aligned} \quad (1.121)$$

The block diagram of the trajectory tracer of the first type is given in Fig. 1.25.

The field to be investigated is simulated in the electrolytic tank by a conventional method. The trajectories of the value E_x and E_y re-

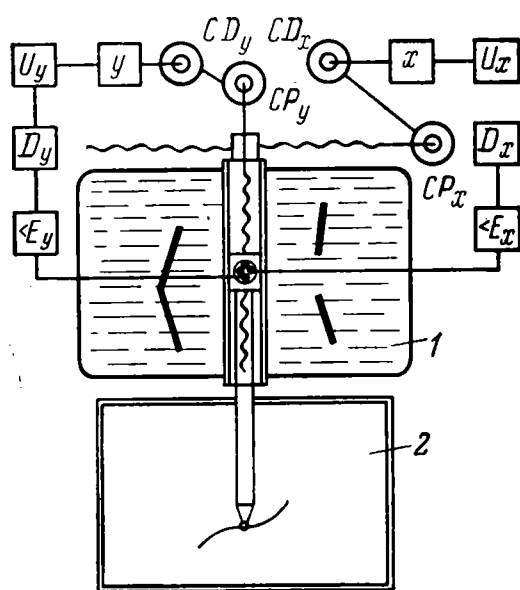


Fig. 1.25. Block diagram of trajectory tracer of the first type
1—electrolytic tank; 2—table

quired for the calculation are "measured" in the tank by means of a four-pin probe fixed on a probe carriage which is capable of moving over the tank in two perpendicular directions (OX , OY). The probe is oriented so that during the movement of the carriage the planes passing through the opposite pins of the probe are always parallel to the selected coordinate axes (OX , OY) (Fig. 1.26).

Since the tank is fed with an alternating current, while the electronic integrators are direct-current devices, special linear demodulators are inserted between the probe head and the integrators. The first pair of integrators generates voltages proportional to v_x and v_y . The signals from the first pair of integrators are fed to the input of

the other pair of integrators the output of which produces signals representing the coordinates of the moving electron in the corresponding scale. The signals from the other pair of integrators are applied to a follow-up system consisting of electronic amplifiers and two pair of synchros. Through the pair synchro-generator and synchro-motor, the data on electron coordinates is transmitted to the feed screws moving the probe carriage and a recorder which is rigidly coupled with it.

During the movement of the probe carriage, in accordance with the data on the coordinates calculated by the electronic integrators, the recorder draws a curve coinciding with the path of the electron moving in the electric field simulated in the tank. A trajectory is drawn within 3-4 minutes. The relative error does not exceed 1-1.5%. When additional units are connected to the system, the tracers can take account for the relativistic corrections and draw the trajectories in crossed electric and magnetic fields.

The principle of operation of the trajectory tracers solving the equations of motion of electrons in the "natural" coordinates is as

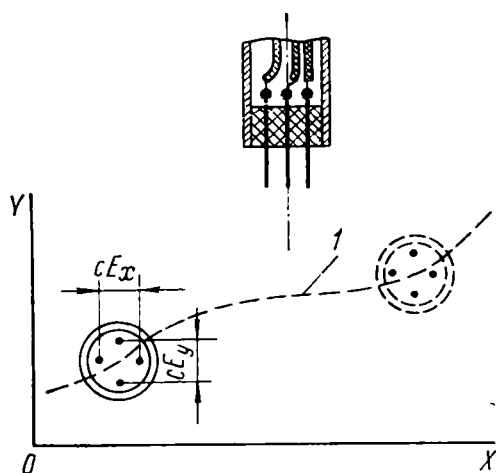


Fig. 1.26. Probe for measuring components of electric field intensity
1—electron trajectory

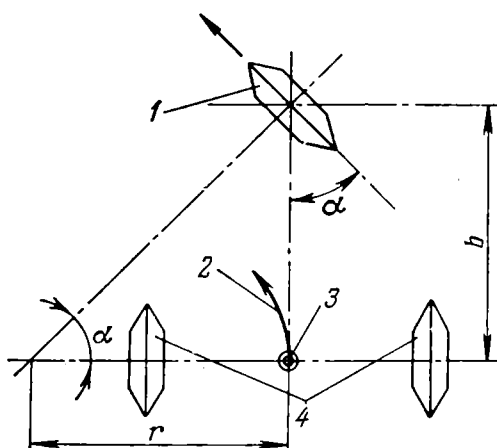


Fig. 1.27. Carriage of trajectory tracer of the second type
1—steering wheels; 2—trajectory; 3—marker; 4—support wheels

follows. Assume that rolling over a plane without friction is a three-wheel carriage having two supporting freely rotating wheels and one guide (steering) wheel whose plane can turn with respect to the longitudinal axis of the carriage (Fig. 1.27).

Mounted on the axle of the supporting wheels is a marker tracing a curve. As shown in the drawing the instantaneous radius of curvature traced by the marker

$$r = \frac{b}{\tan \alpha} \quad (1.122)$$

where b is the distance between the axle of the supporting wheels and the axle of the guide wheel ("base" of the carriage); α is the angle of turn of the steering wheel plane with respect to the longitudinal axis of the carriage.

Comparing the expressions (1.118) and (1.122), we obtain the fundamental equation of the trajectory tracer

$$2U \tan \alpha = bE_n \quad (1.123)$$

The values U and E_n in equation (1.123) can be measured in the electrolytic tank by means of a two-pin probe oriented so that the plane passing through the axes of the pins is normal to the direction of movement of the probe. Through a mechanical coordinate device the probe is connected to the carriage so that the plane of the probe pins is always parallel to the axis of the steering wheel of the carriage.

It is obvious that with such a motion of the carriage, when equation (1.123) is satisfied at any point of the field, the marker traces the curve coinciding with the electron trajectory.

Equation (1.123) is preferably reduced to the form

$$2U \sin \alpha = bE_n \cos \alpha \quad (1.124)$$

The voltages corresponding to the values U and cE_n (c is the distance between the pins of the probe) measured in the electrolytic

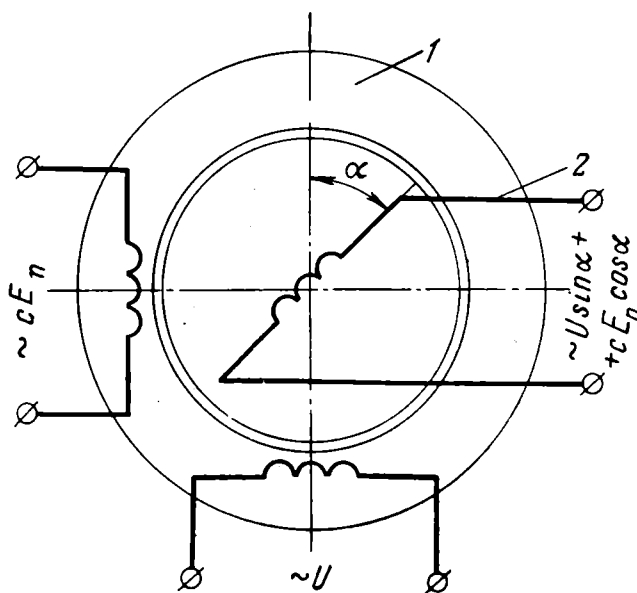


Fig. 1.28. Rotary converter

1—stator; 2—rotor

tank are amplified by electronic amplifiers and fed to a sine-cosine element mounted on the carriage. A sine-cosine rotary converter is used as this element.

The rotary converter (Fig. 1.28) has two perpendicular stator windings. By feeding the stator windings with voltages proportional to U and cE_n , we obtain (in the rotor winding) voltages proportional to $U \sin \alpha$ and $cE_n \cos \alpha$, where α is the angle of turn of the rotor. Thus, at the output of the sine-cosine element there are produced the required voltages proportional to $U \sin \alpha$ and $E_n \cos \alpha$, should the angle α be specified by rotation of the steering wheel of the carriage.

The sum of voltages (with the corresponding coefficients) $U \sin \alpha$ and $cE_n \cos \alpha$ is fed to the amplifier controlling the motor turning the steering wheel of the carriage. It is clear that, when condition (1.124) is satisfied (the phase is correct), this sum vanishes. Should condition (1.124) be unsatisfied, a misalignment voltage is produced which controls the motor turning the steering wheel of the carriage until condition (1.124) is satisfied. The carriage is moved forward by another continuously rotating motor, and since the angle of turn of the steering wheel at each point is determined by condition (1.124), the instantaneous radius of curvature always satisfies condition (1.118) and the marker mounted on the carriage automatically traces the trajectory being sought.

A block diagram of the trajectory tracer of the second type is shown in Fig. 1.29, while the external view of one of the models of this instrument is shown in Fig. 1.30.

The use of trajectory tracers makes it possible to find with a good accuracy the trajectories of charged particles in the fields that cannot be calculated analytically. In this case the time necessary for construction of the trajectories is considerably reduced, while automatic operation of the tracers enables the attention to be concentrated on the essence of the problem in contrast to the analytic and grapho-analytic methods.

Approximate but rather visual picture of the motion of charged particles in plane electric fields can be obtained by use of a rubber diaphragm simulating the field (see Section 1.4). The method of gravitational simulation of the trajectories by means of a rubber diaphragm is based on the agreement of the equations of a charged particle motion in a plane field (1.119) with the equation of motion of a material particle with the mass M over the surface $z(x, y)$ in the gravitational field

$$\left. \begin{aligned} \ddot{x} &= \frac{E_x}{M} = g \frac{\partial z}{\partial x} \\ \ddot{y} &= \frac{F_y}{M} = g \frac{\partial z}{\partial y} \end{aligned} \right\} \quad (1.125)$$

where g is the gravitational acceleration (the direction of the axis OZ coincides with the direction of the gravity force).

Since the trajectories of charged particles moving in an electric field do not depend on the charge and the mass, the agreement of equations (1.119) and (1.125) makes it possible to simulate the paths of the electrons in a plane field $U(x, y)$ by loosely rolling small balls over the surface of the deformed diaphragm. It is only necessary that the magnitude of deformation be proportional to the value of the potential at the corresponding points

$$\left[z = \frac{U}{k}, \text{ see (1.43)} \right]$$

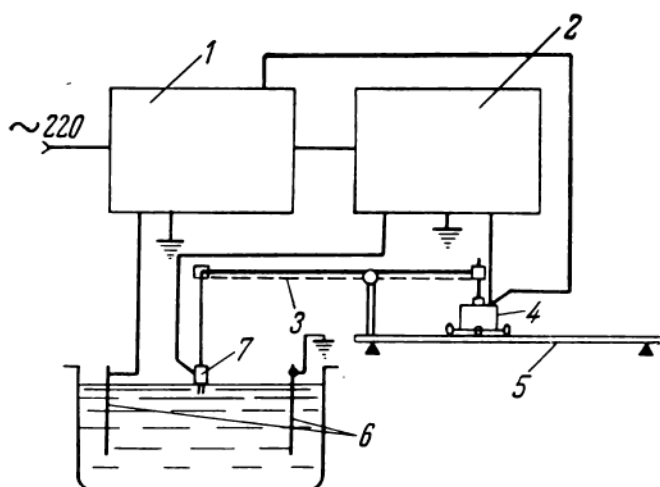


Fig. 1.29. Block diagram of the second-type trajectory tracer
 1—power supply unit; 2—computer unit; 3—transmission of coordinates; 4—carriage;
 5—table; 6—electrodes of model being analyzed; 7—probe

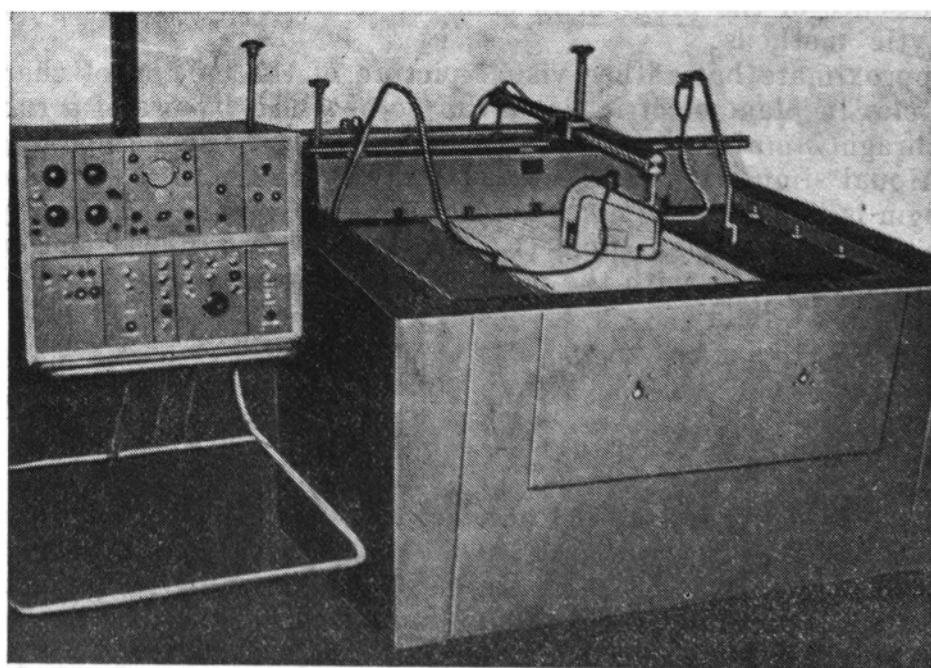


Fig. 1.30. External view of trajectory tracer

Use is usually made of polished steel balls 3-4 mm in diameter freely rolling over the surface of a deformed rubber cloth. The trajectories are obtained by photographing the bright spots on the balls due to intensive illumination. If the shutter of the photographic camera is open during the movement of the balls, the resultant trajectories in the photography are in the form of continuous lines. When the

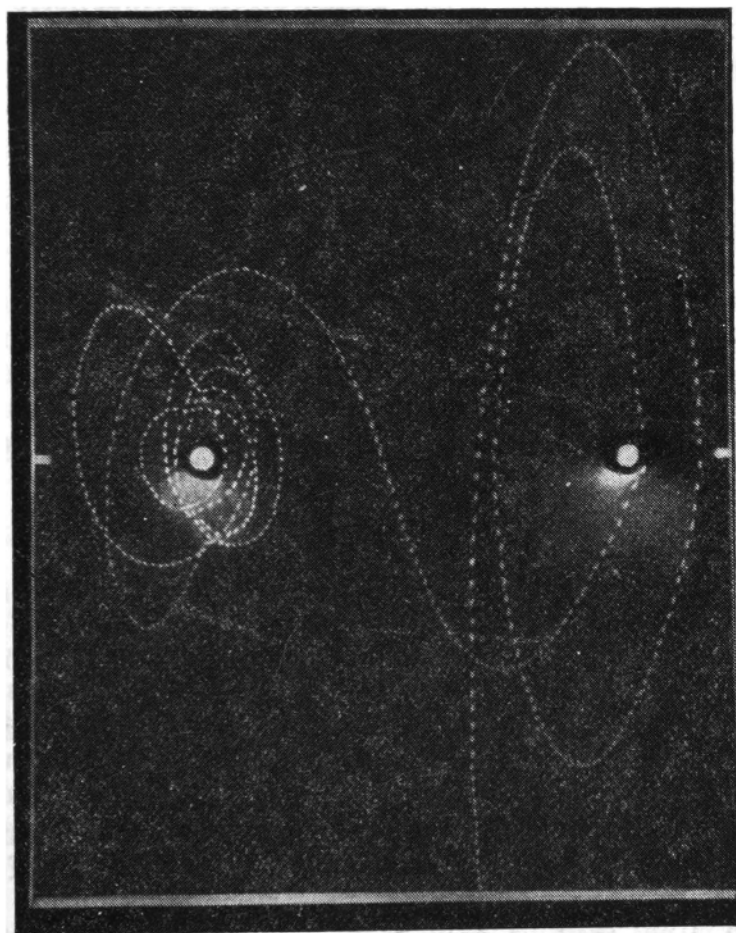


Fig. 1.31. Trajectory obtained by the flexible diaphragm method

installation is illuminated by periodically repeated flashes of light, the trajectory is obtained in the form of a dashed line, the length of each dash (the duration of the flashes must be the same) being a criterion of the ball velocity and, therefore, of the potential at the given point. The photographic picture of a trajectory obtained by the method of gravitational simulation is presented in Fig. 1.31.

The method of gravitational stimulation is practically convenient and simple, however, the error of constructing the trajectories is much greater than in the case of trajectory tracers. Consequently, gravitational simulation is used for preliminary quality investiga-

tions, the obtained results being then corrected by calculations or with the aid of trajectory tracers.

Finding the paths of electrons moving in magnetic fields is often a simple problem as compared to tracing their paths in electrostatic fields. This is due to the fact that the equations of motion of electrons in a magnetic field (1.98) do not contain derivatives of magnetic induction. Therefore, where the distribution of magnetic induction along the axis of an axisymmetric magnetic field is given analytically or the curve of the distribution of magnetic induction is approximated by the analytic function with an adequate accuracy, the system

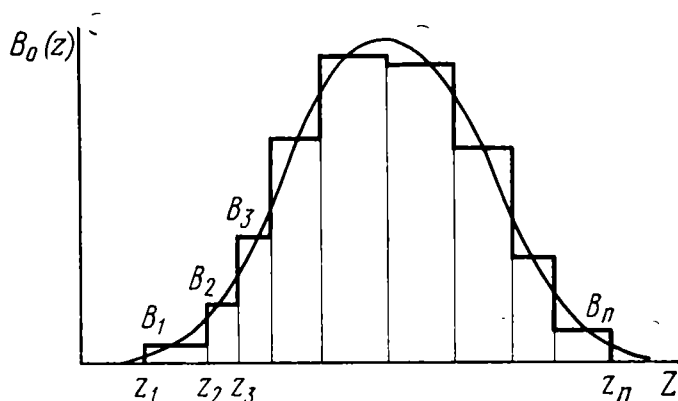


Fig. 1.32. Construction of electron trajectories by the stepped-curve method

of equations (1.98) can be solved precisely. The solution of the first equation of the system (1.98) yields the projection of the trajectory onto a rotating meridian plane, while the solution of the second equation determines the angle of turn of the meridian plane.

Of the approximate methods of solving the equations of motion of electrons in axisymmetric magnetic fields the most common is one based on replacement of the true curve of distribution of magnetic induction along the axis of an axisymmetric magnetic field by a stepped broken line (Fig. 1.32).

As seen from the drawing, the whole area of the field is subdivided into intervals and within each interval the value of magnetic induction at the axis is taken as constant: $B_0(z) |_{z_n, z_{n+1}} = B_0(z_n) = B_n$. In this case the first equation of the system (1.98) assumes the form

$$\frac{d^2 r}{dz^2} + \frac{e}{8mU_0} B_n^2 = 0 \quad (1.125a)$$

It is known that the solution of this equation is expressed through

$$r(z) = A \sin \left[\sqrt{\frac{e}{8mU_0}} B_n (z - z_n) \right] + \\ + C \cos \left[\sqrt{\frac{e}{8mU_0}} B_n (z - z_n) \right] \quad (1.126)$$

where A and C are the constants determined by the starting conditions.

$$r(z)_{z=z_n} = r_n, \quad \left. \frac{dr}{dz} \right|_{z=z_n} = r'_n$$

Simple substitution confirms that

$$A = r'_n \sqrt{\frac{8mU_0}{e}} \times \frac{1}{B_n}, \quad C = r_n$$

Taking into account the starting conditions, we obtain

$$\begin{aligned} r(z)_{z_n, z_{n+1}} = r_n \cos \left[\sqrt{\frac{e}{8mU_0}} B_n (z - z_n) \right] + \\ + r'_n \sqrt{\frac{8mU_0}{e}} \times \frac{1}{B_n} \sin \left[\sqrt{\frac{e}{8mU_0}} B_n (z - z_n) \right] \end{aligned} \quad (1.127)$$

Expression (1.127) is the equation of the trajectory (or rather of the projection of the trajectory) in the meridian plane within the interval from $z = z_n$ to $z = z_{n+1}$.

In order to determine the angle of turn ψ , we have to integrate the second equation of the system (1.98)

$$\psi(z)_{z_n, z_{n+1}} = \psi_n + \sqrt{\frac{e}{8mU_0}} \int_{z_n}^z B_n dz = \psi_n + \sqrt{\frac{e}{8mU_0}} B_n (z - z_n) \quad (1.128)$$

Substituting $z = z_{n+1}$ into (1.127) and (1.128), we obtain respectively, r_{n+1} , r'_{n+1} , ψ_{n+1} , i.e. the values of the departure of the trajectory from the axis, the angle of inclination of the trajectory and the angle of turn about the axis at the end of the n th interval. It is clear that the obtained values are the starting data for calculation of the trajectory in the next $(n + 1)$ interval. Using this technique in succession, we can construct a projection of the whole trajectory on the rotating meridian plane, while summing up the angles ψ calculated for each interval, we can determine the final angle of turn.

The accuracy of the above method of finding the trajectories depends on the selection of the value and the number of intervals into which the area of the field being investigated is divided. When the division is chosen correctly, it is possible to construct a trajectory with a relative error not over 1.5-2% which is usually admissible in solving practical problems.

The grapho-analytic methods of finding the trajectories of electrons in magnetic fields have found not so wide application as the case is with electrostatic fields. This is explained by some complication arising in the calculation and construction due to the necessity of finding the projection of the trajectory onto at least two planes, since in many practically important cases the trajectory of the electron moving in a magnetic field is a spatial (not plane) curve.

As an example, let us consider the method of grapho-analytic finding of the paths of the electrons moving in a magnetic field based on the determination of the radius of curvature of the trajectory projections on two perpendicular planes. It is clear that the projections of the spatial curve on two mutually perpendicular planes uniquely define the whole trajectory.

Take two perpendicular planes XOY and XOZ . In the case of an axisymmetric field it is expedient to select a meridian plane as one of the said planes, while the other plane will be perpendicular to the axis. Assume that the distribution of the magnetic field in the region in question is known (calculated or measured), and at any point (x, y, z) the components B_x, B_y, B_z of the magnetic induction can be defined. Also assume that in the region to be studied the electric field is absent, i.e. $U(x, y, z) = U_0 = \text{const}$. In this case the kinetic energy of the electron will be the same at all points

$$\frac{mv_0^2}{2} = eU_0$$

and the total velocity of the electron

$$v_0 = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{\frac{2e}{m} U_0}$$

If the tangents to the projections of the trajectory in the planes XOY and XOZ constitute angles α and β with regard to the axis OX , the velocity components v_x, v_y, v_z of the electron are described by the expressions

$$\begin{aligned} v_x &= \frac{v_0}{\sqrt{1 + \tan^2 \alpha + \tan^2 \beta}} \\ v_y &= v_x \tan \alpha \\ v_z &= v_x \tan \beta \end{aligned} \quad (1.129)$$

The radii of curvature of the projections of the trajectory on the planes XOY and XOZ can be found from the components of the velocity and acceleration

$$\begin{aligned} R_{xy} &= -\frac{(v_x^2 + v_y^2)^{3/2}}{v_y \frac{dv_x}{dt} - v_x \frac{dv_y}{dt}} \\ R_{xz} &= \frac{(v_x^2 + v_z^2)^{3/2}}{v_z \frac{dv_x}{dt} - v_x \frac{dv_z}{dt}} \end{aligned} \quad (1.130)$$

The accelerations dv_x/dt , dv_y/dt and dv_z/dt are calculated by using the equation of motion of an electron in a magnetic field [see (1.10)]

$$\left. \begin{aligned} \frac{dv_x}{dt} &= -\frac{e}{m} (v_y B_z - v_z B_y) \\ \frac{dv_y}{dt} &= -\frac{e}{m} (v_z B_x - v_x B_z) \\ \frac{dv_z}{dt} &= -\frac{e}{m} (v_x B_y - v_y B_x) \end{aligned} \right\} \quad (1.131)$$

Let the first equation of system (1.131) be multiplied by v_y and the second equation be multiplied by v_x and subtract the second equation from the first one

$$v_y \frac{dv_x}{dt} - v_x \frac{dv_y}{dt} = -\frac{e}{m} [(v_x^2 + v_y^2) B_z - v_z (v_x B_x + v_y B_y)] \quad (1.132)$$

Similarly, by combining the second and third equations of system (1.131), we obtain

$$v_z \frac{dv_x}{dt} - v_x \frac{dv_z}{dt} = -\frac{e}{m} [v_y (v_x B_x + v_z B_z) - (v_x^2 + v_z^2) B_y] \quad (1.132a)$$

Replacing

$$\begin{aligned} \sqrt{v_x^2 + v_y^2} &= v_{xy}, & \sqrt{v_x^2 + v_z^2} &= v_{xz} \\ v_x/v_{xy} &= \cos \alpha, & v_y/v_{xy} &= \sin \alpha \\ v_x/v_{xz} &= \cos \beta, & v_z/v_{xz} &= \sin \beta \end{aligned}$$

in equations (1.132) and substituting (1.132) and (1.132a) into (1.130), we obtain the final expression for the radii of curvature of the projections of the path of the electron moving in a magnetic field on the planes XOY and XOZ

$$\begin{aligned} R_{xy} &= \frac{v_{xy}^2}{-\frac{e}{m} [v_{xy} B_z - v_z (B_x \cos \alpha + B_y \sin \alpha)]} \\ R_{xz} &= \frac{v_{xz}^2}{\frac{e}{m} [-v_{xz} B_y + v_y (B_x \cos \beta + B_z \sin \beta)]} \end{aligned} \quad (1.133)$$

Using formulas (1.133), one can calculate the radii of curvature of the projections of the trajectory on the perpendicular planes by the known values of magnetic induction $B(x, y, z)$ and the electron velocity $v(x, y, z)$. The plot of the projections of the trajectory by the method of radii of curvature is effected in the following order (Fig. 1.33).

An initial point of the projection of the trajectory A_0 is selected on one of the planes, for example XOY . Proceeding from the starting conditions (the potential at the starting point, the entrance angle of the electron), the values of v_0 , v_{x0} , v_{y0} , v_{xy0} , as well as $\sin \alpha_0$ and $\cos \alpha_0$ are calculated. After that R_{xy0} is calculated by

the known value of magnetic induction at the starting point and by formula (1.133). The found value R_{xy0} is plotted on the perpendicular erected from the point A_0 to v_{xy} (point C_0). Taking

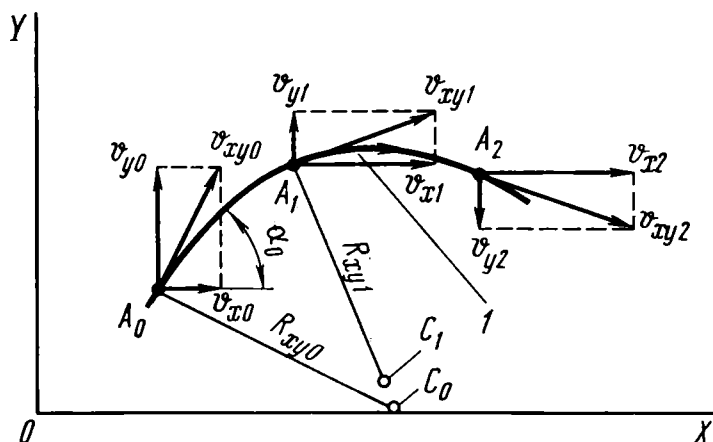


Fig. 1.33. Construction of electron trajectories by the method of radii of curvature
1—trajectory

the point C_0 for the centre of curvature, a circular arc A_0, A_1 is drawn from the starting point A_0 . It will serve as a part of the projection of the trajectory upon the plane XOY . Then the values

$v_{x1}, v_{y1}, v_{xy1}, \sin \alpha_1$ and $\cos \alpha_1$ are calculated for the point A_1 serving as a starting point of the new length of the projection of the trajectory, the value R_{xy1} is determined and the circular arc is drawn to the point A_2 to find R_{xy2} , and so on. Repeating these operations, it is possible to obtain the whole projection of the trajectory upon the plane XOY in the form of arcs of conjugate circles. The projection of the trajectory upon the plane XOZ is obtained in a similar manner.

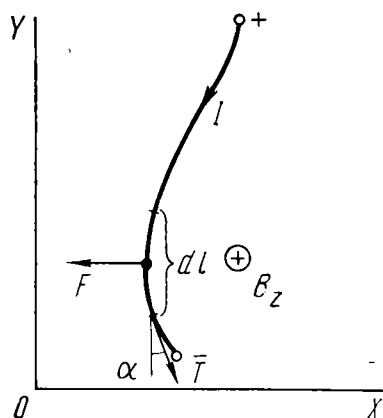


Fig. 1.34. Simulating electron trajectory by the flexible conductor method

The accuracy of plotting the trajectories by the above-considered method can be sufficiently high (the relative error does not exceed 3-5 %) provided that the points of conjugation of separate arcs are chosen correctly. Practically adequate accuracy corresponds to the case when the value of magnetic induction at the final point of each separate arc of the projection differs from the value B at the starting point of this arc for not more than 5-7 %.

One of the experimental methods of finding the electron trajectories in magnetic fields provides for simulating the trajectories by

means of a flexible current conductor. Consider a flexible conductor carrying current I located in a plane XOY (Fig. 1.34).

The direction of the magnetic field B_z is perpendicular to the plane XOY . The conductor element dl is acted upon by a ponderomotive force $F = B_z I dl$ tending to push the conductor out of the magnetic field. If the conductor is tensioned by a force T acting tangentially to the ends of the element dl , this element will be located along a curve whose radius R can be found from the condition of equality of the ponderomotive force and the radial component of the tensile force (by the absolute value). From Fig. 1.34 it is obvious that the radial component of the tensile force is equal to $2T \sin \alpha$. On the other hand, $\sin \alpha = dl/2R$, i.e. the radial force equalizing the ponderomotive force is equal to $T = \frac{dl}{R}$. From the equality of these forces follows the condition of equilibrium of a flexible current-carrying conductor placed into a transverse magnetic field

$$B_z I = \frac{T}{R} \quad (1.134)$$

The "equilibrium" radius of curvature is described by the following expression

$$\frac{1}{R} = \frac{IB_z}{T} \quad (1.135)$$

Formula (1.135) holds for any element of the conductor, therefore, using the known expression (1.59) for the radius of curvature, we obtain the differential equation of the curve positioned along which is the conductor tensioned by the force T and carrying the current I in the transverse magnetic field B_z

$$\frac{\frac{d^2y}{dx^2}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}} = \frac{IB_z}{T} \quad (1.136)$$

In the case of a homogeneous transverse magnetic field ($B_z = \text{const}$) the curve described by equation (1.136) is a circular arc.

Now use expression (1.13) for the radius of curvature of the trajectory of an electron in a transverse magnetic field: $R = mv/eB$. Comparing (1.13) with (1.135), one can see that with the equality $e/mv = I/T$ satisfied, the electron trajectory is described by the same equation (1.136) as the curve along which the current conductor is located. Thus, with the correspondingly selected values of the current I and the tensile force T the conductor placed into a magnetic field will locate itself along a curve coinciding with the electron trajectory having a pulse $p = mv = eT/I$.

It can be shown that the coincidence of the curve along which the current-carrying conductor is located with the electron trajectory

is valid for any magnetic field. The only condition that should be met when simulating the trajectory by a flexible conductor is the stability of the equilibrium of the conductor in the field. The analysis has shown that this condition is satisfied when there is no point of intersection in the region being investigated, i.e. there is no focus.

In practice the paths of the electrons in a magnetic field are simulated by means of a thin (0.03-0.05) mm in diameter copper or silver-plated wire. The small diameter of the flexible conductor is dictated by the necessity to ignore the weight and elasticity of the conductor. Furthermore, the conductor must have the lowest possible specific resistance in order to use measurable currents and fine wires. The use of copper or silver as a material for the conductor is due to this fact either. The wire is tensioned by a weight suspended to a wire carried by a pulley.

The method of flexible conductor is a simple and convenient tool for finding the spatial trajectories of electrons moving in nonhomogeneous magnetic fields. The accuracy of this method depends on many factors (diameter and tension of the wire, intensity of current, etc.) and under otherwise equal conditions it is the higher, the greater the value of magnetic induction. Therefore, where it is necessary to simulate the trajectories in weak fields, it is expedient to increase the magnetic induction and, correspondingly, recalculate the electron pulse.

1.8. Electron-optical Lenses

As it has been shown in Section 1.6, any nonhomogeneous electric or magnetic field having an axial symmetry acts upon electrons moving in the near-axis area as a light-optical lens acts on a light beam. In other words, the electron-optical lenses — nonhomogeneous axisymmetric electric and magnetic fields — can focus electron beams and produce electron-optical images *. The paths of the electrons in a nonhomogeneous axisymmetric field (lens field) can be found either by solving the fundamental equations or by any of the above-discussed approximate methods.

Of practical interest, however, is often not the electron trajectory in the electron lens but the final result of its action, i.e. the point of collection of the electrons having passed the lens; the scale (enlargement) of the object image and so on.

In the general case, as derived from light optics, the optical properties of a centred system are characterized by the position of the four cardinal elements of the lens, two principal foci (F_1 , F_2) and two principal planes (h_1 , h_2) (Fig. 1.35).

* A homogeneous longitudinal magnetic field can produce electron-optical images but, strictly speaking, is not an electron-optical lens (see page 55).

The space to the left from the lens is called the *object space* and that to the right is called the *image space*. If a beam of parallel rays arrives at the lens surface from the object space, then these rays are collected at the point F_2 of the image space focus. Parallel rays incident to the lens at the image space side are collected at the point F_1 or the object space focus. The principal planes h_1 and h_2 have the following property. The ray $abcd$ admitted from the object space to the image space is refracted so that the continuations of its rectilinear sections ab and cd intersect the planes h_1 and h_2 at points n and

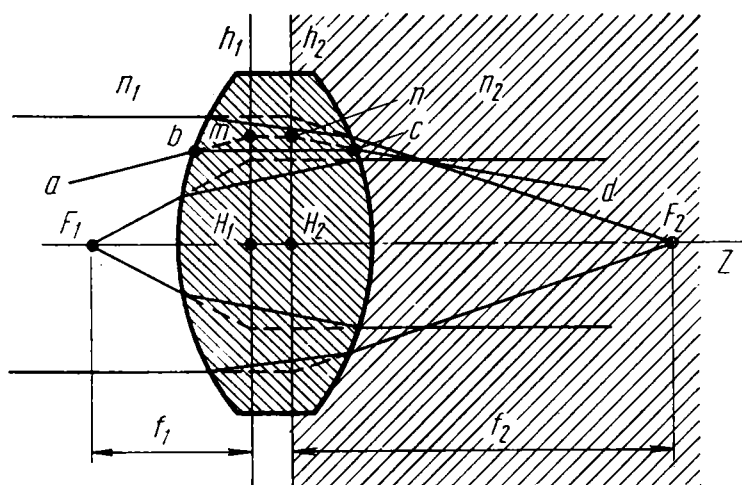


Fig. 1.35. Cardinal elements of lens

m equally spaced from the optical axis. This makes it possible to determine the position of the principal planes from the knowledge of the path of the two rays — one parallel to the axis in the object space and the other parallel to the axis in the image space. By continuing the straight-line portions of the rays lying beyond the lens to their intersection, we obtain in the first case the position of the plane h_2 and in the second case — the position of the plane h_1 . The distances from the principal planes to the corresponding foci are called the focal lengths $f_1 = F_1H_1$, $f_2 = F_2H_2$. The relation between the focal lengths of the object and image spaces is simple.

$$\frac{f_1}{f_2} = \frac{n_1}{n_2} \quad (1.137)$$

where n_1 and n_2 are the refractive indices of the object and image spaces.

Construction of images by the known position of cardinal points is accomplished as follows (Fig. 1.36).

Two rays are drawn from the point A — one parallel to the axis in the object space (ray AB) and the other through the focus F_1 (ray AC) until they intersect the principal planes h_2 and h_1 . Then

from the point B a ray is drawn through the focus F_2 , and from the point C another ray is drawn parallel to the axis in the image space. The point A' of intersection of these rays is an image of the point A . The other points of the object are imaged similarly.

Electron-optical lenses have cardinal points either, their properties being the same. If the area of the field forming a lens is limited

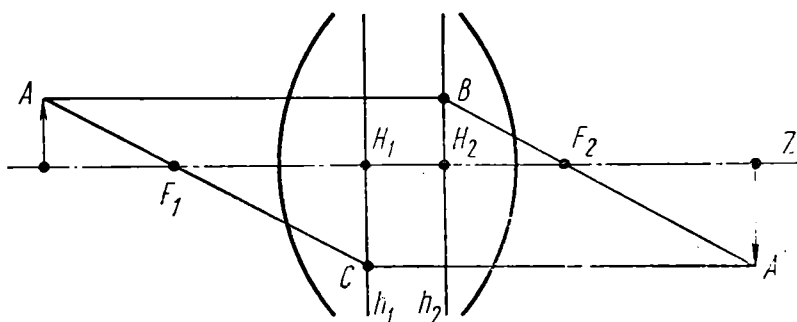


Fig. 1.36. Construction of image by known position of cardinal points

along the axis OZ , the electron trajectories beyond the lens will be sections of straight lines. Assume that an electron moves in the object

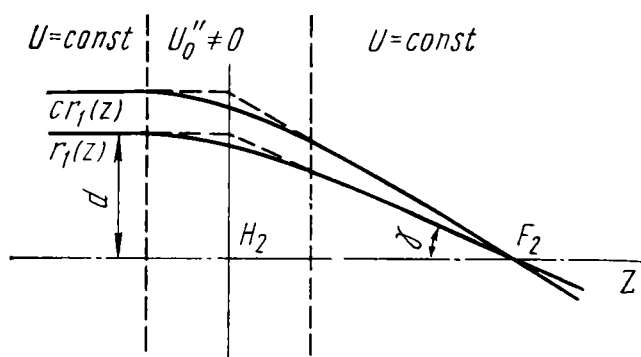


Fig. 1.37. Determining the cardinal points of electron lens

space along a straight-line trajectory $r_1(z)$ parallel to the optical axis (Fig. 1.37).

If the trajectory $r_1(z)$, after having passed the lens, intersects the axis, then owing to the linearity and homogeneity of the fundamental equation (1.82) any trajectory $r(z) = cr_1(z)$ parallel to the axis in the object space will intersect the axis OZ in the image space at the same point as the first trajectory. Thus, the point of intersection of the axis by the beam of trajectories parallel to the axis in the object space is the principal focus of the image space.

Now determine the coordinates of the plane in which the extensions of the straight-line sections of the trajectory $r_1(z)$ will intersect.

From Fig. 1.37 it follows directly that

$$z(F_2) - z(H_2) = -\frac{d}{\tan \gamma} \quad (1.138)$$

Let us take planes z_a and z_b located beyond the field of the lens in the object and image spaces. Then the values d and $\tan \gamma$ contained in (1.138) can be written as $r_1(z_a)$ and $r'_1(z_b)$ and the coordinate $z(H_2)$ is

$$z(H_2) = z(F_2) + \frac{r_1(z_a)}{r'_1(z_b)} \quad (1.139)$$

The plane in which the extensions of the straight-line sections of any trajectory $r(z) = cr_1(z)$ intersect is also determined by the coordinate $z(H_2)$, since the right-hand side of equation (1.139) is independent of c . Thus, the plane H_2 whose position is determined by

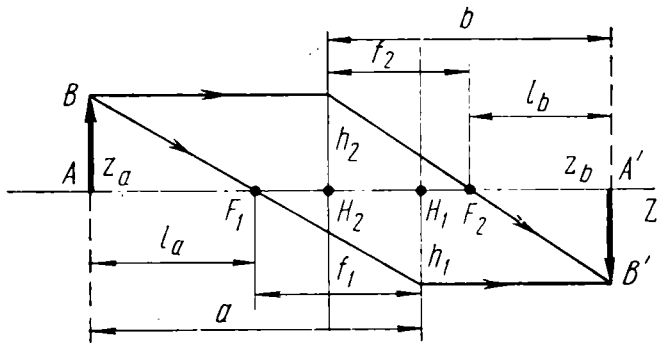


Fig. 1.38. Construction of image (for electron lens)

equation (1.139) is the principal plane of the image space. In a similar manner one can find the position of the principal focus F_1 and the principal plane H_1 of the object space.

It should be noted that in many cases electron lenses have both diverging and converging regions so that the first principal plane is shifted towards the image space and the second principal plane is shifted towards the object space. Thus, both principal planes often lie at one side of the mid-plane, the second principal plane appearing to the left from the first one. This intersection of the principal planes is typical of the most of electron lenses.

Assignment of four cardinal points F_1 , F_2 , H_1 , H_2 of an electron lens is sufficient to form an electron-optical image (Fig. 1.38).

The intersection of the principal planes does not change the sequence of the construction operations. Let the coordinates of the object and image planes be denoted through z_a and z_b , respectively. In this case the distances of the object and image from the corresponding foci are equal to $l_a = z(F_1)$ and $l_b = z_b - z(F_2)$. From Fig. 1.38 it follows that the linear magnification $M = A'B'/AB$ can be presen-

ted in the form

$$M = \frac{f_1}{l_a} = \frac{l_b}{f_2} \quad (1.140)$$

from which follows an expression known as the Newton equation describing the relationship between the positions of the object and the image

$$l_a l_b = f_1 f_2 \quad (1.141)$$

If the positions of the object and the image are determined by the distances a and b from the principal planes, then $a = l_a + f_1$, $b = l_b + f_2$, and from (1.140) we obtain the formula convenient for practical purposes

$$\frac{f_1}{a} + \frac{f_2}{b} = 1 \quad (1.142)$$

It should be shown once again that light-optical relation (1.137) is also valid for electron-optical systems. Assume that we have two linearly independent solutions of equation (1.82), i.e. $r_1(z)$ and $r_2(z)$. Substitute the values r_1 and r_2 into equation (1.81)

$$\left. \begin{aligned} \sqrt{U_0} \frac{d}{dz} \left(\sqrt{U_0} \frac{dr_1}{dz} \right) &= -\frac{1}{4} U_0'' r_1 \\ \sqrt{U_0} \frac{d}{dz} \left(\sqrt{U_0} \frac{dr_2}{dz} \right) &= \frac{1}{4} U_0'' r_2 \end{aligned} \right\} \quad (1.143)$$

Multiply the first equation by $r_2(z)$ and the second one by $r_1(z)$ and subtract the second from the first

$$\sqrt{U_0} \left[r_2 \frac{d}{dz} \left(\sqrt{U_0} \frac{dr_1}{dz} \right) - r_1 \frac{d}{dz} \left(\sqrt{U_0} \frac{dr_2}{dz} \right) \right] = 0 \quad (1.144)$$

Equation (1.144) can be presented in the form

$$\frac{d}{dz} \left[\sqrt{U_0} \left(r_2 \frac{dr_1}{dz} - r_1 \frac{dr_2}{dz} \right) \right] = 0 \quad (1.145)$$

Integration of this equation results in the expression

$$\sqrt{U_0} \left(r_2 \frac{dr_1}{dz} - r_1 \frac{dr_2}{dz} \right) = C \quad (1.146)$$

where C is a constant determined by the starting conditions.

Since $r_1(z)$ and $r_2(z)$ are linearly independent, then on the basis of (1.146) we can write

$$\begin{aligned} \sqrt{U_a} [r_2(z_a) r_1'(z_a) - r_1(z_a) r_2'(z_a)] &= \\ = \sqrt{U_b} [r_2(z_b) r_1'(z_b) - r_1(z_b) r_2'(z_b)] \end{aligned} \quad (1.147)$$

where $r' = \frac{dr}{dz}$; U_a and U_b are constant values of the potential in the object and image spaces; z_a and z_b are the coordinates of the object and image planes.

If the solutions $r_1(z)$ and $r_2(z)$ are selected so that the ray r_1 is parallel to the axis in the object space, while the ray r_2 is parallel to the axis in the image space, then

$$r'_1(z_a) = 0 \quad \text{and} \quad r_2(z_b) = 0$$

and from (1.147) it follows

$$-\sqrt{U_a} r_1(z_a) r'_2(z_a) = \sqrt{U_b} r_2(z_b) r'_1(z_b) \quad (1.148)$$

The expression (1.139) allows the focal length in the image space to be calculated

$$f_2 = z(H_2) - z(F_2) = \frac{r_1(z_a)}{r'_1(z_b)} \quad (1.149)$$

Similarly, the focal length in the object space

$$f_1 = z(H_1) - z(F_1) = \frac{r_2(z_b)}{r'_2(z_a)} \quad (1.150)$$

The relationships (1.149) and (1.150) also stem from the geometric

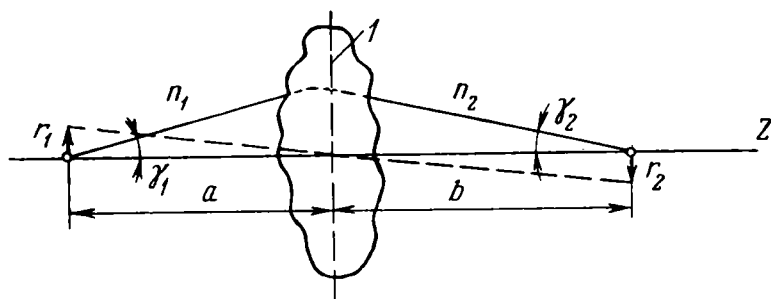


Fig. 1.39. Derivation of the Langrange—Helmholtz equation
1—focusing field

constructions in Fig. 1.38. Setting the ratio f_2/f_1 and using (1.148) we obtain the sought dependence

$$\frac{f_1}{f_2} = \frac{\sqrt{U_a}}{\sqrt{U_b}} \quad (1.151)$$

where $\sqrt{U_a} = n_1$, $\sqrt{U_b} = n_2$ are the refractive indices in the object and image spaces.

Thus, the electron-optical relation (1.151) completely agrees with the light-optical formula (1.137).

Widely used for analysis of light-optical systems is a relation between the refractive indices (n_1 and n_2), the removal of the object points from the axis (r_1 and r_2) and aperture angles (γ_1 and γ_2) in the object and image spaces (Fig. 1.39)

$$r_1 n_1 \gamma_1 = r_2 n_2 \gamma_2 \quad (1.152)$$

called the Lagrange — Helmholtz equation. This relation is a particular case of the known Abbe “sine condition”. It is valid for elec-

tron-optical systems too. Indeed, selecting one of the particular solutions so that $r_1(z_a) = 0$ and $r_1(z_b) = 0$ (see Section 1.5) from (1.147), we obtain

$$\sqrt{U_a} r_2(z_a) r'_1(z_a) = \sqrt{U_b} r_2(z_b) r'_1(z_b) \quad (1.153)$$

Denoting $\sqrt{U_a} = n_1 = \sqrt{U_1}$, $\sqrt{U_b} = n_2 = \sqrt{U_2}$ (the refractive indices in the object and image spaces) and assuming that $r_2(z_a) = r_1$ and $r_2(z_b) = r_2$ (the points of intersection of the ray $r_2(z)$ with the object and image planes), we find the removal of the respective points of the object and image from the axis and, finally, replacing $\tan \gamma_1 = r'_1(z_a)$ and $\tan \gamma_2 = r'_1(z_b)$ by the values of the angles γ_1 and γ_2 (what is quite admissible within the scope of paraxial optics), we obtain the expression

$$\sqrt{U_1} r_1 \gamma_1 = \sqrt{U_2} r_2 \gamma_2 \quad (1.154)$$

coinciding with the Lagrange — Helmholtz equation.

The analytic calculation of the position of the cardinal points of electron-optical lenses in the general case is difficult and often approximate methods are used or the position of the principal focus and the principal planes are found by plotting the trajectories of electrons graphically. This problem, however, is much simplified, if the electron-optical lens is considered as a thin one. In light optics the lens is called thin if its thickness is small as compared to its focal length. Similarly, in electron optics thin electron-optical lenses are those whose thickness, i.e. the length of the field along the axis, is small as compared to the focal length. Of course, division of electron-optical lenses into thin and thick is somewhat arbitrary, since the field diminishes gradually and is not zero even at a considerable distance from the mid-plane due to the continuity of the potential function in a space free from electrical charges. It has been agreed upon that the lens proper is only that portion of the field which has a significant effect on the electron trajectories. Using this criterion, a perceptible number of practically employed electron-optical lenses may be considered as thin ones. A characteristic feature of a thin lens consists in very short distances between both principal planes and between each principal plane and the mid-plane of the lens. Therefore, when studying the thin lenses, it is quite possible to consider both of the principal planes as coinciding with the mid-plane of the lens and to count the focal lengths, as well as the distances to the object and image from the mid-plane of the lens.

In the case of a thin lens we may disregard the path of the electrons within the lens itself and consider that the direction of the electron trajectory abruptly changes when passing the lens. In a similar manner we may regard with approximation the variation of the electron

trajectory distance from the axis inside a thin lens so small that it can be considered constant (Fig. 1.40).

Under these assertions it is possible to integrate the fundamental equation (1.81) within the range from z_a (the object plane) to z_b (the image plane)

$$\sqrt{U_0(z_b)} \frac{dr}{dz} \Big|_{z_b} - \sqrt{U_0(z_a)} \frac{dr}{dz} \Big|_{z_a} = -\frac{r_0}{4} \int_{z_a}^{z_b} \frac{U_0''(z)}{\sqrt{U_0(z)}} dz \quad (1.155)$$

where r_0 is the constant distance between the electron trajectory and the axis within the lens area.

Now let us assume that the electron rays arrive at the lens from the object side in the form of a beam parallel to the axis ($z_a \rightarrow \infty$).

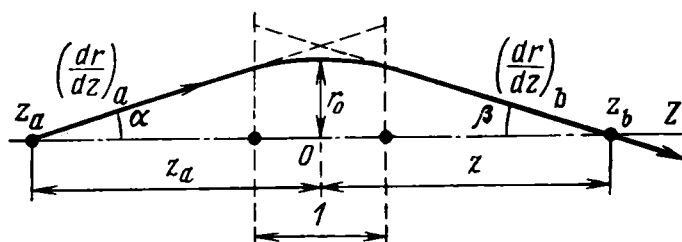


Fig. 1.40. Ray path in thin lens
1—lens

In this case the point of intersection of the converging beam of trajectories with the axis to the right of the lens is the principal focus of the image space, and the distance from the mid-plane to the focus is the focal length f_2 . From Fig. 1.40 it is clear that

$$f_2 = \frac{r_0}{\tan \beta}; \quad \tan \beta = -\frac{dr}{dz} \Big|_{z_b}$$

Thus, the focal length of a thin electrostatic lens is described by the expression

$$\frac{1}{f_2} = \frac{1}{4\sqrt{U_b}} \int_{-\infty}^{+\infty} \frac{U_0''(z)}{\sqrt{U_0(z)}} dz \quad (1.156)$$

where $U_b = U_0(z_b)$ is the potential in the image space and the limits of integration are transferred to infinity since the value $U_0''(z)$ differs from zero only in the domain of the lens and the replacement z_a and z_b by $\pm \infty$ does not change the integral value.

In a similar way, the focal length of the object space (with $z_b \rightarrow \infty$) is described by the formula

$$\frac{1}{f_1} = 4 \frac{1}{\sqrt{U_a}} \int_{-\infty}^{+\infty} \frac{U_0''(z)}{\sqrt{U_0(z)}} dz \quad (1.157)$$

The obtained expressions are starting data for calculation of optical parameters of thin electrostatic lenses. It should be noted that relation (1.151) can be obtained directly by dividing (1.156) by (1.157). Furthermore, combining (1.155), (1.156) and (1.157) and noting that $r_0 \left| \frac{dr}{dz} \right|_{z_a} = a$, $r_0 \left| \frac{dr}{dz} \right|_{z_b} = b$ (the distances from the object and image to the mid-plane), we obtain the Newton equation (1.142).

When using equations (1.156) and (1.157), one has to calculate the values of the second derivative of the axial potential $U_0''(z)$ which sometimes is not easy. For which reason the expression of the optical power ($1/f$) of a thin lens can be transformed by using the known technique of integration by parts. Assume that $1/U_0(z) = u$ and $U_0''(z) \times dz = dv$ and perform integration (1.156) by parts

$$\frac{1}{f_2} = \frac{1}{4 \sqrt{U_b}} \left\{ \frac{U_0'(z)}{\sqrt{U_0(z)}} \right\}_{-\infty}^{+\infty} + \frac{1}{2} \int_{-\infty}^{+\infty} \frac{[U_0'(z)]^2}{[U_0(z)]^{3/2}} dz \right\} \quad (1.158)$$

Since the potentials are constant outside the lens, the derivative $U_0'(z)$ in the first term is equal to zero outside the lens and

$$\frac{1}{f_2} = \frac{1}{8 \sqrt{U_b}} \int_{-\infty}^{+\infty} \frac{[U_0'(z)]^2}{[U_0(z)]^{3/2}} dz \quad (1.159)$$

Using the substitution

$$r(z) = \frac{R(z)}{[U_0(z)]^{1/4}}$$

we can obtain an expression convenient for calculation of the focal length of a thin electrostatic lens.

Such a substitution brings the fundamental equation (1.81) to the form

$$\frac{d^2 R}{dz^2} = -\frac{3}{16} \left[\frac{U_0'(z)}{U_0(z)} \right]^2 R \quad (1.160)$$

Assuming, as before, that R is constant inside the lens and is equal to R_0 , integrate equation (1.160)

$$R'(z_b) - R'(z_a) = -\frac{3}{16} R_0 \int_{-\infty}^{+\infty} \left[\frac{U_0'(z)}{U_0(z)} \right]^2 dz \quad (1.161)$$

With $z_a \rightarrow \infty$ (the parallel beam from the object side) equation (1.161) makes it possible to determine the focal length in the image space

$$\frac{1}{f_2} = \frac{3}{16} \left(\frac{U_a}{U_b} \right)^{1/4} \int_{-\infty}^{+\infty} \left[\frac{U_0'(z)}{U_0(z)} \right]^2 dz \quad (1.162)$$

The focal length in the object space is determined in a similar way.

Equations (1.159) and (1.162) show that the electrostatic electron-optical lenses adjoining at both sides the constant potential domains always are converging lenses. Indeed, the values of potential in these equations cannot be negative since with $U_0 < 0$ the lens turns into a mirror (see page 97), while the only value that can be negative (U'_0) is squared. Thus, $1/f > 0$, the optical power of the system is positive, the lens is converging. In other words, it is impossible to develop a diverging electron-optical lens with domains of constant potential at both sides of this lens. When the object or the image or both are within the field of the lens ($U'_a \neq 0$ or $U'_b \neq 0$), the lens

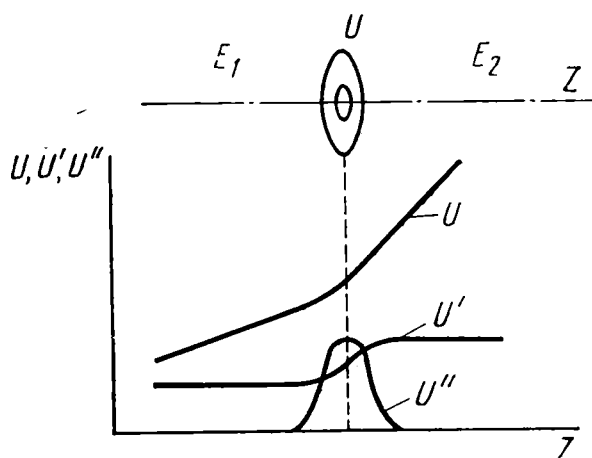


Fig. 1.41. Aperture lens

can be diverging and expressions (1.159) and (1.162) turn to be invalid.

Let actual types of electrostatic electron-optical lenses be considered. In compliance with the character of potential distribution along the system axis there are known three types of electrostatic lenses: (1) aperture lenses, (2) biopotential (immersion) lenses, (3) unipotential lenses.

The *aperture lens* (Fig. 1.41) is formed by a disc-shaped electrode with a circular aperture at a potential U_a . Fields with constant but different potentials E_1 and E_2 act on both sides of the electrode. In some particular case one of these fields may be absent (E_1 or E_2 is zero). It is clear that near the aperture the field intensity will vary along the axis ($U''_0 \neq 0$). It is this region of the field that forms the electrostatic lens proper.

The optical power of an aperture lens can be easily calculated with some approximation. Since the potential near the aperture varies insignificantly, we may in a first approximation transfer the term $\sqrt{U_0}$ in expression (1.156) from under the integral sign. In this case the aperture lens power is calculated by the following approximate

formula

$$\frac{1}{f} \approx \frac{1}{4U_0} [U'_0]_{z_a}^{z_b} = \frac{1}{4U_d} (U'_2 - U'_1) \quad (1.163)$$

where U'_1 and U'_2 are potential gradients to the left and right of the diaphragm.

Taking into account that $U' = dU/dz = -E_z$, formula (1.163) may be conveniently rewritten in the form

$$\frac{1}{f} = \frac{|E_2| - |E_1|}{4U_d} \quad (1.164)$$

where E_1 and E_2 are the intensities of the electric field on the two sides of the aperture.

Depending on the relation of the absolute values of the field intensity, the aperture lens can be converging (positive) or diverging (negative). In this case, when an electron passes from the domain with a lower intensity to the domain of a higher intensity of the field, the lens is converging. The direction of the force acting on the electron in the lens domain is defined by the sign of the second derivative $U''_0(z)$ (see Section 1.6). Figure 1.41 shows the graphs $U_0(z)$, $U'_0(z)$ and $U''_0(z)$ for the case $U'_2 > U'_1$.

As seen from the drawing, with $|E_2| > |E_1|$, $U''_0 > 0$ the lens is converging and with $|E_1| > |E_2|$, $U''_0 < 0$, the lens is diverging.

The potential distribution along the axis of the aperture lens is described with a sufficient accuracy by the expression

$$U_0(z) = U_d + \frac{R_d}{\pi} (E_1 - E_2) \left(1 + \frac{z}{R_d} \arctan \frac{z}{R_d} \right) - \frac{1}{2} (E_1 - E_2) z \quad (1.165)$$

where R_d is the radius of the aperture, while z is measured from the point of intersection of the aperture plane with the axis.

It is clear from expression (1.165) that the potential on the axis in the centre of the aperture ($z = 0$) cannot be equal to the potential of the electrode itself

$$U_0(0) = U_d + \frac{R_d}{\pi} (E_1 - E_2) \quad (1.166)$$

The case $E_1 = E_2$ has no practical interest since in this event the optical power of the lens is zero and the lens vanishes leaving a domain of a homogeneous electric field having no properties of a lens. Since the electrode is adjoined at both sides by the fields with an intensity differing from zero, the positions of the foci calculated by formula (1.164) will not correspond to the points of intersection of the trajectories of the electrons incident to the lens in parallel with the axis. This is explained by the fact that in a homogeneous field the electrons intersecting equipotential surfaces at some angle are subjected to a deflecting force. Therefore, the trajectories beyond

the lens are not straight lines but sections of parabolas (see Section 1.2). Only when the field intensity at one side of the lens is equal to zero, the position of the focus defined by formula (1.164) coincides with the point of intersection of the electron beam trajectories parallel to the axis at the other side of the lens. In the general case the positions of the foci calculated by formula (1.164) coincide with the

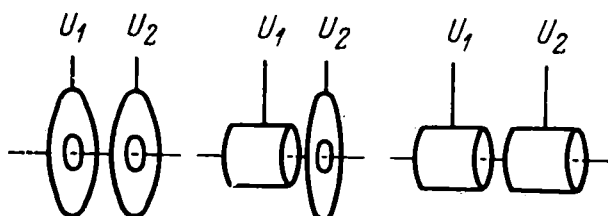


Fig. 1.42. Bipotential lens

points of intersection of the axis by the tangents to the paths of the electrons at the lens outlet.

The aperture lens has a limited application as a self-contained focusing device but it is often used as component of more complicated electron-optical systems.

The *bipotential lens* is usually formed by two coaxial apertures, an aperture and a cylinder or by two coaxial cylinders at different potentials (Fig. 1.42), the potential at both sides of the lens being constant and equal to that of the electrodes forming the lens ($U_1 \neq U_2$; U_1 and $U_2 = \text{const}$).

The diagrams of variations of the potentials, as well as their first and second derivatives are shown in Fig. 1.43.

When the electrons pass through a bipotential lens, their velocity varies: at $U_2 > U_1$ the lens is accelerating and at $U_1 > U_2$ it is decelerating.

From Fig. 1.43 it is clear that the bipotential lens has zones with a positive and a negative second derivative of the potential $U_0''(z)$, i.e. it has a converging and diverging portions of the field. However, the particle velocity determined by the potential value is higher in that zone where $U_0''(z) < 0$, i.e. the diverging region is passed by the electron in a shorter time with the result that an impulse imposed upon this electron radially towards the axis turns to be greater than that directed from the axis. Thus the converging action is predominant. Therefore, a typical bipotential lens is a converging lens. In some cases the action caused by the converging region can be reduced, i.e. it is possible to produce a diverging bipotential lens by reshaping

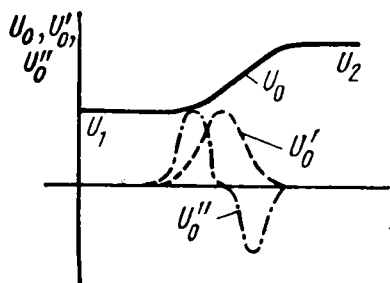


Fig. 1.43. Field of bipotential lens

the electrodes or placing into the lens a fine-mesh grid at a specified potential.

The optical power of a bipotential lens can be calculated by the general formulas provided that the potential distribution along the axis is known. The potential distribution for bipotential lens formed by two cylinders with a similar radius R at potentials U_1 and U_2 at a distance between the cylinders much less than the radius was given in Section 1.3 [see (1.31)]. Designate the relation U_1/U_2 as γ . In this case expression (1.31) assumes the form

$$U_0(z) = \frac{U_1 + U_2}{2} \left[1 + \frac{1 - \gamma}{1 + \gamma} \tanh \left(1.315 \frac{z}{R} \right) \right] \quad (1.167)$$

Substitution of (1.167) into (1.82) results in a differential equation which cannot be solved in its general form. However, making some simplifications, we can show that with the potential distribution of the form (1.167) it is possible to obtain an accuracy sufficient for practical purposes with γ ranging from $\gamma \geq 0.1$ to $\gamma \leq 10$ when determining the optical power of the lens formed by two cylinders similar in radius by the following formulas

$$\left. \begin{aligned} \frac{1}{f_1} &= R\gamma^{-1/4} \left(\frac{1}{2} \times \frac{\gamma+1}{\gamma-1} \ln \gamma - 1 \right) \\ \frac{1}{f_2} &= R\gamma^{1/4} \left(\frac{1}{2} \times \frac{\gamma+1}{\gamma-1} \ln \gamma - 1 \right) \end{aligned} \right\} \quad (1.168)$$

Since in the lens in question the field permeates the cylinders rather deeply, this lens cannot be always considered as thin. It is expedient, therefore, to read the focal lengths not from the mid-plane of the lens but from the principal planes whose position can be calculated in the following manner

$$\left. \begin{aligned} z(H_1) &= 0.19 \ln \gamma - \frac{\gamma^{-1/4} - 1}{\frac{1}{2} \times \frac{\gamma+1}{\gamma-1} \ln \gamma - 1} \\ z(H_2) &= 0.19 \ln \gamma - \frac{1 - \gamma^{1/4}}{\frac{1}{2} \times \frac{\gamma+1}{\gamma-1} \ln \gamma - 1} \end{aligned} \right\} \quad (1.169)$$

Expressions (1.169) show that in the bipotential lens the principal planes intersect one another and lie at one side of the lens mid-plane.

The bipotential lens formed by two apertures having an opening with the radius R , spaced at a distance d , and having potentials U_1 and U_2 (Fig. 1.44) in a first approximation can be regarded as a combination of two aperture lenses.

In this case the potential distribution in the space between the apertures can be considered as linear

$$U_0(z) \Big|_{z_1}^{z_2} = U_1 + \frac{U_2 - U_1}{d} z \quad (1.170)$$

and the field intensity between the apertures, constant

$$E \Big|_{z_1}^{z_2} = \frac{U_1 - U_2}{d} = -U'_{12} = \text{const} \quad (1.171)$$

Outside the lens, i.e. at both sides of the apertures, the field is zero, while the potentials are constant and equal respectively to U_1 and U_2 . There are transition regions near the apertures in which $U_0''(z) \neq 0$ and $r'(z) \neq 0$. However, due to small extent of the transition regions along the axis OZ we may consider the potentials U_1 and U_2 in these zones constant as well as the distance between the electron and the axis (r_1 and r_2).

Let the boundaries of the transition zones be denoted z_{11} , z_{12} , and z_{21} , z_{22} . In this case the angles of inclination of the trajectories to the axis at the inlet of the first transition region (near the first aperture) are equal to r'_{11} , at the outlet of the first transition region to r'_{12} , at the inlet and outlet of the second transition region (near the other aperture) to r'_{21} and r'_{22} . Since $U_0(z)$ is considered constant in the transition regions, the fundamental equation (1.82) for these regions assumes the form

$$\frac{d^2 r}{dz^2} = -\frac{1}{4} \times \frac{U_0''(z)}{U_0(z)} r \quad (1.172)$$

Integration of this equation within the limits z_{11} , z_{12} yields the following equations:

for the first transition region

$$r'_{12} - r'_{11} = -\frac{r_1}{4U_1} U'_{12} \quad (1.173)$$

for the second transition region

$$r'_{22} - r'_{21} = \frac{r_2}{4U_2} U'_{12} \quad (1.174)$$

Let us study the trajectory of an electron entering the lens area parallel to the axis. In this case $r'_{11} = 0$. Then

$$r'_{12} = -\frac{r_1}{4U_1} U'_{12} \quad (1.175)$$

Between the apertures (between the transition regions) $U_0'(z) = U'_{12} = \text{const}$ and $U_0'' = 0$. Consequently, the equation (1.81)

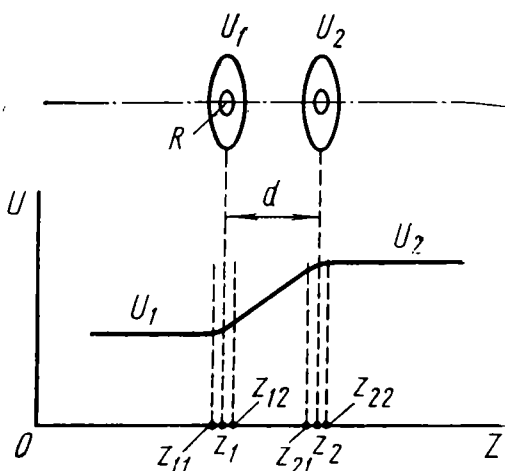


Fig. 1.44. Bipotential lens of two apertures

takes the form

$$\frac{d}{dz} \left(\sqrt{U_0} \frac{dr}{dz} \right) = 0 \quad (1.176)$$

and

$$\sqrt{U_0} \frac{dr}{dz} \bigg|_{z_1}^{z_2} = \text{const} = \sqrt{U_1} r'_{12} \quad (1.177)$$

Using the linearity condition of potential distribution (1.170), transform equation (1.177) into the form

$$dr = \frac{2 \sqrt{U_1}}{U'_{12}} \times \frac{dU_0}{2 \sqrt{U_0}} r'_{12} = \frac{2 \sqrt{U_1}}{U'_{12}} d(\sqrt{U_0} r'_{12}) \quad (1.178)$$

Integration of equation (1.178) within the limits z_1, z_2 results in the expression for the departure of the electron from the axis at the lens outlet

$$r_2 = r_1 + \frac{2 \sqrt{U_1}}{U'_{12}} (\sqrt{U_2} - \sqrt{U_1}) r'_{12} \quad (1.179)$$

Substituting the value of r'_{12} from (1.175), we obtain

$$r_2 = \frac{r_1}{2} \left(3 - \frac{\sqrt{U_2}}{\sqrt{U_1}} \right) \quad (1.180)$$

On the basis of equations (1.175) and (1.177) we can write

$$\sqrt{U_2} r'_{21} = \sqrt{U_1} r'_{12} = -\frac{1}{4} \times \frac{U'_{12}}{4 \sqrt{U_1}} r_1 \quad (1.181)$$

The focal length can now be regarded equal to $f_2 = r_2 / \tan \gamma_2 = r_2 / r'_{22}$ for the image space and to $f_1 = -r_1 / \tan \gamma_1 = r_1 / r'_{11}$ for the object space.

Determining r_2, r'_{22} from (1.180) and (1.174) with due regard to (1.181), we obtain the final expression for the optical power of a bi-potential lens formed by two apertures

$$\left. \begin{aligned} \frac{1}{f_1} &= \frac{3}{8d} \times \frac{U_1 - U_2}{U_1} \left(\sqrt{\frac{U_1}{U_2}} - 1 \right) \\ \frac{1}{f_2} &= \frac{3}{8d} \times \frac{U_2 - U_1}{U_2} \left(\sqrt{\frac{U_2}{U_1}} - 1 \right) \end{aligned} \right\} \quad (1.182)$$

The position of the principal planes can be calculated by the formulas

$$\left. \begin{aligned} z(H_1) &= z_2 - \frac{4d}{3} \times \frac{U_2}{U_2 - U_1} \\ z(H_2) &= z_1 - \frac{4d}{3} \times \frac{U_1}{U_2 - U_1} \end{aligned} \right\} \quad (1.183)$$

The above-mentioned simple expressions for the optical parameters of a bipotential lens give satisfactory practical accuracy when

there is even a weak homogeneous field between the apertures. This condition is satisfied the better, the greater is the distance between the apertures. Formulas (1.182) and (1.183) can practically be employed when $d \gg 2R$.

Formulas (1.182) can be simplified for a weak bipotential lens. Since the optical power of a bipotential lens depends substantially on the electrode potentials ratio U_2/U_1 , it is clear that for a weak lens this ratio is close to unity, i.e. $(U_2 - U_1)/U_1 \ll 1$. In this case the right-hand side of equations (1.182) can be expanded in a series by the powers of the ratio $(U_2 - U_1)/U_1$ and the terms containing this ratio in powers higher than the first one may be neglected. Then the optical power of the weak immersion lens formed by two apertures is given by the expression

$$\frac{1}{f} \approx \frac{3}{16d} \left(\frac{U_2 - U_1}{U_1} \right)^2 \quad (1.184)$$

The approximate expression (1.184) can be used at $\frac{U_2 - U_1}{U_1} \leq 0.2$. From (1.184) it follows directly that the bipotential lens is always a converging lens, $1/f > 0$, regardless of the sign of the difference $U_2 - U_1$.

It is interesting to note that approximate estimation of the focal length of a bipotential lens formed by two cylinders of the same radius R at the potentials U_1 and U_2 can be made by using a semiempiric formula similar to the analytic expression (1.184):

$$f = kR \left(\frac{U_1}{U_2 - U_1} \right)^2 \quad (1.185)$$

where k is a constant close to 10.

Analytic calculation of bipotential lenses formed by two cylinders of different radii or by a cylinder and an aperture is rather difficult, since the Laplace equation for such systems is solved analytically in a very complicated way. Therefore, on using such lenses, it is necessary to refer either to experimental data or to the diagrams obtained for identical bipotential lenses of some other types by the method of numerical integration. Widely used in practice are the so-called p - q curves making it possible to determine the distance to the image (q) and the transverse magnification of the lens (M) by the specified distance from the object (p) and the electrode potentials ratio (U_2/U_1). Given in Fig. 1.45 as an example are the p - q curves for a bipotential lens formed by two cylinders of different radii. Figure 1.46 illustrates the same curves for a bipotential lens formed by a cylinder and an aperture, the opening radius of the aperture being equal to $0.38 R_1$ (cylinder radius) and the width of the gap between the cylinder and the aperture being equal to $1.33 R_1$.

The given formulas and diagrams show that the optical power of bipotential lenses greatly depends on the potential ratio U_2/U_1 .

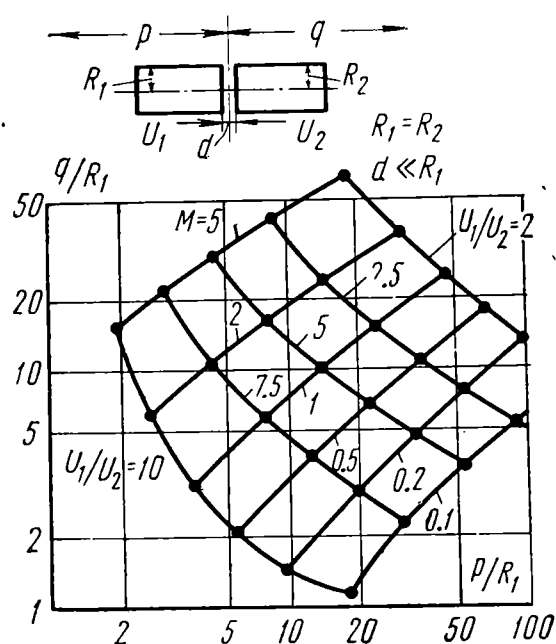


Fig. 1.45. Graphs for determining parameters of bipotential lens formed by two cylinders

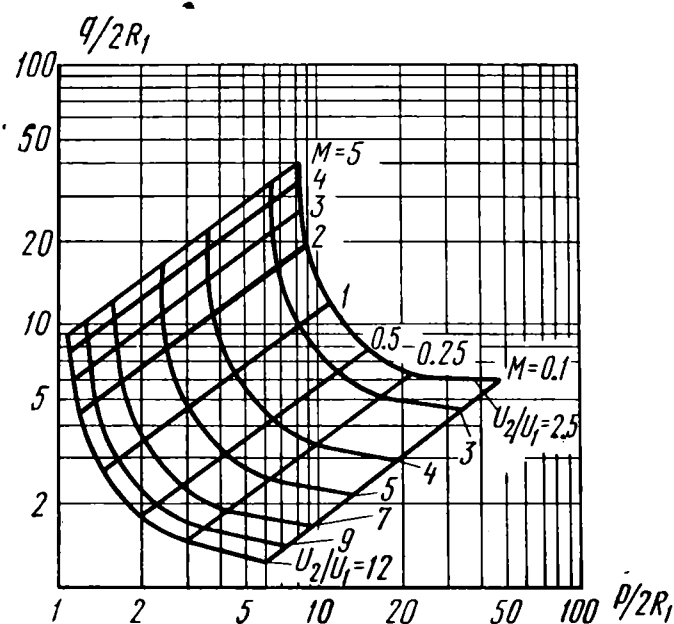


Fig. 1.46. Graphs for determining parameters of bipotential lens formed by aperture and cylinder

of the electrodes. The optical properties of the lenses are effected by the geometric proportions, particularly the radius of the cylinder (or the aperture opening), at a lower potential. In the case of lenses formed by two cylinders the focal length varies almost in proportion to the variation of the radius of the cylinder at a lower potential. At the same time, a change in the width of the gap between the cylinders

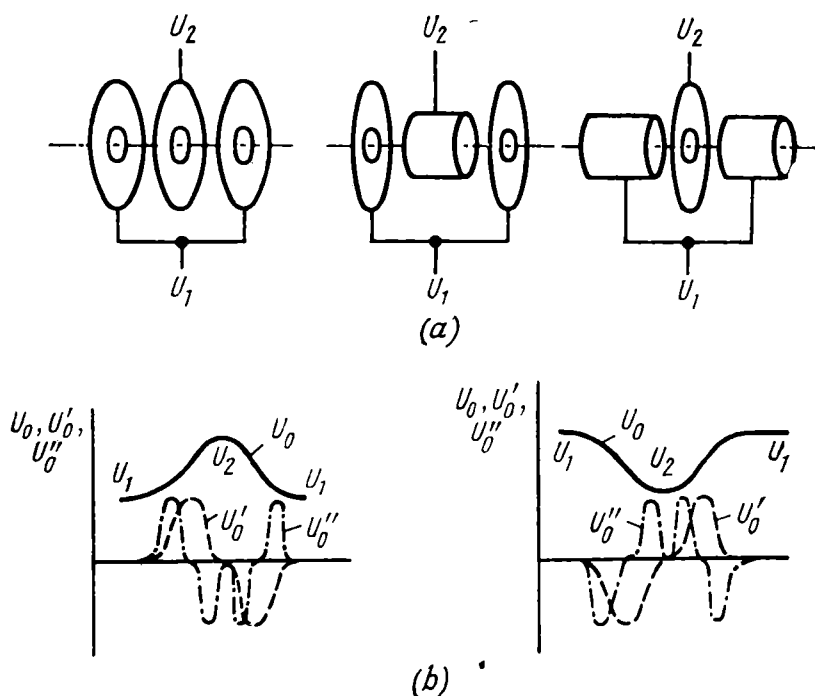


Fig. 1.47. Unipotential lens

(a) electrode systems; (b) distribution of potential and its derivatives

(or between the cylinder and aperture) has a far less influence on the optical properties of the lens up to the values of the gap width commensurable with the radius of the cylinders.

Widely used in electron-beam devices is a bipotential objective, which is a variety of a bipotential lens. A characteristic feature of this device is that the object (usually an electron emitter) is positioned in the field area of the lens. Since the bipotential objective is one of the essentials of an electron gun, it is discussed in detail in Chapter 4.

The *unipotential lens* (Fig. 1.47a) is formed by three coaxial electrodes (apertures or cylinders), the outer electrodes being at a common potential U_1 , and the centre electrode at a different, lower or higher potential U_2 . If the outer electrodes are of the same shape and are located at the same distance from the centre electrode, then the field of the lens is symmetric relative to the mid-plane of the lens. Such a unipotential lens is called symmetric. The electrode systems

shown in Fig. 1.47a form symmetric unipotential lenses. When the outer electrodes are not identical or are unequally spaced from the centre electrode, the unipotential lens is asymmetrical. The unipotential lens is characterized by the equality of the potentials on the outer electrodes. As a result the energy of electrons passing through this lens does not change. It is the velocity vector direction that changes. The equality of the potentials on the outer electrodes results in the opposite directions of the electric field at the two sides of the mid-plane (plane of symmetry) of the lens. Therefore, the unipotential lens field always features a saddle-shaped point (see Fig. 1.16). In the case of a symmetric unipotential lens this point coincides with the point of intersection of the mid-plane and the axis.

As shown in Fig. 1.47b the unipotential lens has converging and diverging regions ($U_0'' > 0$ and $U_0'' < 0$). However, as the case is with the bipotential lens, the electron velocity is higher in the diverging regions and the converging action is predominant. The result is that the optical power of the unipotential lens is always positive.

The optical parameters of a unipotential lens greatly depend on the potential ratio U_2/U_1 of the centre and outer electrodes. The greater this ratio differs from unity, the "stronger" is the lens. With $U_2/U_1 = 1$ the optical power of a unipotential lens is equal to zero. Since the equipotential regions lie at both sides of the unipotential lens, the focal lengths in the object and image spaces are equal to each other. The principal planes of the unipotential lens are also intersecting, but in contrast to the bipotential lens, the principal planes of the symmetric unipotential lens are located at the same distance from the mid-plane.

The positions of the foci and the principal planes of a unipotential lens can be determined by the general formulas. Since the analytic expressions for the potential distribution along the axis of a unipotential lens are too complicated, the unipotential lenses are usually calculated by means of approximate expressions dealing with axial potential distribution and focal length.

For a unipotential symmetric lens of three apertures with opening radii R_1 (the outer electrodes) and R_2 (the centre aperture), provided the distance d between the apertures is not less than the radii R_1 , R_2 of the apertures, the potential at the centre of the lens and at the centres of the outer apertures can be expressed by the following approximations

$$\left. \begin{aligned} U_0(0) &\approx U_2 + \frac{U_1 - U_2}{1 + \frac{d}{R_2} \arctan \frac{d}{R_2}} \\ U_0(z_d) &\approx U_1 - \frac{U_1 - U_2}{2R_2} \times \frac{R_1}{1 + \frac{d}{R_2} \arctan \frac{d}{R_2}} \end{aligned} \right\} \quad (1.186)$$

Expressions (1.186) are more convenient since the relation $\kappa = \frac{U_0(0)}{U_0(z_d)}$ determines the focal length of a symmetric unipotential lens formed by three apertures with an accuracy sufficient for practical purposes. This focal length is expressed by

$$f = \frac{8}{3} \times \frac{\kappa d}{(1-\kappa)^2} \quad (1.187)$$

The experimentally determined value of the focal length of a unipotential lens consisting of three apertures satisfactorily complies with the value f calculated by the formula (1.187) when $f > d$, i.e. in case of comparatively weak lenses. Obviously the requirement that $f > d$ results in $\kappa > 0.2$ which is usually the criterion for judgment if formula (1.187) is applicable. When constructing images, we may approximately consider that the principal planes of a unipotential lens coincide with the mid-plane (the plane of symmetry).

Finally, we may handle the unipotential lens formed of three apertures as a combination of three aperture lenses, regarding $U_0(z)$ inside the lens as a linear function of z and taking the potentials on the axes in the planes of the apertures to be equal to the potentials of the apertures themselves. Like in the case of a bipotential lens [see equations (1.170)-(1.182)] we can obtain a formula for the optical power of a unipotential symmetric lens formed by three apertures at potentials U_1 , U_2 , U_1 and the distance d between the apertures as follows

$$\frac{1}{f} = \frac{3}{8d} \times \frac{U_2 - U_1}{\sqrt{U_1 U_2}} \left(3 + \frac{U_2}{U_1} - 4 \sqrt{\frac{U_2}{U_1}} \right) \quad (1.188)$$

Using the expansion into a series by the powers of the ratio $(U_2 - U_1)/U_1$ and, rejecting the terms containing this ratio in the powers higher than the first, we obtain an approximate expression for a weak unipotential lens $\left(\left| \frac{U_2 - U_1}{U_1} \right| \ll 1 \right)$ which is

$$\frac{1}{f} \approx \frac{3}{8d} \left(\frac{U_2 - U_1}{U_1} \right)^2 \quad (1.189)$$

This expression is similar to formula (1.187).

The potential distribution along the axis of a unipotential symmetric lens formed by three cylinders of the same radius R with a length of the medium cylinder equal to $2l$ and with a cylinder gap $d \ll R$ is described with an adequate accuracy by the equation

$$U_0(z) = U_1 - \frac{(U_1 - U_2) \sinh \left(2.63 \frac{l}{R} \right)}{\cosh \left(2.63 \frac{l}{R} \right) + \cosh \left(2.63 \frac{z}{R} \right)} \quad (1.190)$$

where U_1 and U_2 are the potentials of the outer and the central electrodes respectively.

Substitution of (1.190) into (1.162) results in an equation whose solution can be presented as follows

$$\frac{1}{f} = \frac{2}{R} \left\{ \left(3 \cosh \alpha - \frac{2U_1}{U_1 - U_2} \sinh \alpha \right) \times \right. \\ \times \left[F \left(\cosh \alpha - \frac{U_1 - U_2}{U_1} \sinh \alpha \right) - F(\cosh \alpha) \right] + \alpha \coth \alpha - 1 - \\ \left. - \frac{U_1 - U_2}{U_1} \sinh \alpha F \left(\cosh \alpha - \frac{U_1 - U_2}{U_1} \sinh \alpha \right) \right\} \quad (1.191)$$

where $\alpha = 2.63l/R$

$$\left. \begin{aligned} F(x) &= \frac{1}{2\sqrt{1-x^2}} \arccos x \quad \text{at } |x| < 1 \\ F(x) &= \frac{1}{2\sqrt{x^2-1}} \operatorname{arccosh} x \quad \text{at } x > 1 \end{aligned} \right\} \quad (1.192)$$

When the difference between U_1 and U_2 is small formula (1.191) is inconvenient as it includes small differences of high values.

When $(U_1 - U_2/U_1) \leq 0.4$, the optical power of a unipotential symmetric lens formed by three cylinders can be calculated by the formula

$$\frac{1}{f} = \frac{1}{l} \left(\frac{U_1 - U_2}{U_1} \right)^2 \Phi \left(\frac{l}{R} \right) \times \\ \times \left[1 + \frac{U_1 + U_2}{U_1} \Psi \left(\frac{l}{R} \right) \right] \quad (1.193)$$

The functions of the ratios of the geometrical dimensions Φ and Ψ are presented diagrammatically in Fig. 1.48.

From this figure it is evident that when $\frac{l}{R} \geq 2$, the $\Psi=1$, and the $\Phi \left(\frac{l}{R} \right)$ becomes a linear function and we may consider approximately that $\Phi|_{l/R \geq 2} = 0.165 (l/R)$.

Formulas (1.191) and (1.193) are suitable for calculation of com-

paratively weak (long-focus) unipotential lenses; when the value of f is commensurable with the length of the central electrode, the error considerably increases. Formula (1.191) can be used for approximate calculation of a unipotential lens with high (commensurable with R) gaps between the outer and central electrodes, but in this case the value of $2l$ should be presented by the distance between the centres of the gaps rather than by the length of the central electrode.

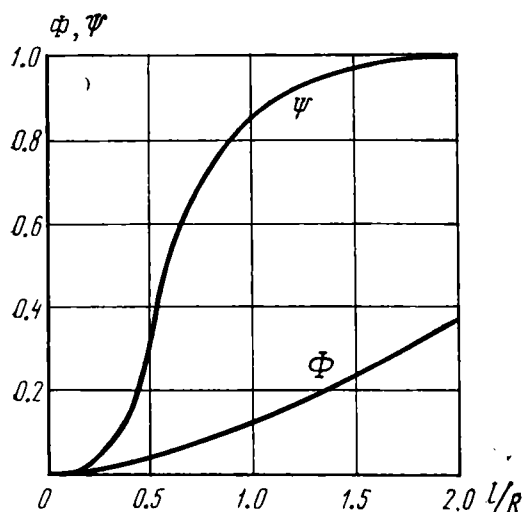


Fig. 1.48. Function curves for calculation of unipotential symmetric lens

With asymmetric unipotential lenses it is impossible to obtain analytic expressions for calculation of optical parameters. Therefore, the optical power of such lenses is to be found either experimentally or by simulating the field in the electrolytic bath with approximation of $U_0(z)$ by a suitable analytic function and determining the optical parameters by solving the general equations.

The unipotential lens can be used both with $U_1 > U_2$ and with $U_2 > U_1$. The ratio $U_1 > U_2$ is used practically more often since the outer electrodes are at a higher potential. Such a potential ratio is advantageous for several reasons. When $U_2 < U_1$ an additional high-voltage power source is not necessary for feeding the medium electrode (with a specified final energy of the electrons). With the same absolute value of the difference ($U_1 - U_2$) the lens is stronger when $U_1 > U_2$. Finally, with $U_1 > U_2$ the lens has lower aberrations (see Section 1.9), since, in this case, the converging area is near the mid-plane of the lens and, therefore, the electron beam within the lens has a smaller diameter.

In the general case the optical power of a unipotential lens with a low-voltage central electrode is most dependent on the diameter of the aperture of the central electrode and increases with a decrease in the diameter.

More complicated electrostatic electron-optical systems formed by several electrodes can be regarded as a combination of aperture lenses and bipotential or unipotential lenses. The total optical power of a complex electron-optical system can be roughly estimated by the light-optical formula

$$\frac{1}{f_{\Sigma}} \approx \frac{1}{f_1} + \frac{1}{f_2} + \dots + \frac{1}{f_n} \quad (1.194)$$

where $f_1, f_2, \dots, f_n$ are the focal lengths of the lenses forming the electron-optical system.

The bipotential and unipotential electrostatic lenses with their electrodes at certain potential differences can be used for reflecting an electron flow, i.e. be converted into electron mirrors. Depending on the sign of curvature of the equipotential surfaces in the reflection area, the mirror can be either converging (concave) or diverging (convex). Shown in Fig. 1.49 is a bipotential lens with the right-hand electrode at a potential $U_2 < 0$.

As shown in the figure, the electrons incident along a normal to the reflecting (zero) equipotential surface reverse their direction of motion and fly from the mirror along a trajectory coinciding with the inlet trajectory, while the electrons incident at a certain angle to the same surface describe narrow parabolas. Of course, the electron beam near the reflecting surface cannot be regarded as paraxial, because the angles formed by the electron trajectories relative to the axis in the reflection area cannot be considered small even at

very rough approximation. Therefore, when determining the optical parameters of electron mirrors, the equations of paraxial optics may be used only for a very rough estimation. Furthermore, it should be noted that electron mirrors have a considerable chromatic aberration (see Section 1.9) due to the commensurability of the natural scatter of electron velocities and the velocity determined by the reflection area potential.

A unipotential lens with the central electrode potential $U_2 < 0$ can be also used as an electron mirror. Varying this potential makes it possible to change the sign of the optical power of the mirror (to obtain converging and diverging mirrors) and control the optical power within a wide range. When the central electrode potential of

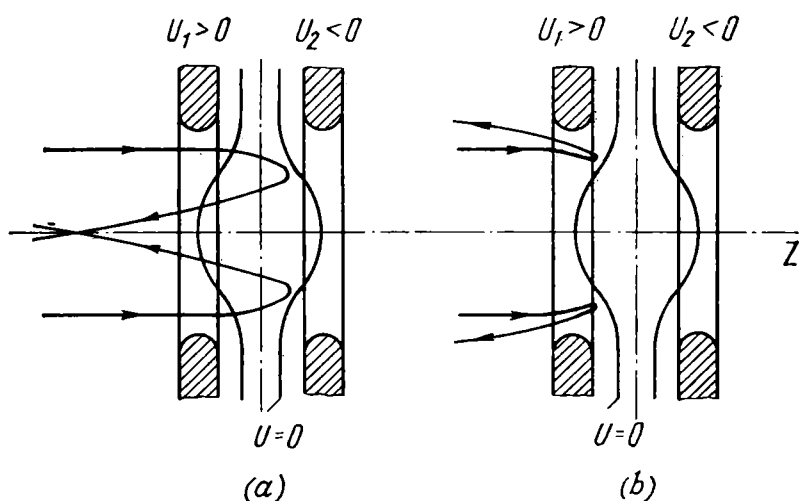


Fig. 1.49. Bipotential electron mirror
(a) converging; (b) diverging

a unipotential lens is reduced, the area having a potential below zero and reflecting the electrons is first formed near the electrode itself, while the potential along the lens axis remains positive. In this case the lens serves as a lens for near-axis electrons and changes into a mirror for periphery electrons. This property of a unipotential lens can be used for controlling the intensity of the electron flow passing through the lens. By varying the potential of the central electrode from zero to such a negative value that the equipotential $U = 0$ lies at the saddle point of the field, it is possible to smoothly vary the electron beam current from its maximum determined by the parameters of the electron-beam device to zero. Under these conditions the unipotential lens is similar to the "iris" diaphragm widely used in optical devices.

Used sometimes in the electron-beam optics as individual elements are electron mirrors formed by plane or concave electrodes at zero or subzero potential which are placed into a cylinder at a posi-

tive potential. The electron mirrors are seldom used in electron-beam devices mainly due to large aberrations, particularly chromatic aberration, which interferes with the formation of good electron-optical images.

In addition to the electrostatic lenses, electron-beam devices widely employ magnetic lenses. More frequent are thin lenses with electric fields produced by short current-carrying coils. Long magnetic lenses (homogeneous longitudinal magnetic field) are used in some television camera tubes for image transfer (see Chapter 12) and for limiting intensive electron beams (see Chapter 2).

Consider a thin magnetic lens (Fig. 1.50a) formed by a short circular coil having n turns with a mean radius R_{mean} through which an electric current I flows. The distribution of field on the axis of such a coil is described by expression (1.45) and is presented as a diagram in Fig. 1.50b.

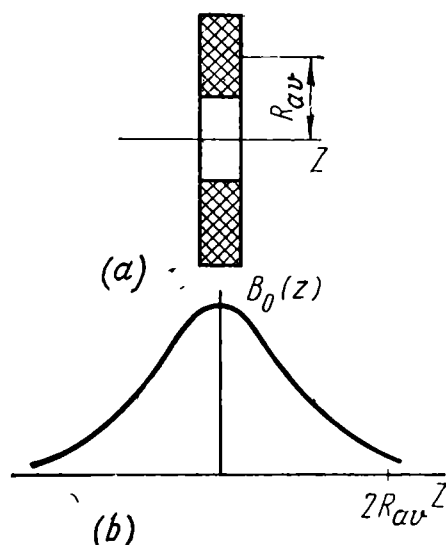


Fig. 1.50. Thin magnetic lens

As shown in the drawing, at a distance $2R_{mean}$ from the mid-plane of the coil the magnetic induction drops down more than 10 times the maximum value. Thus, with a focal length exceeding two diameters of the coil the lens may be roughly considered thin. In this case the distance between the electron and the axis within the lens can be considered as constant [$r(z) = r_0 = \text{const}$]. Next integrate the equation of motion (1.98) within the limits from z_a to z_b

$$\left. \frac{dr}{dz} \right|_{z_b} - \left. \frac{dr}{dz} \right|_{z_a} = - \frac{er_0}{8mU_0} \int_{z_a}^{z_b} B_0^2 dz \quad (1.195)$$

$$\psi_b - \psi_a = \sqrt{\frac{e}{8mU_0}} \int_{z_a}^{z_b} B_0 dz$$

Using the same technique as in the case of a thin electrostatic lens, we obtain an expression for the optical power of a thin magnetic lens

$$\frac{1}{f} = \frac{e}{8mU_0} \int_{-\infty}^{+\infty} B_0^2 dz \quad (1.196)$$

In equation (1.196) the integration limits are transferred to $\pm\infty$, since beyond the lens the field rapidly drops down practically to

zero. Since the field produced by the circular coil is symmetric with respect to its mid-plane, while the potentials at both sides of the lens are equal ($U_a = U_b = U_0$), the principal foci of the magnetic lens are located symmetrically relative to the mid-plane ($|f_1| = |f_2|$) and the principal planes lie at an equal distance from the mid-plane coinciding with this plane to a first approximation.

The optical power of the magnetic lens formed by a short circular coil can be calculated rather simply. Substituting expression (1.45) for magnetic induction into (1.196), we obtain

$$\begin{aligned} \frac{1}{f} &= \frac{e}{8mU_0} \times \frac{\mu_0^2 R_{mean}^2 (nI)^2}{4} \int_{-\infty}^{+\infty} \frac{dz}{(R_{mean}^2 + z^2)^3} = \\ &= \frac{3}{256} \times \frac{\pi e \mu_0^2}{m} \times \frac{(nI)^2}{R_{mean} U_0} \quad (1.197) \end{aligned}$$

Substituting the numerical values e , m and μ_0 , we obtain a formula convenient for practical calculation of the focal length of thin magnetic lenses

$$f = 98 \frac{U_0 R_{mean}}{(nI)^2} \quad (1.198)$$

Hence, substituting expression (1.45) into the second equation (1.195), to obtain the angle of image turn, gives us

$$\psi = 10.7 \frac{nI}{\sqrt{U_0}} \quad (1.199)$$

The values of U_0 , I and R in formulas (1.198) and (1.199) should be in volts, amperes and centimetres respectively to obtain f in centimetres and ψ in angular degrees.

Often used in practice is an approximate expression for ampere-turns calculation of the focusing coil by a given value of the focal length f and the electron energy U_0 . This expression follows directly from (1.198)

$$nI \approx 10 \sqrt{\frac{R_{mean} U_0}{f}} \quad (1.200)$$

As it has been mentioned formulas (1.183) and (1.185) are valid for long-focus lenses only. It is difficult to create short-focus (strong) magnetic lenses by means of circular coils without a ferromagnetic envelope, since to increase the optical power of a lens calls for an increase in the ampere turns, i.e. either in the current or in the number of turns. An increase in the current, as well as an increase in the number of turns, dictates an increase in the cross section of the coil with resultant increase in the mean radius of the coil, i.e. reduction of the optical power. More than that when the number of ampere turns is highly increased and the field along the axis is of perceptible

length the trajectories of electrons may intersect the axis still in the area of a sufficiently strong field, i.e. the focus (or the image) turns to be inside the lens. The beam diverging behind the focus will again be focused by the remaining portion of the field and another focus (or image) appears behind the lens. At a very high magnetic induction the second image may also appear inside the lens with a third image behind the lens, etc. The use of the lens under such conditions is not practicable as the second focal length will be much greater than the first one, since only a portion of the field is involved by the focusing. Furthermore, the second and subsequent images will be much worse than the first one due to high aberrations.

An increase in the optical power of a magnetic lens with the same ampere turns can be achieved by means of

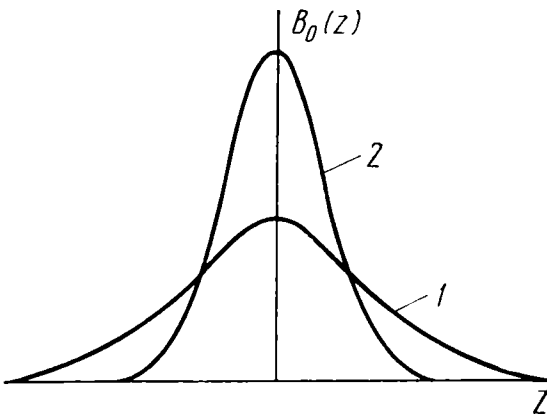


Fig. 1.51. Distribution of lens magnetic field

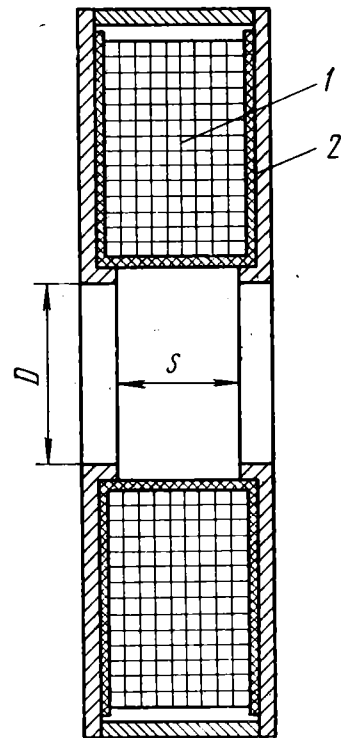


Fig. 1.52. Focusing coil with ferro-magnetic case

1—winding; 2—case

“compression” of the field area along the axis (Fig. 1.51). According to the law of total current,

$$\int_{-\infty}^{+\infty} B_0(z) dz = nI \quad (1.201)$$

i.e. the areas bounded by the curves 1 and 2 in Fig. 1.51 are the same at a fixed value of nI . However, the optical power of a magnetic lens is proportional to B_0^2 and, therefore, when a field is distributed as illustrated by the curve 2 in Fig. 1.51, the lens is stronger. Thus, a decrease in the length of the field along the axis and an increase in the maximum value of $B_0(z)$ raise the optical power of

a magnetic lens. In practice short-focus magnetic lenses can be obtained by placing a focusing coil into case of ferromagnetic material (Fig. 1.52).

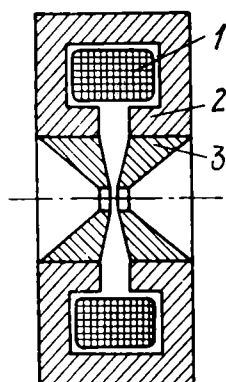


Fig. 1.53. Lens with pole pieces
1—winding; 2—ferromagnetic case; 3—pole pieces

Reducing the gap s of the magnetic core, makes it possible to “compress” the distribution curve of the magnetic field along the axis and correspondingly increase the optical power of the lens. Still greater effect is obtained from application of special pole pieces placed into the magnetic circuit (Fig. 1.53).

The lenses with pole pieces used in electron microscopes have a focal length not greater than a few millimetres. The influence of the ferromagnetic case on the distribution of the magnetic field along the axis of the magnetic lens is shown graphically in Fig. 1.54.

Also shown in Fig. 1.54d is the distribution of field in the so-called double magnetic lens formed by two coils enclosed by a ferromagnetic case with a partition, the currents in the coil having opposite directions. If that is the case the optical powers of the lenses formed by each coil are added to each other since the value $1/f$

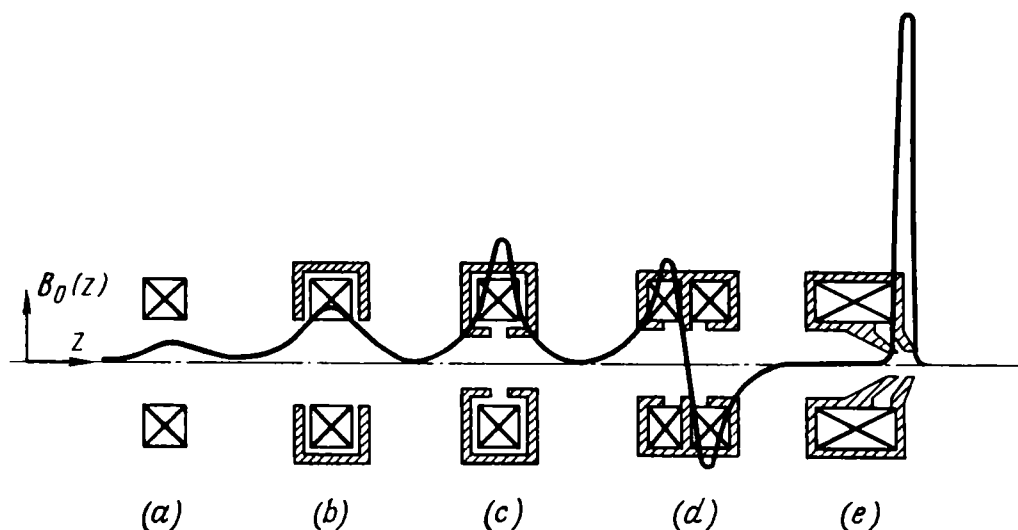


Fig. 1.54. Influence of ferromagnetic case on magnetic field distribution
(a) coil without case; (b), (c) coils with ferromagnetic cases; (d) double magnetic lens; (e) coil with ferromagnetic case and pole pieces

is proportional to B_0^2 , i.e. it is independent of the direction of the magnetic induction. At the same time the angles of image rotation are subtracted, since $\psi \sim B_0$ and the angle of rotation changes its sign with the reversal of the direction of the magnetic field (current in the coil).

Thus, a double magnetic lens formed by similar coils has a double optical power compared with the optical power of each of the lenses constituting it and does not rotate the image ($\psi = \psi_1 - \psi_2 = 0$). Such lenses find applications when the rotation of the image is undesirable for some reason.

Calculation of the parameters of magnetic lenses with ferromagnetic cases is rather difficult. The problem can be solved by selecting the function $B_0(z)$ well approximating the real distribution of the magnetic induction along the lens axis and allowing the fundamental equations to be solved analytically. Examples of such approximating functions were given in Section 1.5 [see formulas (1.76) and (1.77)].

Investigation of encased magnetic lenses has shown that the value s/D , the ratio of the gap width to the inner diameter of the ferromagnetic case (see Fig. 1.52), is the basic geometrical relationship determining the optical parameters of the lens (with the ampere turns and electron energy prescribed). Figure 1.55 presents graphical illustrations of the focal length of an encased magnetic lens *versus*

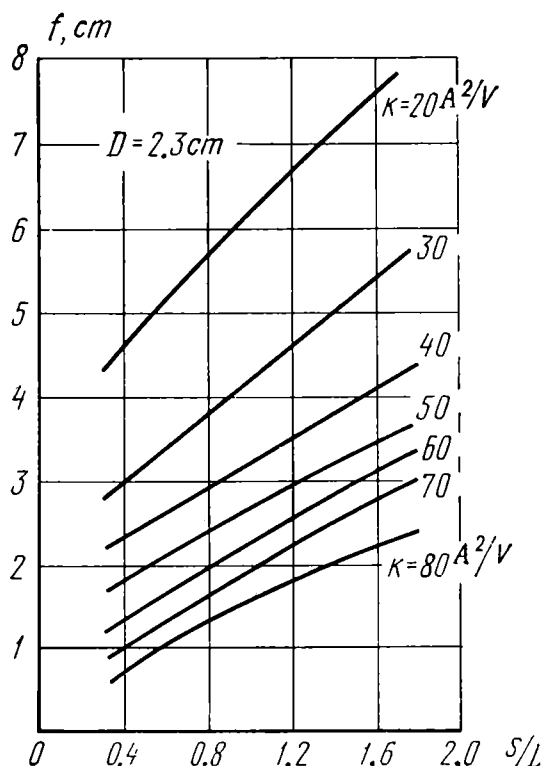


Fig. 1.55. Influence of the ratio of gap width to inner diameter of ferromagnetic case on optical properties of lens

the s/D ratio at different values of $(nI)^2/U_0 = k$. As shown in the drawing, a change in the s/D ratio causes a greater effect on the optical power at lower numbers of ampere turns. This is explained by the fact that the magnetic circuit becomes saturated with an increase in $(nI)^2$ and the curve $B_0(z)$ "expands" along the axis. The influence of the ratio of the gap width to the internal diameter of the ferromagnetic case on the optical power is more pronounced with $s/D < 1$. When s/D is greater than 1.5, a change in this ratio has a comparatively small effect on the optical power.

The encased magnetic lenses can be roughly calculated by means of formula (1.185) with the use of a coefficient $k < 1$

$$nI \approx 10k \sqrt{\frac{2RU_0}{f}} \quad (1.202)$$

where $2R$ is the internal diameter of the magnetic circuit, while $k = 0.6$ to 0.75 .

The coefficient k does not remain constant with a change in nI but rises with an increase in the ampere turns and attains unity with the saturation of the ferromagnetic.

Magnetic lenses with a ferromagnetic case have found wide application since they allow small focal lengths to be obtained. An advantage of the encased lenses is in their economy due to the concentration

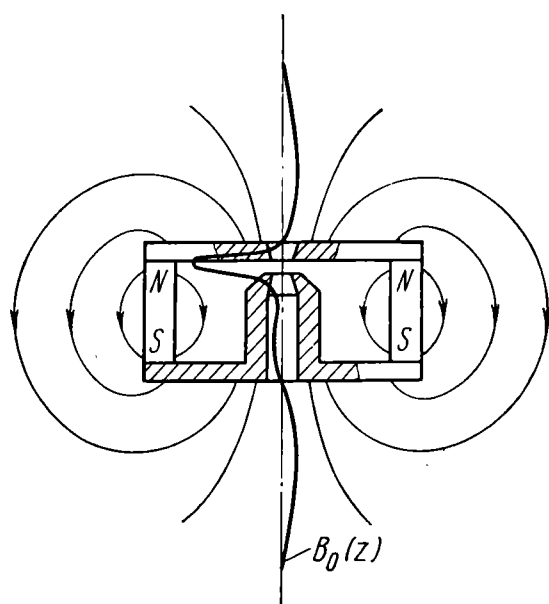


Fig. 1.56. Magnetostatic lens

of magnetic field by the ferromagnetic. None the less, even when ferromagnetic envelopes and pole pieces are employed in electron-optical systems, a perceptible amount of energy is spent to produce a magnetic field. It is evident that such a field can be produced by means of permanent magnets. In this case we have a magnetostatic electron-optical lens which needs no power supply source. The most simple magnetostatic lens is formed by a magnetized ring. In order to concentrate the magnetic field in the region of the electron beam path, the magnetic ring is provided with a magnetic circuit having a small gap

(Fig. 1.56). Further improvement in this field is a double magnetostatic lens formed by two rings magnetized in the opposite direction.

Magnetostatic lenses do not enjoy wide application in the electron-beam engineering due to a number of intrinsic disadvantages. First, control of the optical power of magnetostatic lenses calls for application of mechanical devices for moving magnetic shunts short-circuiting a part of the magnetic flux. Second even in the presence of magnetic circuits there are noticeable stray fields near the magnetostatic field which interferes with installation of a device furnished with magnetostatic lenses inside an apparatus. Finally, any change in the ambient temperature causes a change in the magnetic induction and, therefore, in the optical parameters of the lens. There are some other less significant disadvantages which restrict the application of magnetostatic lenses.

1.9. Image Distortion (Aberration) in Electron-optical Systems

On deducing the fundamental equations of electron optics, we discussed only the rays close to the axis, i.e. paraxial beams. Formally the condition of paraxiality was expressed in that we restricted ourselves only to the first terms of the expansion in series (1.53) and (1.74). In real electron-optical devices use is often made of wide beams of charged particles. In this case neglecting the subsequent terms of the expansion results in considerable errors.

As seen from equations (1.81) and (1.96), the condition for obtaining an ideal image is the proportionality of the inclination angle variation of the first degree r trajectory.

Use of the subsequent terms of the expansion results in appearance of terms proportional to the third, fifth, etc. degrees of r or dr/dz in equations (1.81) and (1.96). Thus, for example, if the term with r^4 is taken into account in expansion (1.53), the equation of motion of an electron in an axisymmetric field assumes the form

$$\frac{d^2r}{dz^2} + \frac{U'_0}{2U_0} \left[1 + r^2 \left(\frac{U''_0}{4U_0} - \frac{U''_0}{4U'_0} \right) + \left(\frac{dr}{dz} \right)^2 \right] \frac{dr}{dz} + \\ + \frac{U'''_0}{4U_0} \left[1 + r^2 \left(\frac{U''_0}{4U_0} - \frac{U''_0}{8U'_0} \right) + \left(\frac{dr}{dz} \right)^2 \right] r = 0 \quad (1.203)$$

In contrast to (1.82), equation (1.203) has additional terms proportional to r^3 , $r^2 dr/dz$, $r (dr/dz)^2$ and $(dr/dz)^3$, i.e. the third-order terms. The presence of these terms results in appearance of errors in the image (aberrations) called the third-order aberrations. If the terms with r^6 , r^8 , etc. are taken into account in expansion (1.56) the result is that the aberrations of fifth-order, seventh-order, etc. respectively are taken into account too.

In the process of calculation and analysis of electron-optical systems only the third-order aberrations are taken into consideration, as the aberrations of higher orders are not high when use is made of beams not too wide and their calculation is associated with considerable difficulties. The image aberrations of higher orders must be taken into account only in some special cases such as calculation of electron-optical systems of electronic microscopes and electron-optical devices of high resolution.

In equation (1.203) the terms containing r characterize the distance between the trajectory and the axis, while the terms containing dr/dz characterize the trajectory inclination angle relative to the field axis. Both these values can be expressed through r_a , i.e. the distance between the object points and the axis, and r_d , the radius of the aperture limiting the beam in the lens area (Fig. 1.57).

Assume that a narrow paraxial beam nn is emitted from the point a of an object and creates an undistorted (accurate) image of the

point a in the plane z_b . For the total wide beam mm emitted from the point a , a certain aberration figure is formed in the plane z_b , rather than a point. The same result will be when the object point is spaced from the axis (point a' in Fig. 1.57).

The aberration figures in the image plane are not circles, therefore, in order to estimate the distortion of the image, it is expedient to use the Cartesian coordinates in the object and image planes, the origin of these coordinates coinciding with the point where the planes intersect the axis, and characterize the aberrations by the values

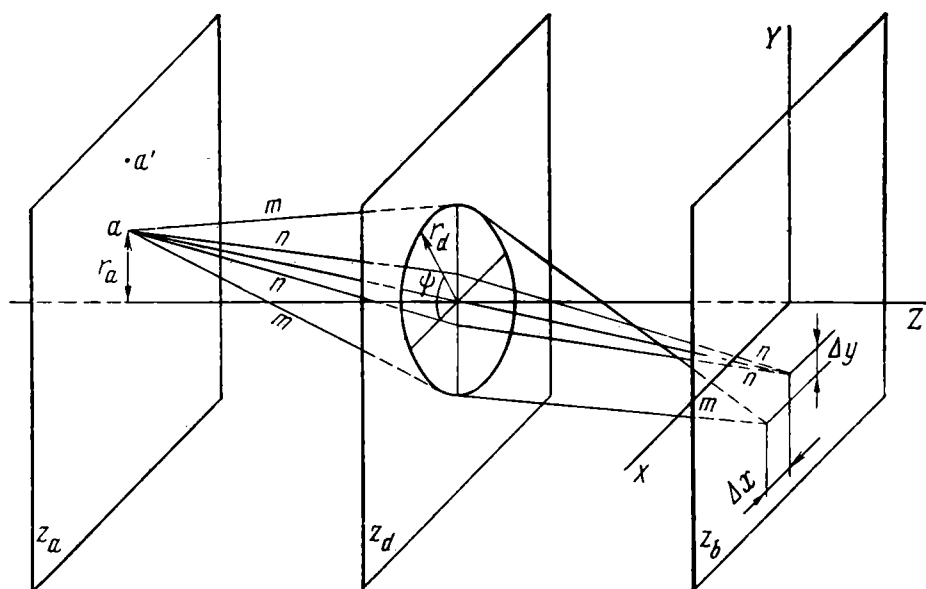


Fig. 1.57. Geometric aberration origin

Δx and Δy of the deviation of the points of the aberration figure from the point of the image formed by the paraxial beam.

Since the aperture limiting the beam must be circular, the coordinates of the aperture edges must be preferably expressed in the polar system of coordinates

$$x_d = r_d \cos \psi, \quad y_d = r_d \sin \psi$$

With such a selection of the coordinates the deviations of the beams in the image plane from the point of the image formed by paraxial beams are defined by the expressions

$$\begin{aligned} \Delta x = & Br_d^3 \cos \psi + F(2 + \cos 2\psi) r_d^2 x_a + \\ & + (2C + D) \cos \psi r_d x_a^2 + E x_a^3, \\ \Delta y = & Br_d^3 \sin \psi + F \sin 2\psi r_d^2 x_a + D \sin \psi r_d x_a^2, \end{aligned} \quad (1.204)$$

where the coefficients B, C, D, E, F are the functions of the axial potential and its derivatives of the first, second, third, and fourth

orders. These coefficients characterize five geometrical aberrations of the third order.

From (1.204) it is clear that for the points of the object lying on the system axis ($x_a = 0$) all the terms, except for the first, vanish and there remains only one aberration determined by the coefficient B . This error is known as *spherical aberration* and is most important for electron-beam devices since these devices usually employ electron beams passing from the points located near the axis. In the presence of spherical aberration, obtained in the image plane in place of the object point lying on the axis, is an aberration figure, which, as it follows from (1.204), is a circle with a radius Δr

$$\Delta r = \sqrt{(\Delta x)^2 + (\Delta y)^2} = Br_d^3 \quad (1.205)$$

From the physical point of view the origination of spherical aberration can be explained as follows. The beams passing within the lens area at a considerable distance from the axis (near the aperture edge limiting the beam) are more (or less) refracted than the paraxial beams so that they intersect closer to (or farther from) the plane z_b . As a result, a circle with a radius determined by expression (1.205) is formed in the image plane instead of the point. In the most of actual systems the potential distribution is such that the nonparaxial beams intersect the axis nearer than the image plane.

Since the radius of the circle of spherical aberration scatter is proportional to r_d^3 , this error can be reduced by decreasing the aperture radius, i.e. using narrower beams. This method, however, is not always possible, since a decrease in the aperture radius limits the beam current. By selecting proper potential distribution, one can reduce the spherical aberration by decreasing the coefficient B . Theoretically it is possible to design a field for which $B = 0$, but in practice the value of B always differs from zero and some aberration always takes place.

The radius r_d of the stop aperture limiting the beam is proportional to the tangent of the aperture angle γ_1 (Fig. 1.57). Taking into account that for small angles $\tan \gamma_1 \approx \gamma_1$, equation (1.205) can be presented in the form

$$\Delta r = B' \gamma_1^3 \quad (1.206)$$

where B' is the coefficient of the spherical aberration of the lens.

The investigation of spherical aberration of electrostatic electron-optical lenses has shown that the ratio B'/r_d (with the similar values of r_d/f) differs but little in lenses of different types. Bipotential lenses with $U_1/U_2 < 1$ (decelerating lenses) feature the least spherical aberration, while accelerating bipotential lenses, on the contrary, have the highest spherical aberration as compared with other electrostatic lenses. Bipotential lenses formed by two cylinders of the same

radius have comparatively low spherical aberration as compared with nonsymmetric bipotential lenses.

Unipotential lenses formed by three cylinders also have minimum spherical aberration provided the cylinders are of the same radius. The coefficient of spherical aberration of a unipotential lens can be approximated by the following empirical formula

$$B' \approx k \frac{f^3}{R_2^2} \quad (1.207)$$

where f is the focal length of the lens, R_2 is the radius of the internal electrode (cylinder or aperture), $k = 2.5$ to 5 , the smaller values corresponding to the lenses with an internal electrode in the form of a cylinder and the greater values corresponding to the electrode in the form of an aperture.

Generally spherical aberration increases with focal length, but it somewhat reduces with an increase in the distance to the image.

The coefficient F in (1.204) characterizes the image error called the *coma*. This kind of aberration gets its name due to the resemblance of the aberration figure with a comet. It is clear that this kind of aberration is characteristic only of those points of the object that do not lie on the axis ($x_a \neq 0$). The aberration figure produced by this error also has a form of a circle but with a centre which does not coincide with the point of the image formed by the paraxial beam. Taking all the coefficients but F , in equations (1.204) equal to zero, let these equations be squared and summed up

$$(\Delta x + 2Fx_ar_d^2)^2 + (\Delta y)^2 = F^2x_a^2r_d^4 \quad (1.208)$$

Equation (1.208) is an equation of a circle with a radius $\Delta r = \sqrt{F^2x_a^2r_d^4}$ and with the centre spaced from the image point at a distance $2Fx_ar_d^2$.

Since the aberration figures are formed by the beams passing through the entire aperture and not only by the "edge" beams, we have in the image plane a combination of superimposed circles with gradually increasing radii and with centres located at a greater distance from the point of the image created by the paraxial beam.

The coma can be reduced by arising and by bringing the object nearer to the axis. Naturally, the coefficient F can be also reduced by selecting the potential distribution.

The coefficients C and D characterize the image error known as *astigmatism*. Combining the terms with the coefficients C and D in equations (1.204), we obtain the ellipse equation

$$\frac{(\Delta x)^2}{[(2C + D)r_dx_a^2]^2} + \frac{(\Delta y)^2}{(Dr_dx_a^2)^2} = 1 \quad (1.209)$$

whose centre coincides with the point of undistorted image and with the axis values proportional to $r_dx_a^2$. With $D = 0$ the ellipse degene-

rates into a straight line with a length $4Cr_a x_a^2$. With $C = 0$ the aberration figure transforms into a circle, the cone of the beams passing through the aperture and the edges of the aberration figure having an apex. We can therefore select a surface on which the image will be free of astigmatism. This surface, however, is not flat and simply touches the plane at the point where it is intersected by the axis.

The image error appearing with $C = D$ and $D \neq 0$ is usually regarded as separate-type aberration called *curvature of the image surface*.

The astigmatism is of importance for imaging the points of the object which are remote from the axis, i.e. when using "skewed" beams. By curving the object or surface on which the image is reproduced, one can considerably reduce this aberration.

The above-discussed image error should not be confused with aberrations appearing when disturbing the axial symmetry of the fields focusing the electron beam. For example, if the apertures forming an electrostatic lens or the coils producing a magnetic field are not accurately circular, the image created by such systems is distorted, in which case even the near-axial points of the objects are represented by ellipses or short lines, i.e. we obtain astigmatic aberration figures. Similar errors take place in the case of inaccurate assembly (lack of alignment or displacement from the axis) of the components forming the electron-optical system. In contrast to the "true" astigmatism appearing during a disturbance of the axial symmetry of the fields, this phenomenon is referred to as *near-axial astigmatism*.

The final geometric error determined by the coefficient E in (1.204) is called *distortion*. This aberration results in an error of the image scale — the magnification of the system turns to be dependent on the removal of the points of the object from the axis; $\Delta x = Ex_a^3$. Depending on the distribution of the field the image of more remote parts of the object can be larger or smaller than that of the near-axial parts. Distortion that is positive, the magnification increasing with the field angle, is called pincushion because the image of a square has concave sides and thus looks like a pincushion. The opposite type, negative distortion, is called barrel distortion because the image square has bulging sides.

The aberration figures corresponding to the above-described geometric image errors of electrostatic lenses are given in Fig. 1.58.

The above-described geometric aberrations are also inherent in magnetic lenses. However, due to the "twisting" action of the magnetic field, the magnetic lenses have their own "anisotropic" aberrations appearing owing to the fact that, when using the nonparaxial beams, the angle of turn of the image depends on the distance between the points of the object and the axis. This group of aberrations includes an anisotropic coma, anisotropic astigmatism and anisotropic distort-

tion. An image obtained by means of a magnetic lens and having aberrations of all types is shown in Fig. 1.59.

As can be seen in the figure the object points removed from the axis seem blurred, indistinct due to the presence of such aberrations as astigmatism and coma. More than that the entire picture is, as if "twisted" because of anisotropic image errors.

It should be noted that the use of magnetic lenses often makes it possible to obtain images with small aberrations. This is due to the fact that the focusing magnetic fields are usually formed by coils

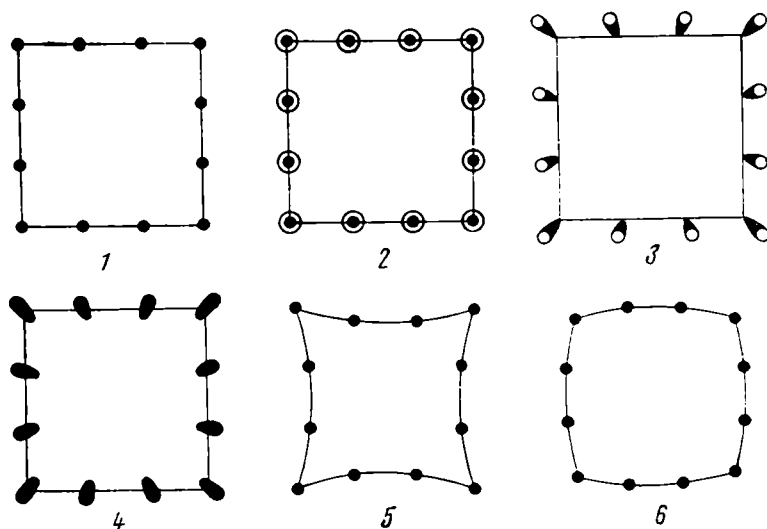


Fig. 1.58. Geometric aberrations

1—no distortion; 2—spherical aberration; 3—coma; 4—astigmatism; 5—pincushion distortion; 6—barrel distortion

of comparatively great diameter located outside cathode-ray tubes, in which an electron flow takes place. In this case the electron beam occupies only a small near-axial space of the field and, therefore, the conditions of paraxiality are met much better.

The above-described geometrical aberrations are dependent on the fields forming electron lenses and are independent of the electron beam parameters. So far we assumed that all the electrons of the beam at a given point of the field have the same energy uniquely determined by the potential of this point. This assumption is valid only for the electrons entering the accelerating field at the zero starting velocity (or at the same velocities). As a rule, real electron beams have different amounts of energy, since the use of different cathodes (thermoelectric, photoelectric, etc.) as an electron source causes natural scattering of starting velocities. In this case the electron energy at a field point at a potential U is as follows

$$\frac{mv^2}{2} = e(U + u) \quad (1.210)$$

where u is the starting energy of the electron expressed through the equivalent potential difference.

When focusing beams having different amounts of energy additional errors occur which are called *chromatic aberrations* in analogy

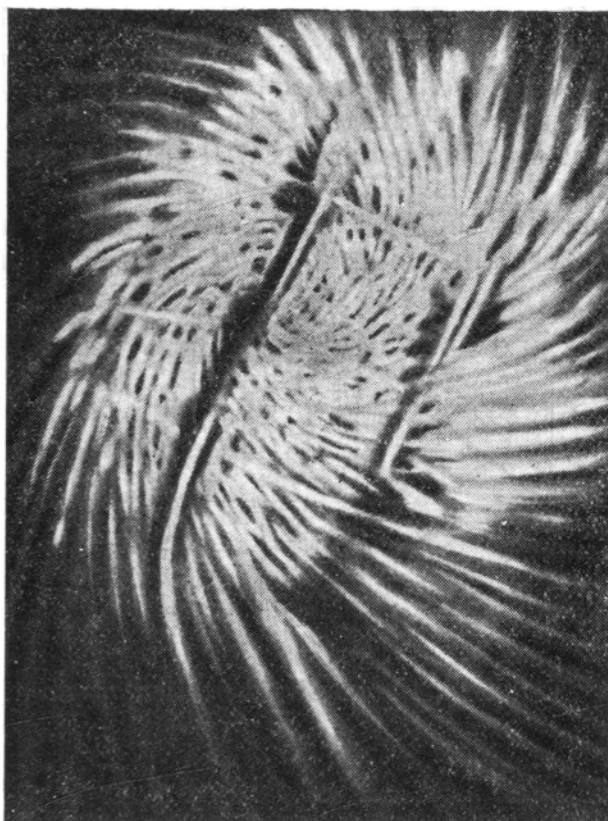


Fig. 1.59. Image distorted by all types of magnetic lens aberrations

with light optics. From the physical point of view the chromatic aberrations result from the dependence of the optical parameters of the electron lenses on the electron energy of the beam being focused. This dependence directly follows from formulas (1.156) and (1.196) in which the values U_b and U_0 placed before the integral have the meaning of electron energy.

Consider the trajectories of two electrons, one of which moves from the cathode at a zero starting velocity ($u = 0$), and the other — at non-zero starting velocity ($u > 0$) (Fig. 1.60).

It is clear that the first, slower electron, will be within the lens field a larger interval of time and, having obtained a greater impulse, will intersect the axis nearer to the lens than the other, faster electron. Considering the combination of the electrons with starting energies distributed within the interval from zero to a certain value eu , we

can easily see that the beam of electrons being injected from the same point on the axis but having their starting velocities scattered cannot be brought to one point in any plane square with the optical axis.

Assume that the electrons having zero starting velocities emerge from the point z_a on the lens axis, pass through the lens and gather at the point z_b on the axis. Then, obviously, the electrons having other than zero starting velocity leaving the same point z_a will intersect the axis at the points with coordinates $z_b + \Delta z$. In that the higher the starting velocity, the greater is the value of Δz . The electrons of other than zero starting velocities will create a circle of confusion in the plane z_b .

Next, suppose that the most of electrons have a starting energy not exceeding a certain value eu . The value Δz corresponding to

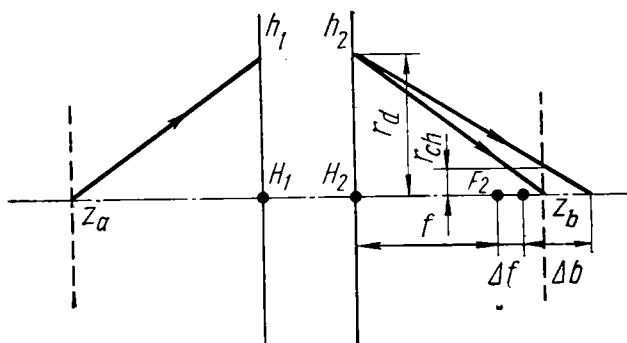


Fig. 1.60. Chromatic aberration origin

this energy is denoted as Δb . Then according to Fig. 1.60 the radius of the confusion circle (the radius of the chromatic aberration circle) is equal to

$$r_{ch} = \Delta b \tan \gamma_2 = r_d \frac{\Delta b}{b + \Delta b} \approx r_d \frac{\Delta b}{b} \quad (1.211)$$

where r_d is the radius of the aperture and $\Delta b \ll b$.

Taking into account expression (1.142) and assuming that the potentials at both sides of the lens are equal, determine the position of the images created by the slow ($u = 0$) and fast ($u > 0$) electrons, respectively

$$\left. \begin{aligned} \frac{1}{a} + \frac{1}{b} &= \frac{1}{f} \\ \frac{1}{a} + \frac{1}{b + \Delta b} &= \frac{1}{f + \Delta f} \end{aligned} \right\} \quad (1.212)$$

where Δf is the value which allows for the increase in the focal length of the lens for fast electrons.

Subtracting the first equation of system (1.212) from the second one and neglecting the value of Δf as compared to f , will give us

$$\frac{\Delta b}{b} = \frac{b}{f} \times \frac{\Delta f}{f} \quad (1.213)$$

From (1.213) it follows that the radius of the circle of chromatic aberration is proportional to the ratio $\Delta f/f$. Using the formula for the optical power of electrostatic lenses (1.156), we find the ratio $\Delta f/f$

$$\left. \begin{aligned} \frac{1}{f} &= \frac{1}{4 \sqrt{U_b}} \int_{-\infty}^{+\infty} \frac{U_0''(z)}{\sqrt{U_0(z)}} dz \\ \frac{1}{f + \Delta f} &= \frac{1}{4 \sqrt{U_b + u}} \int_{-\infty}^{+\infty} \frac{U_0''(z)}{\sqrt{U_0(z)}} dz \end{aligned} \right\} \quad (1.214)$$

Dividing the first equation of system (1.214) by the second one and making some simple operations, we obtain

$$\frac{\Delta f}{f} = \frac{1}{2} \times \frac{u}{U_b} \quad (1.215)$$

Thus the ratio $\Delta f/f$ is proportional to the ratio of the starting electron energy (eu) to the energy (eU_b) acquired by the electron in an accelerating field. Finally, using equations (1.211), (1.213) and (1.215), the radius of the chromatic aberration circle can be expressed as follows

$$r_{ch} = C_{ch} r_d \frac{u}{U} \quad (1.216)$$

Expression (1.216) shows that the chromatic aberration, like the spherical aberration, takes place at the object points lying on the axis. This aberration is known as the *chromatic aberration of the image plane position* or *aperture chromatic aberration*, as the value r_{ch} is proportional to the aperture radius r_d .

As follows from (1.216), it is possible to reduce the aperture chromatic aberration by reducing the aperture, i.e. the diameter of the electron beam, and increasing the accelerating voltage. This aberration, however, cannot be eliminated completely.

When electrons having different energies depict the object points that do not lie on the axis, there chromatic aberration appears and causes distortion of the magnification scale. This type of aberration is called *chromatic aberration of linear magnification*. Finally, when using magnetic lenses, the scatter of the electron velocities results in *chromatic aberration of the angle of turn of the image*. The last two types of chromatic aberration also depend on the ratio u/U , but in contrast to the aperture aberration, can be corrected by the corresponding selection of the fields and the position of the aperture.

Since the diffusion circle radius in case of chromatic aberrations is determined by the ratio u/U , this type of image errors is significant when focusing slow electrons, i.e. those accelerated by a small potential difference. In the same manner, chromatic aberration causes noticeable deterioration in the image resolution when using electron mirrors, since in the reflection field the space potential is close to zero and the ratio u/U can be rather great. The accelerating voltage employed in many electron devices exceeds 1 kV. In this case the chromatic aberration is less important, since in the case an oxide cathode is used at 1000° K ($eu \approx 0.17\text{ eV}$) the value of u/U turns to be less than 10^{-3} .

When the lens field adjoins the electron source (bipotential objective, see Section 3.2), the presence of electron starting energies and the scatter of the starting velocities in magnitude and direction can have a perceptible effect on the quality of focusing. In this case, one should take into account not only the absolute value of the starting velocities, but the direction of the electron escape as well. It is clear that the lower the intensity of the accelerating field near the cathode, the higher is the angle of divergence of the paths of electrons escaping from the same point of the emitting surface at velocities different in magnitudes and direction. Owing to the different directions of the initial velocities the electrons escaping from the same point of an object enter the field of the lens at different aperture angles. The scatter of these angles results in ordinary spherical aberration, while the scatter in the magnitudes of the velocities causes chromatic aberration. Thus, the focusing of a bipotential objective is affected by combined spherochromatic aberration.

The radius of the diffusion circle due to spherochromatic aberration in the plane of the Gaussian image is determined by the expression

$$r_{sph.ch} = 2M \frac{u_0}{E} \quad (1.216a)$$

where M is the magnification of the electron-optical system; u_0 is the most probable starting electron energy expressed through an equivalent potential difference; E is the electric field intensity near the cathode surface.

Expression (1.216a) is known in literature as the Reknagel formula. In the plane of the best sharpness not coinciding with the plane of the Gaussian image the radius of the circle of confusion is somewhat lower. According to L. Artsimovich, it is determined by the formula

$$r'_{sph.ch} = 1.2M \frac{u_0}{E} \quad (1.216b)$$

Distortion of an electron-optical image is observed when the supply voltage is not stable. This phenomenon should be also regarded as

chromatic aberration, since the optical parameters of the lenses vary with changes in the electrode potentials of the electrostatic lenses and the current producing the magnetic induction of the magnetic lenses.

1.10. Focusing by Fields Without Axial Symmetry. Cylindrical and Quadrupole Lenses

In addition to the above-mentioned axisymmetric electric and magnetic fields, used to focus electron beams, may be fields having no axial symmetry. These can be exemplified by electrostatic and magnetic flat fields, i.e. the field that does not depend on one of the Cartesian coordinates (for example, x). For flat fields $U(x, y, z) \equiv U(0, y, z)$ and $B(x, y, z) \equiv B(0, y, z)$. It is clear that such fields have symmetry planes perpendicular to the axis OX . Flat electrostatic fields are produced by slit-aperture diaphragms or by a pair

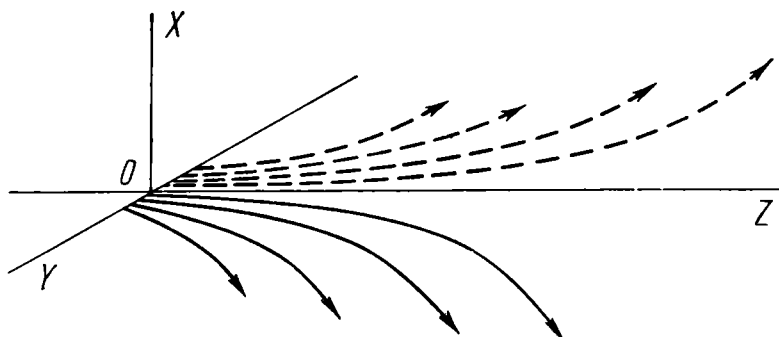


Fig. 1.61. Plane field

of flat plates and flat magnetic fields, by elongated coils which can be placed into slotted electromagnetic cases.

The equipotential surfaces of the flat electrostatic field are cylinders with generating lines parallel with the axis OX . Since the equipotential surfaces of electric field are used in the electron optics as an analog of a refracting surface in the light optics, an electron beam passes through a flat field like a light beam passes through a cylindrical lens. Owing to this analogy, electron-optical systems formed by flat field are called cylindrical electron lenses.

Generally, an electrostatic electron lens is a portion of a flat field whose symmetry plane is parallel to the axis OX (Fig. 1.61).

Find the origin of coordinates in this plane. In this case the path of an electron moving from the origin of coordinates in the direction normal to the plane XOY is linear and coincides with the axis OZ . This follows from the fact that along the axis OZ we have zero intensity components E_x (since U is independent of x) and E_y of the field (because of the symmetry of the field with respect to the plane

XOZ). Thus, the trajectory coinciding with the axis OZ serves as an axis for the electron beam.

Consider the motion of an electron in the field of a cylindrical lens (Fig. 1.61). Since the field is plane, the field intensity component $E_x \equiv 0$. The force acting on the electron in a transverse direction (along the axis OY) is equal to

$$m \frac{d^2 y}{dt^2} = -eE_y \quad (1.217)$$

To determine the field intensity component E_y , use should be made of the Ostrogradsky — Gauss theorem. Plot a parallelepiped with edges parallel to the coordinate axes (Fig. 1.62).

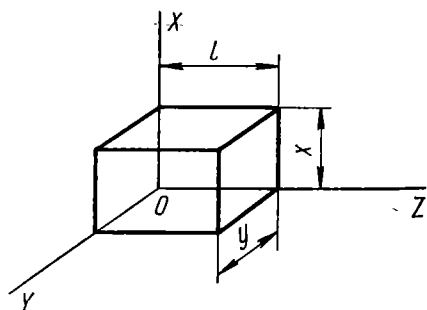


Fig. 1.62. Elementary parallelepiped

Let the vector $\mathbf{E}$ through the parallelepiped surface be equal to

$$xE_y + xy \int_0^l \frac{\partial E_z}{\partial z} dz = -xyl \frac{\rho}{\epsilon_0} \quad (1.218)$$

where x, y, l are respectively the width, height and length of the parallelepiped; ρ is the space charge density (it is supposed that $E_x = 0$).

With small values of the beam current we may consider that the field is not distorted by the charge of the beam electrons, i.e. it is uniquely determined by the shape, location and potentials of the electrodes. In this case we may suppose that $\rho = 0$. With l of small value the derivative $dE_z/dz = -U''$ (the primes stand for differentiation with respect to z) has a small variation along the length l and can be removed from under the integral sign. In this case we have

$$E_y = U''y \quad (1.219)$$

Substituting the value E_y into (1.217) leads to the equation

$$m \frac{d^2 y}{dt^2} = -eU''y \quad (1.220)$$

Restricting ourselves to the paraxial region, we can proceed from differentiation with respect to t to differentiation with respect to z (see Section 1.6). Using the differential operator $\frac{d}{dt} = \frac{dz}{dt} \times \frac{d}{dz}$ two times and noting that $dz/dt = v_z \approx \sqrt{\frac{2e}{m}} U$, we obtain

$$\frac{d}{dz} \left(\sqrt{U} \frac{dy}{dz} \right) = -\frac{U''}{2\sqrt{U}} y \quad (1.221)$$

or after the differentiation we have

$$y'' + \frac{U'}{2U} y' + \frac{U''}{2U} y = 0^* \quad (1.222)$$

The obtained equation is the fundamental equation of the cylindrical optics. It shows that in one direction — in the plane YOZ — a cylindrical lens acts in the same manner as an axisymmetric lens [the difference between equations (1.222) and (1.82) is only in the constant multiplier in the last term], while in the perpendicular direction — in the plane XOZ — the lens field has no effect on the motion of electrons ($E_x = 0$). Therefore, passing a cylindrical lens, the electron beam parallel to the axis OZ creates in its focal plane an image in the form of a straight line parallel to the axis OX.

The method used for axisymmetric lenses can also be employed for determining the optical power of a cylindrical lens in paraxial approximation

$$\frac{1}{f} = \frac{1}{2\sqrt{U_b}} \int_{-\infty}^{+\infty} \frac{U''}{\sqrt{U}} dz \quad (1.223)$$

or after the integration by parts

$$\frac{1}{f} = \frac{1}{4\sqrt{U_b}} \int_{-\infty}^{+\infty} \frac{(U')^2}{U^{3/2}} dz \quad (1.224)$$

where U_b is the value of the potential outside the lens in the image space.

The comparison of expressions (1.223) and (1.224) with formulas (1.156) and (1.159) indicates that with a similar distribution of the potential along the axis the optical power of cylindrical lenses is twice as high as that of axisymmetric ones. Like the axisymmetric lenses the cylindrical electrostatic lenses can be formed by a slit aperture, two slit apertures or by two pairs of flat plates, three slit apertures or three pairs of flat plates. A cylindrical lens can also be built by combining slit apertures and pairs of flat plates. Thus we obtain cylindrical aperture lenses, cylindrical bipotential lenses and cylindrical unipotential lenses.

To calculate the optical parameters of cylindrical lenses, we can use either the general formulas (1.223), (1.224) or approximate expressions identical to those given above for axisymmetric lenses with a corresponding change in the numerical coefficient. For example, the focal length of a cylindrical diaphragm lens is approximated by the formula

$$f \approx \frac{2U_d}{|E_2| - |E_1|} \quad (1.225)$$

* This method is suitable for obtaining the basic equation (1.82) for an axisymmetric field.

where U_d is the potential of the slit aperture, while E_1 and E_2 are the field intensities to the right and to the left from the aperture.

The examples of the electrode systems of cylindrical electrostatic lenses are given in Fig. 1.63.

It is interesting to note that in a cylindrical field which has a special saddle point (for example, in the centre of a unipotential cylindrical lens), in contrast to an axisymmetric field, the equipotentials at this point intersect at right angles.

A cylindrical magnetic lens also creates a line focus, but due to the "turning" effect of the magnetic field the focal line is turned

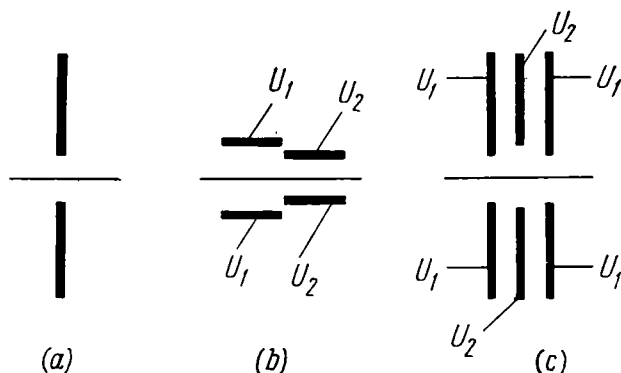


Fig. 1.63. Electrode systems of cylindrical electrostatic lenses
(a) slit aperture; (b) bipotential lens; (c) unipotential lens

through 90° , i.e. lies in the plane XOY . The optical power of a cylindrical magnetic lens is

$$\frac{1}{f} = \frac{e}{2mU} \int_{-\infty}^{+\infty} B^2 dz \quad (1.226)$$

i.e. the cylindrical magnetic lens is 4 times as powerful as the axisymmetric lens [see formula (1.196)].

Cylindrical lenses have geometrical aberrations similar to the aberrations of axisymmetric lenses, as well as chromatic aberration.

Cylindrical lenses are sometimes used in electron-beam devices when it is necessary to obtain a line focus ("hatch" focus). Cylindrical lenses formed in the electron-beam deflection space result in additional errors (distortion) of the deflection.

In axisymmetric and cylindrical lenses the electron trajectories in the lens space constitute small angles relative to the direction of the electric field intensity or magnetic induction. In other words, in these optical systems the electrons move approximately along the lines of force of the field. Therefore, the above-considered focusing systems are referred to as longitudinal systems. In the longitudinal

systems electron beams are focused by a small (compared with the longitudinal) transverse component of the field. Much more effective are transverse systems, in which the lines of force of the field are directed across the beam. The transverse electron-optical systems have gained wide use, particularly for focusing high-energy electrons. The transverse systems have new electron-optical properties. In particular, such fields can be used for design of aberration-free focusing systems.

The transverse focusing fields are usually created by four electrodes or four magnetic coils located about the axis of the system, the

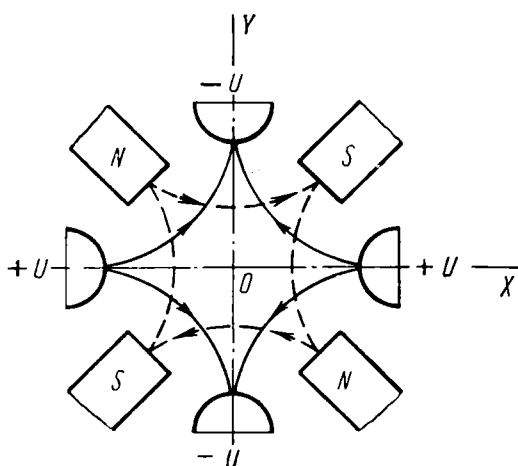


Fig. 1.64. Transverse focusing fields

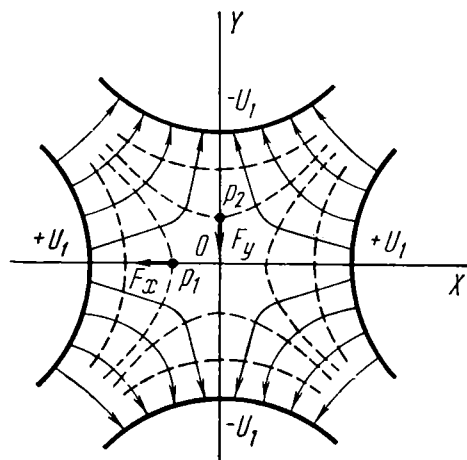


Fig. 1.65. Quadrupole electrostatic lens

diametrically opposite electrodes or magnets being of similar polarity and the adjacent ones of different polarity (Fig. 1.64).

Such four-pole systems having two symmetrical planes are known as *quadrupole lenses*. A specific feature of the quadrupole lenses is in that their axial component of the field is equal to zero.

As an example, consider a quadrupole electrostatic lens formed by four identical electrodes in the form of hyperbolic cylinders located symmetrically and equally spaced from the axis of potentials $\pm U_1$ (Fig. 1.65).

Potential distribution in such a system is described by the equation

$$U(x, y) = \frac{1}{2} k (x^2 + y^2) \quad (1.227)$$

where $k = 2U_1/a^2$ (a is the distance from the axis to the electrode surface).

In the potential distribution such as described by (1.227), the field intensity components E_x and E_y are linear functions of the

coordinates

$$\begin{aligned} E_x &= -\frac{\partial U}{\partial x} = -kx \\ E_y &= -\frac{\partial U}{\partial y} = ky \\ (E_z &= 0) \end{aligned} \quad (1.228)$$

i.e. the gradients of the field intensity are constant. For which reason, the quadrupole lens with hyperbolic electrodes is called *lens with a constant gradient*.

Consider the motion of an electron entering such a lens parallel to the axis OZ . One can easily see that at the point p_1 (Fig. 1.65) the electron is acted on by the force $F_x = -eE_x = ekx$ directed from the lens axis, while at the point p_2 it is under the action of the force $F_y = -eE_y = -eky$, directed towards the axis. Thus, in the plane XOZ the lens is diverging while in the plane YOZ it is converging. In any point of the field not lying in the symmetry planes the electron is acted on by a force always having a component forcing the electron towards the plane XOZ and from the plane YOZ .

Set up an equation of electron motion in the above field with due account for formulas (1.228).

$$\left. \begin{aligned} m \frac{d^2x}{dt^2} &= -eE_x = \frac{2eU_1}{a^2} x \\ m \frac{d^2y}{dt^2} &= -eE_y = -\frac{2eU_1}{a^2} y \end{aligned} \right\} \quad (1.229)$$

As there is no field on the lens axis, it is quite permissible to consider the longitudinal component of the electron velocity as constant for the near-axial region (paraxial approximation), i.e. $v_z = \sqrt{\frac{2e}{m} U_0} = \text{const}$ (U_0 is the constant potential on the axis) and to proceed from differentiation with respect to t in (1.229) to differentiation with respect to z :

$$\begin{aligned} m \sqrt{\frac{2e}{m} U_0} \frac{d}{dz} \left(\sqrt{\frac{2e}{m} U_0} \frac{dx}{dz} \right) &= \frac{2eU_1}{a^2} x \\ m \sqrt{\frac{2e}{m} U_0} \frac{d}{dz} \left(\sqrt{\frac{2e}{m} U_0} \frac{dy}{dz} \right) &= -\frac{2eU_1}{a^2} y \end{aligned}$$

or

$$\left. \begin{aligned} x'' - \kappa_E^2 x &= 0 \\ y'' + \kappa_E^2 y &= 0 \end{aligned} \right\} \quad (1.230)$$

where $\kappa_E = \frac{U_1}{a^2 U_0}$ and the primes indicate the differentiation with respect to z .

Now consider a symmetric quadrupole magnetic lens (Fig. 1.66).

With the given coordinate axes the planes XOZ and YOZ are the antisymmetry planes of the magnetic field *. Assume that the electrons enter the lens as a beam parallel to the axis OZ at the velocity $v_z = \sqrt{\frac{2e}{m}} U_0$. At the point p_1 the electron is acted on by the force $F_x = ev_z B_y$ directed from the axis, while at the point p_2 the electron is acted on by the force $F_y = -ev_z B_x$ directed towards the lens axis. At any point not lying in the antisymmetry planes the electron is acted on by a force having a component forcing the electron towards

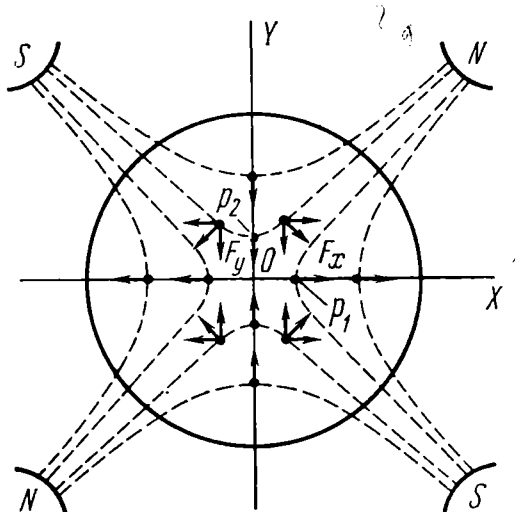


Fig. 1.66. Quadrupole magnetic lens

the plane XOZ and from the plane YOZ . Thus, the magnetic quadrupole lens will be converging in the plane YOZ and diverging in the plane XOZ .

Write the equations of electron motion in a quadrupole magnetic lens as follows:

$$\left. \begin{aligned} m \frac{d^2 x}{dt^2} &= ev_z B_y \\ m \frac{d^2 y}{dt^2} &= -ev_z B_x \end{aligned} \right\} \quad (1.231)$$

The magnetic induction components, like the case was with the axisymmetric field (see Section 1.5), can be presented in the form of series

$$\left. \begin{aligned} B_x(x, y, z) &= F'(z)y - \frac{1}{12} F''(z)(3x^2y + y^3) + \dots \\ B_y(x, y, z) &= F(z)x - \frac{1}{12} F''(z)(x^3 + 3xy^2) + \dots \end{aligned} \right\} \quad (1.232)$$

* The rotation of the axes OX and OY through 45° as compared to the system shown in Fig. 1.64 is convenient when studying magnetic quadrupole lenses, since in this case the equations of the trajectories in the magnetic lens agree with the similar equations for the electrostatic quadrupole lens.

where $F(z) = \left(\frac{\partial B_x}{\partial y}\right)_0 = \left(\frac{\partial B_y}{\partial x}\right)_0$ is the function of z equal to the value of the partial derivatives $\partial B_x/\partial y$ and $\partial B_y/\partial x$ on the system axis.

Within the limits of paraxial approximation it is quite admissible to restrict ourselves to the first terms in series (1.232). In this case the substitution of B_x and B_y from (1.232) into (1.231) leads to the following relationships

$$\left. \begin{aligned} \frac{d^2x}{dt^2} &= \frac{e}{m} v_z \left(\frac{\partial B_y}{\partial x}\right)_0 x \\ \frac{d^2y}{dt^2} &= -\frac{e}{m} v_z \left(\frac{\partial B_y}{\partial x}\right)_0 y \end{aligned} \right\} \quad (1.233)$$

Assuming that in the paraxial region $v_z = \sqrt{\frac{2e}{m} U_0} = \text{const}$, proceed from differentiation with respect to t to differentiation with respect to z

$$\left. \begin{aligned} \frac{d^2x}{dz^2} &= \sqrt{\frac{e}{2mU_0}} \left(\frac{\partial B_y}{\partial x}\right)_0 x \\ \frac{d^2y}{dz^2} &= -\sqrt{\frac{e}{2mU_0}} \left(\frac{\partial B_y}{\partial x}\right)_0 y \end{aligned} \right\} \quad (1.234)$$

or

$$\left. \begin{aligned} x'' - \kappa_M^2 x &= 0 \\ y'' + \kappa_M^2 y &= 0 \end{aligned} \right\} \quad (1.234a)$$

where

$$\kappa_M^2 = \sqrt{\frac{e}{2mU_0}} \left(\frac{\partial B_y}{\partial x}\right)_0$$

The system of equations (1.234) coincides with system (1.230) describing the motion of electrons in an electrostatic quadrupole lens. Therefore, in the general case the equations of electrons motion in the near-axial region of a symmetric electrostatic or magnetic lens can be presented in the form

$$\left. \begin{aligned} x'' - \kappa^2 x &= 0 \\ y'' - \kappa^2 y &= 0 \end{aligned} \right\} \quad (1.235)$$

where $\kappa = \kappa_E$ for the electrostatic lens and $\kappa = \kappa_M$ for the magnetic lens.

Assume that the lens has a length L , i.e. the lens field is concentrated between the planes $z = -L/2$, $z = L/2$ and abruptly drops down to zero outside this region. If that is the case we can approximately consider that the field within the lens does not depend on z and equations (1.235) transform into equations with constant coefficients having the following solutions

$$\left. \begin{aligned} x &= A \cosh \kappa z + B' \sinh \kappa z \\ y &= C \cos \kappa z + D \sin \kappa z \end{aligned} \right\} \quad (1.236)$$

Here the constants A, B, C, D are determined by the starting conditions.

For example, if in the plane $z = -L/2$ (in the "inlet" plane of the lens) $x = x_0, y = y_0, x' = x'_0, y' = y'_0$, then

$$\left. \begin{aligned} x &= x_0 \cosh \kappa \left(z + \frac{L}{2} \right) + \frac{x'_0}{\kappa} \sinh \kappa \left(z + \frac{L}{2} \right) \\ y &= y_0 \cosh \kappa \left(z + \frac{L}{2} \right) + \frac{y'_0}{\kappa} \sinh \kappa \left(z + \frac{L}{2} \right) \end{aligned} \right\} \quad (1.237)$$

To determine the optical parameters of a quadrupole lens, let us consider the path of the electron entering the lens parallel to the axis OZ . In this case $x'_0 = y'_0 = 0$ and

$$\left. \begin{aligned} x &= x_0 \cosh \kappa \left(z + \frac{L}{2} \right) \\ y &= y_0 \cosh \kappa \left(z + \frac{L}{2} \right) \end{aligned} \right\} \quad (1.238)$$

The focal lengths (see Section 1.8) are equal to

$$f_x = -\frac{x_0}{x' \left(\frac{L}{2} \right)}, \quad f_y = -\frac{y_0}{y' \left(\frac{L}{2} \right)} \quad (1.239)$$

Differentiate (1.238) and determine x' and y' when $z = L/2$ (in the "outlet" plane of the lens)

$$\left. \begin{aligned} x' \left(\frac{L}{2} \right) &= x_0 \kappa \sinh \kappa L \\ y' \left(\frac{L}{2} \right) &= -y_0 \kappa \sinh \kappa L \end{aligned} \right\} \quad (1.240)$$

Substituting (1.240) into (1.239) leads to expressions:

$$f_x = -\frac{1}{\kappa \sinh \kappa L}, \quad f_y = \frac{1}{\kappa \sinh \kappa L} \quad (1.241)$$

The expressions for the focal lengths (1.241) show that, indeed, in the plane YOZ the lens is converging ($f_y > 0$, real focus) and in the plane XOZ the lens is diverging ($f_x < 0$, imaginary focus).

Thus, upon passing through the quadrupole lens the electron beam parallel to the axis is deformed in such a way that a line focus is produced in the focal plane of the image space, i.e. a section of a straight line passing through the point of intersection of the focal plane by the axis and a parallel to the axis OX (Fig. 1.67).

The length of this section is equal to $2x(z_{fy}) = 2 \left[x_0 + f_y x' \left(\frac{L}{2} \right) \right]$. Substituting $x' (L/2)$ from (1.240) and f_y from (1.241), we obtain

$$l_f = 2 \left(x_0 + \frac{\sinh \kappa L}{\sin \kappa L} \right) \quad (1.242)$$

In the general case the focal lengths should be measured from the principal planes whose positions are determined by the expressions

$$\left. \begin{aligned} z(H_x) &= \frac{L}{2} - \frac{1 - \cosh \kappa L}{\kappa \sinh \kappa L} \\ z(H_y) &= \frac{L}{2} + \frac{1 - \cos \kappa L}{\kappa \sin \kappa L} \end{aligned} \right\} \quad (1.243)$$

In the case of a thin weak quadrupole lens, $\kappa L \ll 1$. In that $\sin \kappa L \approx \sin h \kappa L \approx \kappa L$ and the expressions for the focal lengths are simplified to

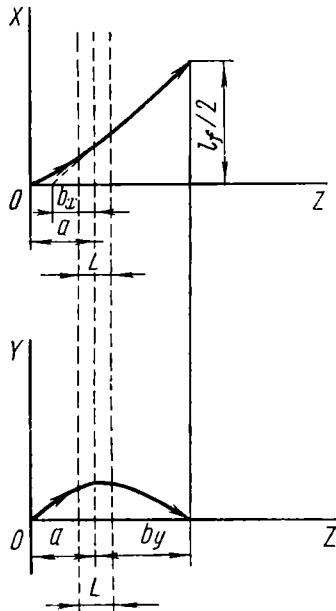
$$f_x \approx -\frac{1}{\kappa^2 L}, \quad f_y \approx \frac{1}{\kappa^2 L} \quad (1.244)$$

i.e. for the thin weak lenses the focal lengths are equal in magnitude and the principal planes coincide with the centre plane of the lens.

The approximations (1.244) can practically be used for $\kappa L \leq 0.5$. At higher values of κL the focal lengths perceptibly differ in magnitude, as $\sinh \kappa L$ rises faster than $\sin \kappa L$.

The given equations for the optical parameters, strictly speaking, are valid only for lenses with a constant field gradient. This condition is satisfied when using electrostatic lens electrodes or pole pieces of a magnetic lens in the form of hyperbolic cylinders positioned symmetrically with respect to the lens axis. In practice often used are electrodes and pole pieces having the shape of circular cylinders (from the side facing the axis), since they are easier to make. In case of replacing hyperbolic cylinders with circular ones the field changes, but little, provided the radii of the circular cylinders are chosen correctly. For example, with electrostatic lenses the best approximation to the field having a constant gradient is obtained

Fig. 1.67. Path of rays in quadrupole lens



when the radius of the circular cylinder $R = (1.1 \text{ to } 1.15)a$, where a is the distance from the electrode axis. With magnetic lenses the optimum value of R is $1.15a$.

In the same manner an assumption of an abrupt drop of the field at the boundary of the electrodes or pole pieces is a rough approximation. Satisfactory results, however, may be obtained, if the value L found in the expression for the optical parameters is regarded as a "length" of the lens with due consideration of the stray field outside the electrodes or pole pieces. It has been found experimentally that

good agreement of the design data with the experimental ones is obtained when substituting into the formula $L = l + 1.1a$, where l is the length of the electrodes or pole pieces, and a is the distance from the surface of the electrode (pole) to the axis.

The calculation of electrostatic quadrupole lenses is rather simple, since the fields with approximately constant gradients can easily be realized in practice. The calculation of magnetic lenses is much more complicated because of the necessity to calculate or determine experimentally the value of the magnetic induction and then its derivative. As an example, let an approximate formula be given for the focal length of a quadrupole magnetic lens formed by two rectangular saddle-shaped coils having n turns each and being arranged over a cylindrical surface with a radius R :

$$f = \frac{4R^2}{l} \times \frac{\sqrt{U_0}}{nI} \quad (1.245)$$

where I is the current in the coils in amperes and U_0 is the accelerating voltage in volts.

Quadrupole lenses reflect a point object as a length of straight line ("stroke" focus). However, two quadrupole lenses located along the axis under certain conditions can produce a point image of a point object. Such a system consisting of two quadrupole lenses is called the *doublet*. It is clear that each lens of a doublet produces a linear image of a point object. However, if the field of the second lens of the doublet is turned through 90° with respect to the field of the first lens, the optical parameters of the lenses can be selected in such a way that in a certain plane the linear images produced by each a lens will merge into one point image.

The doublet is not quite identical to an axisymmetric lens. In particular, magnification of a doublet of two quadrupole lenses is not the same in the directions OX and OY ($M_x \neq M_y$). Therefore, in reflection of an elongated object with axial symmetry, for example a small circle, the image is in the form of an ellipse. It could be shown that in this case the small semiaxis of the ellipse is less than the radius of the image created by an axisymmetric lens, while the great semiaxis exceeds this radius. More than that, a point image can be obtained only with the object, both lenses and the image plane in a certain position. Therefore, during the motion of the object along the axis, the merging images move apart and become linear. The same departure is observed when changing the optical parameters of the doublet lenses.

Generally the quadrupole lenses have the same aberrations as axisymmetric lenses. But in contrast to the axisymmetric lenses, the spherical aberration may be compensated for in some types of quadrupole lenses. It is also possible to build an achromatic combined quadrupole lens with electric and magnetic fields superimposed in

one field. If the polarity of the electrodes and magnetic poles of a combined lens is such that the forces acting on the electron at the expense of the electric and magnetic fields are opposing then, within a certain interval of the electron energies, the optical power of this lens is independent of the magnitude of this energy. Thus, a combined lens is free from chromatic aberration in a certain interval of accelerating voltage.

It should be noted that focusing by means of quadrupole lenses is more "rigid", i.e. the electron-focusing forces rise at a higher rate than in axisymmetric lenses, should the electron deviate from the equilibrium trajectory. Therefore, the use of quadrupole lenses is very promising when focusing fast electrons (i.e. those moving at velocities commensurable with the velocity of light).

1.11. Electron Beam Focus Fundamentals

The above-considered methods of calculation and experimental study of electron-optical systems are indirect. These methods, taken as the first stage of solving electron-optical problems, include assignment, calculation or experimental finding of the electric and magnetic fields used for focusing electrons. Next, the electron trajectories in these fields are found and their position being used for determining the optical parameters of the electron-optical systems. If the found optical parameters do not satisfy the assignment, the field must be changed and the calculation is repeated. As it has been shown in the preceding sections, in many practically important cases real finding of electron trajectories or determination of optical parameters of the lenses is a complicated and time-consuming process. Therefore, the interest, that is presently shown in the direct methods of electron optics, is well understood. By *direct* methods is meant the calculation of electron-optical systems in which used as the starting data are the configuration of the beam to be focused (assignment of several trajectories) or the optical parameters for the focusing system. Then these starting data are used for finding the electric or magnetic fields providing for beams of the predetermined shape or for the specified electron-optical parameters of the focusing systems.

Both the direct and indirect electron-optical problems are covered by the general theory of focusing charged particles developed by G. Grinberg.

Consider the problem of focusing an electron beam in an electrostatic field by using the fundamentals of the Grinberg theory. To this end introduce some simplifying assumptions. First, restrict yourself to narrow beams. Second, neglect the intrinsic field of the electron beam, i.e. the action of the space charge of the electrons and, finally, consider the electrons moving at a velocity much

lower than the velocity of light without taking into account their starting velocities.

Under the given assumptions the electron velocity at any point of the field is $v = \sqrt{\frac{2e}{m} U}$ in accordance with the law of conservation of energy. Assume that an electron beam is initiated at a point A (Fig. 1.68) and one of the trajectories, for example AB , is known and specified as $\mathbf{R} = \mathbf{R}(s)$, where $\mathbf{R}$ is the radius-vector of the trajectory point, while s is the length measured along the trajectory from the initial point A .

Let this trajectory be principal. Then the position of all other trajectories can be characterized by the vector $\mathbf{p}$ connecting a point

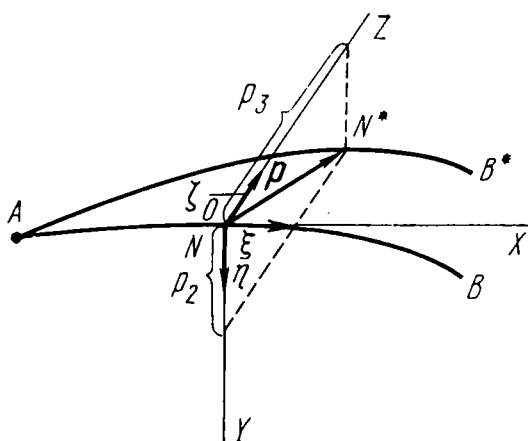


Fig. 1.68. Main (AB) and adjacent (AB^*) trajectories

of the principal trajectory with a point of an adjacent trajectory, the vector $\mathbf{p}$ lying in a plane normal to the principal trajectory. All the values relating to the adjacent trajectory will be designated by an asterisk.

Introduce a "natural" system of coordinates and align the axes OX , OY , OZ with the tangent, normal and binormal of the principal trajectory respectively at the point N . The position of the adjacent trajectory AB^* will be completely determined, if the vector $\mathbf{p}(s)$ is prescribed, since the radius-vector of the point N^* of the adjacent trajectory is as follows

$$\mathbf{R}^* = \mathbf{R} + \mathbf{p} = \mathbf{R}(s) + \mathbf{p}(s) \quad (1.246)$$

The lines p_1 , p_2 and p_3 serve as the projections of $\mathbf{p}$ onto the coordinate axes. Since the vector $\mathbf{p}$ lies in the plane YOZ , then $p_1 = 0$, $\mathbf{p} = p_2 \boldsymbol{\eta} + p_3 \boldsymbol{\zeta}$ ($\boldsymbol{\xi}$, $\boldsymbol{\eta}$, $\boldsymbol{\zeta}$ are the unit vectors of the coordinate system) and

$$\mathbf{R}^* = \mathbf{R}(s) + p_2(s) \boldsymbol{\eta} + p_3(s) \boldsymbol{\zeta} \quad (1.247)$$

The equation of electron motion along the adjacent trajectory is written as

$$m \frac{d}{dt} \left(\frac{d\mathbf{R}}{dt} + \frac{d\mathbf{p}}{dt} \right) = -e\mathbf{E}^* \quad (1.248)$$

where $\mathbf{E}^*$ is the intensity of the electric field on the adjacent trajectory.

Transform the left-hand side of equation (1.248) and proceed from differentiation with respect to time to differentiation with respect to the arc, using the ratio $dt = ds^*/v^* = (ds^*/ds) (ds/v^*)$:

$$\begin{aligned} m \frac{d}{dt} \left(\frac{d\mathbf{R}}{dt} + \frac{d\mathbf{p}}{dt} \right) &= m \left[\frac{d}{ds} \left(\frac{d\mathbf{R}}{ds} + \frac{d\mathbf{p}}{ds} \right) \frac{ds}{ds^*} v^* \right] \frac{ds}{ds^*} v^* = \\ &= m [\mathbf{R}''(s) + \mathbf{p}''(s)] \left(\frac{ds}{ds^*} v^* \right)^2 + \\ &\quad + \frac{m}{2} [\mathbf{R}'(s) + \mathbf{p}'(s)] \frac{d}{ds} \left(\frac{ds}{ds^*} v^* \right)^2 \end{aligned} \quad (1.249)$$

where the primes denote the differentiation with respect to s .

Determine the differential of the arc ds^* of the adjacent trajectory through the differential of the arc ds of the principal trajectory:

$$ds^* = \sqrt{\left(\frac{d\mathbf{R}}{ds} + \frac{d\mathbf{p}}{ds} \right)^2} ds = \sqrt{(\xi + \mathbf{p}')^2} ds$$

For narrow beams ($\mathbf{p}$ is a low value) we can restrict ourselves to the terms of the first order of smallness with respect to $\mathbf{p}$, $\mathbf{p}'$, $\mathbf{p}''$. Then

$$ds^* \approx \sqrt{1 + 2(\xi \mathbf{p}')} ds \approx [1 + (\xi \mathbf{p}')] ds$$

Using the Frenet formulas which connect the derivatives of the unit vectors of the tangent, principal normal and binormal with the radius of curvature ρ and a radius of torsion τ

$$\frac{d\xi}{ds} = \frac{\eta}{\rho}, \quad \frac{d\eta}{ds} = -\frac{\xi}{\rho} + \frac{\zeta}{\tau}, \quad \frac{d\zeta}{ds} = -\frac{\eta}{\tau} \quad (1.250)$$

Taking into account that $\mathbf{p} = p_2\eta + p_3\zeta$, determine $\mathbf{p}'$

$$\mathbf{p}' = \frac{d}{ds} (p_2\eta + p_3\zeta) = -\frac{p_2}{\rho} \xi + \left(p_2' - \frac{p_3}{\tau} \right) \eta + \left(p_3' + \frac{p_2}{\tau} \right) \zeta \quad (1.251)$$

Then

$$ds^* = \left(1 - \frac{p_2}{\rho} \right) ds, \quad \frac{ds}{ds^*} = 1 + \frac{p_2}{\rho}, \quad \left(\frac{ds}{ds^*} \right)^2 = 1 + \frac{2p_2}{\rho} \quad (1.252)$$

The electron velocity along the adjacent trajectory v^* (according to the law of conservation of energy) is given as $v^* = \sqrt{\frac{2e}{m} U^*}$.

The potential distribution (1.252) along the adjacent trajectory $U^*(s^*)$ can be expressed through the known distribution of the

potential $U_0(s)$ along the principal trajectory. To this end, expand into a series the potential near the point N of the principal trajectory

$$\begin{aligned}
 U(x, y, z) = & U_0(N) + U_x(N)x + U_y(N)y + \\
 & + U_z(N)z + \frac{1}{2}U_{xx}(N)x^2 + \frac{1}{2}U_{xy}(N)xy + \dots \quad (1.253) \\
 U_x = & \frac{\partial U}{\partial x}, \quad U_y = \frac{\partial U}{\partial y}, \quad U_z = \frac{\partial U}{\partial z}, \quad U_{xx} = \frac{\partial^2 U}{\partial x^2}, \dots
 \end{aligned}$$

Using this expansion, present the potential at the point of the adjacent trajectory in the form

$$U^*(N^*) = U_0(N) + U_y(N)p_2 + U_z(N)p_3 \quad (1.254)$$

since the coordinates of the point N $x = 0$, $y = p_2$, $z = p_3$ and for a close adjacent trajectory (value p is small) we can restrict ourselves to the terms of the first order of smallness.

Since the point N is selected arbitrarily, relation (1.253) makes it possible to connect the potential distribution along the adjacent trajectory with the potential distribution along the principal trajectory

$$U^* = U_0 + U_y p_2 + U_z p_3 \quad (1.255)$$

On the basis of relationships (1.252) and (1.255) expression (1.249) can be transformed so that the left-hand side of the equation of motion assumes the form

$$\begin{aligned}
 2e(U_0 + U_y p_2 + U_z p_3) [R''(s) + p''(s)] \times \left(1 + \frac{2p_2}{\rho}\right) + \\
 + e[R'(s) + p'(s)] \frac{d}{ds} \left[(U_0 + U_y p_2 + U_z p_3) \left(1 + \frac{2p_2}{\rho}\right) \right] \quad (1.256)
 \end{aligned}$$

The vector values in this expression can be expressed through their projections onto the unit vectors

$$\begin{aligned}
 R' = \frac{dR}{ds} = \xi, \quad R'' = \frac{d\xi}{ds} = \frac{\eta}{\rho} \quad [\text{see (1.250)}] \\
 p' = -\frac{p_2}{\rho}\xi + \left(p_2' - \frac{p_3}{\tau}\right)\eta + \left(p_3' + \frac{p_2}{\tau}\right)\zeta \quad [\text{see (1.251)}] \quad (1.257) \\
 p'' = \frac{d}{ds}(p') = \left(-\frac{2p_2'}{\rho} + \frac{p_2}{\rho^2}\rho' + \frac{p_3}{\rho\tau}\right)\xi + \\
 + \left(p_2'' - \frac{2p_3}{\tau} + \frac{p_3}{\tau^2}\tau' - \frac{p_2}{\rho^2} - \frac{p_2}{\tau^2}\right)\eta + \\
 + \left(p_3'' + \frac{2p_2}{\tau} - \frac{p_2}{\tau^2}\tau' - \frac{p_3}{\tau^2}\right)\zeta
 \end{aligned}$$

Substitution of the expressions for R' , R'' , p' and p'' into (1.256) makes it possible to obtain the projections of the left-hand side of the equation of motion on the direction of the axes OY and OZ .

Now transform the right-hand side of equation (1.248)

$$-eE^* = e(\text{grad } U)^* \quad (1.258)$$

Find the projections of the potential gradient on the directions of the axes OY and OZ at the point N^* of the adjacent trajectory. To this end, it is necessary to obtain a scalar product of $\text{grad } U$ and vectors η and ζ :

$$(\text{grad } U\eta)_{N^*} = \left(\frac{\partial U}{\partial y}\right)_{N^*} = \left(\frac{\partial U}{\partial y}\right)_N + \left(\frac{\partial^2 U}{\partial x \partial y}\right)_N x + \left(\frac{\partial^2 U}{\partial y^2}\right)_N y + \left(\frac{\partial^2 U}{\partial y \partial z}\right)_N z + \dots \quad (1.259)$$

Restricting ourselves to the terms of the first order of smallness and substituting the coordinates of the point N^* ($0, p_2, p_3$), we obtain the projection of the right-hand side of equation (1.259) on the direction of the axis OY :

$$(\text{grad } U\eta)_{N^*} = U_y + U_{yy}p_2 + U_{yz}p_3 \quad (1.260)$$

where we used the designations of (1.253).

In a similar way obtain the projection on the direction of the axis OZ :

$$(\text{grad } U\zeta)_{N^*} = U_z + U_{yz}p_2 + U_{zz}p_3 \quad (1.261)$$

Equating the projections to the similar directions of the left-hand and right-hand sides of equation (1.248) and making some transformations, we obtain

$$\left(p'_2 - \frac{p_3}{\tau}\right) \frac{dU_0}{ds} + 2 \left[p''_2 + p_2 \left(\frac{1}{\rho^2} - \frac{1}{\tau^2} \right) - \frac{2p'_3}{\tau} + \frac{p_3\tau'}{\tau^2} \right] U_0 + p_2 \left(\frac{2}{\rho} U_y - U_{yy} \right) + p_3 \left(\frac{2}{\rho} U_z - U_{zz} \right) = 0 \quad (1.262)$$

$$\left(p'_3 + \frac{p_2}{\tau}\right) \frac{dU_0}{ds} + 2 \left(p''_3 - \frac{p_3}{\tau^2} + \frac{2p_2}{\tau} - \frac{p_2\tau'}{\tau^2} \right) U_0 - (p_2 U_{yz} + p_3 U_{zz}) = 0$$

Equations (1.262) can be transformed by using some additional relations. Since at the point N of the principal trajectory the direction of the axis OX coincides with ξ , then

$$U_x(N) = \left(\frac{\partial U}{\partial x}\right)_N = \left(\frac{dU}{ds}\right)_N$$

Using relations (1.250), we obtain the expression respectively for d^2U/ds^2 and U_y :

$$\left(\frac{d^2U}{ds^2}\right)_N = U_{xx}(N) + \frac{1}{\rho} U_y(N) \quad (1.263)$$

$$U_y(N) = \frac{2U_0}{\rho} \quad (1.264)$$

Since we do not take into account the intrinsic charge of the electrons of the beam, the potential distribution satisfies the Laplace equation:

$$U_{xx} + U_{yy} + U_{zz} = 0$$

Therefore, $U_{yy} + U_{zz} = -U_{xx}$. Express U_{xx} from (1.263). Then

$$U_{yy} + U_{zz} = -\frac{d^2U}{ds^2} - \frac{2U_0}{\rho} \quad (1.265)$$

From (1.265) it follows that when the field is not specified through the whole space and what is known is only the distribution of the potential $U_0(s)$ along the principal trajectory, the value U_{yy} or U_{zz} can be specified arbitrarily. Besides, the value U_{yz} is determined by no additional relationships and can be a function of s . Thus, we may assume that

$$U_{zz} = \Phi(s), \quad U_{yz} = \Psi(s) \quad (1.266)$$

where Φ and Ψ are any arc functions.

With due account for (1.264)-(1.266) equations (1.263) take the form

$$\begin{aligned} \frac{d}{ds} \left(p_2 \frac{dU_0}{ds} \right) - \frac{p_3}{\tau} \times \frac{dU_0}{ds} + 2 \left[p_2'' + p_2 \left(\frac{2}{\rho^2} - \frac{1}{\tau^2} \right) - \right. \\ \left. - \frac{2p_3'}{\tau} + \frac{p_3\tau'}{\tau^2} \right] U_0 + \Phi(s) p_2 - \Psi(s) p_3 = 0 \\ \left(p_3' + \frac{p_2}{\tau} \right) \frac{dU_0}{ds} + 2 \left(p_3'' - \frac{p_3}{\tau_2} + \frac{2p_2'}{\tau} - \frac{p_2\tau'}{\tau^2} \right) U_0 - \\ - \Phi(s) p_2 - \Psi(s) p_3 = 0 \quad (1.267) \end{aligned}$$

This system of two equations linear with respect to p_2 , p_3 and U_0 governs the behaviour of a narrow beam of electrons moving through an electrostatic field. Specifying the field and the principal trajectory, it is possible to find all the adjacent trajectories, i.e. to determine the shape of the beam, by solving the system of two linear differential equations. On the contrary, by specifying the shape of the beam (the principal and adjacent trajectories), the potential distribution can be found, i.e. the set problem can be solved. It should be remembered, however, that the number of adjacent trajectories determining the shape of the beam, which can be specified arbitrarily, is limited. For example, when directly solving a problem for an electrostatic field, in addition to the principal trajectory, one can specify only "one and a half" trajectory, i.e. $p_2(s)$ and $p_3(s)$ can be specified for one adjacent trajectory (besides the principal one), while for another adjacent trajectory there can be specified either p_2 or p_3 but not the two trajectories together. Setting of "two and a half" (together with the principal) trajectories allows one to solve uniquely

the problem of the potential distribution near the beam by means of system (1.267).

Equations (1.267) can be obtained from the general Grinberg equations, in which $B = 0$ (without a magnetic field) and $v \ll c$ (nonrelativistic case). The Grinberg equations make it possible to find either the trajectories by specified electric and magnetic fields or the electric and magnetic fields by specified trajectories, i.e. this theory is of the most general form. It should be remembered that the exact solution of a system of equations is not always possible and approximate methods are used in a number of practically important cases. The problem is much simplified, if the fields (or a beam) have axial or plane symmetry. In the presence of axial symmetry the beam axis is conveniently selected as a principal trajectory. In this case no curvature and torsion are present. ($1/\rho = 1/\tau = 0$) and when substituting $r = \sqrt{p_2^2 + p_3^2}$ the second equation of system (1.257) coincides with the fundamental equation of paraxial optics (1.82).

Electron Optics of Intensive Beams

2.1. Space Charge in Electron Beams

In Chapter One we studied the motion of electrons in electric and magnetic fields, assuming that the electric field is uniquely determined by the shape, arrangement and electrode potentials, while the magnetic field is determined by the size and shape of the magnetic coils, as well as by the current flow in these coils, or by the arrangement of permanent magnets with a known magnetizing force. In other words, considering the focusing of electron beams, we did not take into account the electric and magnetic fields produced by the electron beam itself. These fields can be ignored only with certain restrictions imposed on the magnitude of the beam current and the accelerating voltage. It is evident that the higher the current density of the beam, the higher the density of the space charge of the electrons and the greater is the influence of the inherent fields produced by the moving electrons on the characteristics of the electrons motion. On the other hand, the higher the accelerating voltages, the higher electron energy and the lower is the influence of the inherent fields of the electron beam on the characteristics of the motion.

Thus, the action of the space charge in the beams is conveniently characterized by the coefficient of the space charge taking into account the magnitudes of the beam current and the accelerating voltage. The space charge coefficient P also called the *perveance* is defined as the ratio of the beam current to the voltage as a $3/2$ power

$$P = \frac{I}{U^{3/2}} \left[\frac{\text{A}}{\text{V}^{3/2}} \right] \quad (2.1)$$

Since in real devices forming intensive electron beams the value of P is not higher than $10^{-5} \text{ A/V}^{3/2}$, the concept of “microperveance” is often used when $p = P \times 10^6 [\mu\text{A/V}^{3/2}]$.

The study of electron beams has shown that the influence of the intrinsic electric field of the beam has a perceptible effect on the motion of the electrons when $P \geq 10^{-8} \text{ A/V}^{3/2}$ ($p \geq 10^{-2} \mu\text{A/V}^{3/2}$). Therefore, if in any section of the beam $p < 10^{-2} \mu\text{A/V}^{3/2}$, what takes place in the most of electron-beam devices, the action of the space

charge may be neglected and the formulas given in Chapter One may be used. On the contrary, electron beams used in microwave devices (klystrons, travelling wave tubes, backward wave tubes, etc.) feature a value of p that usually considerably exceeds $10^{-2}/\mu\text{A}/V^{3/2}$ and the action of the space charge should be taken into account. Electron beams, having $p > 10^{-2} \mu\text{A}/V^{3/2}$ though at one section area, relate to intensive beams and the calculation of electron-optical systems forming such beams must be carried out by means of formulas taking into account the intrinsic field of the beam electrons. The perceptible action of the intrinsic magnetic fields of moving electrons, as it will be shown below, starts to manifest itself only at relativistic velocities and this action may be ignored in many practical cases.

The action of the space charge in the beams can be characterized by the following three factors: expansion of the electron beam in a space having no field, a drop in the beam potential, and limitation of the beam current. It is clear that the third factor results from the second one since considerable potential drop materially decreases the electron velocity and, if the potential goes to zero (to the cathode potential) in a section area of the beam, the latter becomes cut off.

Complete mathematical description of electron flows shaped into beams of a certain kind is associated with considerable difficulties. This, first of all, is explained by the fact that a real electron flow is a complex physical object consisting of an aggregate of moving discrete negatively charged particles, electrons. Interactions between electrons forming a beam cannot practically be taken into consideration. Therefore, when solving the problems associated with formation of intensive beams, a number of assumptions must be made, i.e. we should consider a certain simplified model of a real beam. The first simplifying assumption is the replacement of a sum of forces acting on the selected electron on the part of the adjacent electrons with a force of some electrically charged medium having continuously distributed density of space charge acting on this electron.

The laminar flow condition, i.e. a flow having no intersecting electron trajectories, is an important assumption. Under such an assumption the electron moving along the beam boundary will remain all the time within this boundary. Thus, the calculation of the beam contour which is one of the most important problems of the intensive beams optics is reduced to the calculation of the trajectory of the outer electron. Of course, a real electron beam has no sharp boundary, since the current density at the edge of the beam drops down gradually, however, the assumption of the presence of a clear-cut boundary and an outer electron of the beam considerably simplifies the solution of the basic problems of the intensive beams optics and is widely used in practice.

The intensive electron beams employed in many devices are "long", i.e. their "length" or duration along one of the coordinate

axes (axis 0Z) considerably exceeds the radius of an axisymmetric beam or the thickness of a "strip" (flat) beam. In these cases the problem is often considerably simplified by using the fact that electron trajectories are paraxial. Remaining within the limits of the paraxial optics, we may consider with a sufficient accuracy that the longitudinal (along the axis 0Z) component of the electron velocity is equal to the total velocity v_t determined by the law of conservation of energy. Really, at a low angle of inclination of the trajectory to the axis the transverse velocity component $v_{\perp} \ll v_z$ and

$$v_z = v_t \sqrt{1 - \left(\frac{v_{\perp}}{v_t}\right)^2} \approx v_t = \sqrt{\frac{2e}{m} U} \quad (2.2)$$

The calculation shows that the angle of inclination of the trajectory to the axis $\gamma \ll 8^\circ$ gives an error within 1%, when using equation (2.2). Even with comparatively high values of γ ($\gamma = 15^\circ$) the error is below 5% which is allowable in a number of practical cases.

On solving specific problems, some additional assumptions are made. For example, we may assume that the current density is uniform at all points of a cross section area of the beam. Since adequately high accelerating voltages are usually employed for shaping intensive beams, we may neglect the initial velocities of the electrons emitted by the cathode, i.e. to consider that with $U = 0$ (on the cathode) the electron velocity $v = 0$.

Most widely used in practice are intensive beams with a circular or rectangular cross section area (in this case one side of the rectangle is considerably greater than the other) and a ring-like one. Accordingly, we shall consider axisymmetric (sometimes inaccurately referred to as cylindrical or round), band and tubular beams. In all cases it is assumed that the length of the beam is considerably greater than its cross-sectional dimensions. It should be noted that the influence of the space charge is not the same in the beams of different configurations. Therefore, the analysis of the action of the space charge must be conducted separately for intensive beams of a specific type.

Consider an intensive axisymmetric beam. Assume that a cylindrical electron beam having current I and radius r_0 (Fig. 2.1) is inserted into a space at a constant potential U_0 .

Next, assume that the beam current density is constant at all points of the cross-sectional area and that all electrons of the beam have the same axial velocity component v_z determined by the potential U_0 (i.e. the potential drop across the beam is not taken into account).

The equation of electron motion in the radial direction is as follows

$$m \frac{d^2 r}{dt^2} = -eE_r \quad (2.3)$$

where E_r is the radial component of the field intensity created by the space charge.

Note that in the space in which the beam electrons move $U = U_0 \neq f(z)$, $E_z = -dU/dz = 0$, and $E_\psi = 0$ due to the axial symmetry of the beam, i.e. the character of motion of the electrons will be determined only by the radial component of the field intensity E_r .

To determine E_r , let a small cylinder having radius r and length l

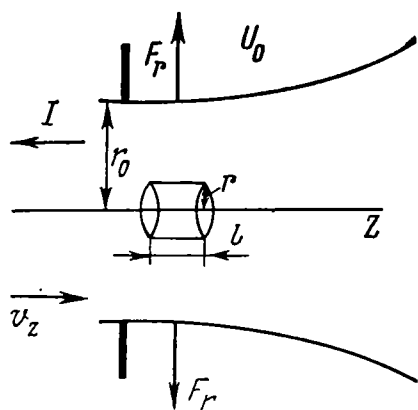


Fig. 2.1. Expansion of cylindrical beam

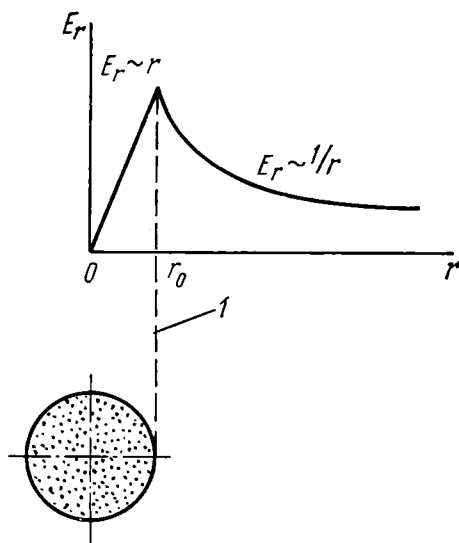


Fig. 2.2. Distribution of radial component of electric field intensity
1—beam boundary

be taken from the beam and use the Ostrogradsky—Gauss theorem for the electric field intensity flow vector. Then

$$2\pi r l E_r + \pi r^2 \int_0^l \frac{\partial E_z}{\partial z} dz = \pi r^2 l \frac{\rho}{\epsilon_0} \quad (2.4)$$

where ρ is the space charge density.

Since $E_z = 0$, the second term in the left-hand side of equation (2.4) is also equal to zero. Expressing the density of the space charge through the current

$$\rho = -\frac{j}{v_z} = -\frac{I}{\pi r_0^2 \sqrt{\frac{2e}{m} U_0}} \quad (2.5)$$

we obtain the following expression for the radial component of the electric field intensity

$$E_r = -\frac{I r}{2\pi \epsilon_0 \sqrt{\frac{2e}{m} U_0} r_0^2} \quad (2.6)$$

Expression (2.6) shows that the radial component of the intensity of the electric field produced by a space charge inside the beam is directly proportional to the radius r , i.e. to the distance from the axis.

At the boundary of the beam ($r = r_0$) the radial component of the field is

$$E_r|_{r=r_0} = - \frac{I}{2\pi\epsilon_0 \sqrt{\frac{2e}{m}} U_0 r_0} \quad (2.7)$$

From (2.7) it directly follows that at a constant current in an axisymmetric beam the field intensity at its boundary is the higher the thinner is the beam. The electron trajectories of a laminar axisymmetric beam theoretically cannot intersect the axis, as when the boundary electrons approach the axis, the field intensity and, therefore, the repulsive forces increase without restriction.

Formula (2.7) can be used for calculation of the field near the outer boundary of the beam (when $r > r_0$). As seen from (2.7), the field intensity outside the beam drops in proportion to $1/r$. The distribution of the radial component intensity of an electric field produced by a space charge is graphically presented in Fig. 2.2.

As seen from the drawing, the maximum radial field intensity corresponds to the beam boundary.

Thus, the electron moving along the beam boundary is acted on by a radial force $F_{re} = -eE_r$, directed from the beam axis, i.e. tending to increase the beam radius.

Substituting the value E_r from (2.7) into the expression for the radial force, we obtain the value of the Coulomb repulsion force defined through the beam current, radius and accelerating voltage

$$F_{re} = \frac{eI}{2\pi\epsilon_0 \sqrt{\frac{2e}{m}} U_0 r_0} = \frac{eI}{2\pi\epsilon_0 v_z r_0} \quad (2.8)$$

In addition to this force tending to widen the beam, the electrons are acted on by the Lorentz force due to the magnetic field produced by the moving electrons, which can be regarded as linear parallel currents. According to the Bio-Savarr law the magnetic induction on the beam surface is

$$B = \frac{\mu_0 I}{2\pi r_0} \quad (2.9)$$

while the radial force acting on the electrons due to this magnetic field

$$F_{rM} = - \frac{e\mu_0 I v_z}{2\pi r_0} \quad (2.10)$$

Expression (2.10) shows that the radial force created by the intrinsic magnetic field of the beam is directed to the axis, i.e. it tends to compress the beam.

The resultant radial force acting on the boundary electron is obtained by summing up expressions (2.8) and (2.10):

$$F_{r\Sigma} = F_{re} + F_{rM} = \frac{eI}{2\pi r_0} \left(\frac{1}{\varepsilon_0 v_z} - \mu_0 v_z \right) \quad (2.11)$$

Taking into account that ε_0 and μ_0 are connected through the ratio $\varepsilon_0/\mu_0 = 1/c^2$, equation (2.11) can be transformed into

$$F_{r\Sigma} = \frac{eI}{2\pi\varepsilon_0 v_z r_0} \left(1 - \frac{v_z^2}{c^2} \right) \quad (2.12)$$

From (2.12) it follows that

$$F_{rM} = -F_{re} \frac{v_z^2}{c^2} \quad (2.13)$$

i.e. at $v_z \ll c$ $|F_{rM}| \ll |F_{re}|$ and the effect of the intrinsic magnetic field is taken into account only for relativistic electron velocities. In a similar way it might be shown that noticeable influence of the intrinsic magnetic field on the motion of electrons in beams of another type will manifest itself only at relativistic electron velocities. Practically at accelerating voltages up to 25-30 kV the magnitude of the "collecting" magnetic force is only a few per cent the magnitude of the repulsive electrostatic force and the action of the beam magnetic field can be ignored in most of systems employed for shaping intensive beams.

To determine the contour of an axisymmetric electron beam inserted into an equipotential space, set up an equation of motion of the outer (moving along the beam boundary) electron. To this end, substitute the value E_r from (2.7) into equation (2.3)

$$m \frac{d^2 r}{dt^2} = \frac{eI}{2\pi\varepsilon_0 \sqrt{\frac{2e}{m} U_0} r} \quad (2.14)$$

Restricting ourselves to paraxial approximation (considering $v_t = v_z = dz/dt = \sqrt{\frac{2e}{m} U_0}$) let us transform equation (2.14) by differentiating with respect to z in place of t (see Section 1.6)

$$\frac{d^2 r}{dz^2} = \frac{1}{4\pi\varepsilon_0 \sqrt{\frac{2e}{m}}} \times \frac{I}{U_0^{3/2}} \times \frac{1}{r} \quad (2.15)$$

This first term in the right-hand side of equation (2.15) is a constant coefficient and is numerically equal to 1.52×10^4 , the second term in the right-hand side is the perveance P . Thus, equation (2.15)

can be written in the form

$$r'' - 1.52 \times 10^4 P r^{-1} = 0 \quad (2.16)$$

where the primes denote the differentiation with respect to z .

The analysis of the solution of equation (2.16) is more clear when we use dimensionless variables R and Z :

$$R = \frac{r}{r_0}, \quad Z = \frac{z}{l} \quad (2.17)$$

where r_0 is the initial radius of the beam; l is a certain scalar value, "length" of the beam in the units of which the distances along the axis OZ are measured.

With such a replacement of the variables we have

$$r'' = \frac{r_0}{l^2} R''$$

and equation (2.16) is reduced to the form

$$R'' - \frac{\kappa}{R} = 0 \quad (2.18)$$

where $\kappa = 1.52 \times 10^4 P (l/r_0)^2$.

Generally a beam can be inserted into an equipotential space when it has a converging, cylindrical or diverging initial form. Therefore, equation (2.18) will be solved under the following starting conditions: $Z = 0$, $R = 1$, $R' = R'_0$, i.e. when the measurement reference point coincides with the boundary where the beam is inserted into the equipotential space (it is evident that if the beam before entering the equipotential space had a cylindrical form, then $R'_0 = 0$). Multiply all the terms of equation (2.18) by $2R'$ and use $\frac{d}{dz} (R')^2 = 2R'R''$.

In this case the equation assumes the form

$$\frac{d}{dZ} (R')^2 = \frac{2\kappa R'}{R} \quad (2.19)$$

Integrate the equation with due account for the accepted starting conditions

$$R' = \sqrt{2\kappa \ln R + (R'_0)^2} \quad (2.20)$$

In the integration the ratio $(\ln R)' = R'/R$ is taken into consideration.

Transform equation (2.20)

$$dZ = \frac{dR}{\sqrt{2\kappa \ln R + (R'_0)^2}} \quad (2.21)$$

and integrate it

$$Z = \int_1^R \frac{dR}{\sqrt{2\kappa \ln R + (R'_0)^2}} \quad (2.22)$$

The integral at the right-hand side of equation (2.20) can be reduced to the tabulated value by using the substitution

$$2\kappa \ln R + (R'_0)^2 = 2\kappa u^2 \quad (2.23)$$

After that the solution of equation (2.19) finally takes the form

$$Z = \frac{e^{-\alpha}}{\sqrt{\frac{\kappa}{2}}} \int_{\sqrt{\alpha}}^{\pm \sqrt{\ln R + \alpha}} e^{u^2} du \quad (2.24)$$

where $\alpha = (R'_0)^2/2\kappa = 326 (r'_0)^2/p$ (p is the microperveance).

The integral in (2.24) is tabulated. Taking its value from the tables one can plot a graph of $R = f(Z)$ versus different values of R'_0 .

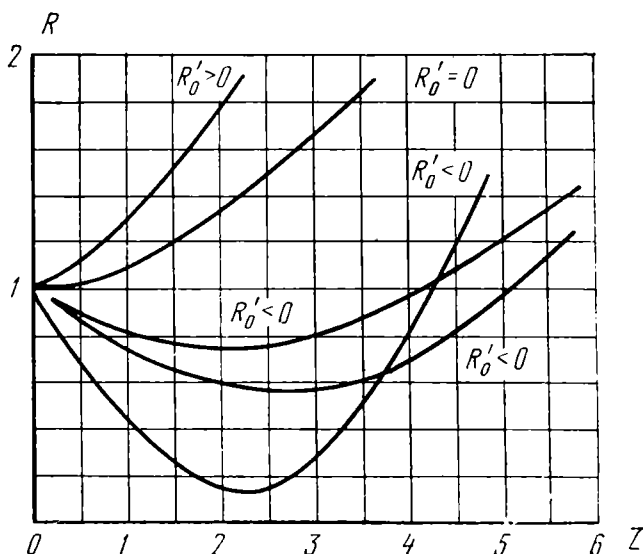


Fig. 2.3. Envelopes of axisymmetric beams

When $R'_0 = 0$ (cylindrical beam at $Z = 0$), formula (2.24) is simplified to

$$Z = \sqrt{\frac{2}{\kappa}} \int_0^{\sqrt{\ln R}} e^{u^2} du \quad (2.25)$$

As an example Fig. 2.3 shows graphs of R versus Z at different values of R'_0 .

It is obvious that these curves determine the contours of axisymmetric beams at different inclinations of the outer trajectories with regard to the axis when the beam enters the equipotential space. As is seen from the graphs, the beams which, before entering the field-free space, have outer electron trajectories parallel to the axis (cylindrical beam) or form positive angles of inclination to the axis

(diverging beam), infinitely widen in the equipotential space due to the repulsive action of the space charge. The beams which before entering the equipotential space have negative angles of inclination of the outer trajectories (converging beam), attain their minimum radius in the field-free space and then also start expanding. The plane of the smallest beam cross section area is called the *crossover plane*, though the concept of "crossover" applied to intensive beams having no trajectories intersecting the axis is purely conventional. Since the curves determining the beam contour are symmetric with respect to the crossover plane, the initially converging beam attains its initial radius at a distance of $2z_{cr}$, where z_{cr} is the distance from the initial plane ($Z = 0$) to the plane where the beam radius is minimum.

In the crossover plane $r'_{cr} = 0$ ($R'_{cr} = 0$). By making zero the right-hand side of equation (2.20), we determine the minimum radius of the beam (crossover radius)

$$r_{cr} = r_{\min} = r_0 e^{-(R'_0)^2/2\kappa} = r_0 e^{-326(\tan \gamma_0)^2/p} \quad (2.26)$$

where γ_0 is the initial angle of inclination of the outer trajectory of the beam to the axis.

It is obvious that the more the outer trajectory of the converging beam is inclined towards the axis (negative angle), the smaller is the beam crossover plane radius.

The graphs also show that as the absolute value of the angle of inclination rises up, the crossover plane first moves away from the initial plane (z_{cr} increases) and then starts returning to it (z_{cr} decreases). Owing to the symmetry of the beam with respect to the crossover plane, the plane in which the beam assumes its initial radius, also first moves away from the initial plane with an increase in $|\gamma_0|$ and then moves closer to it. Thus there is a certain optimum initial angle of inclination of the outer trajectory of the converging beam in which the crossover plane and, therefore, the plane in which the beam radius is equal to the initial one is most remote from the initial plane. The optimum angle of inclination can be calculated by the formula

$$\tan \gamma_{opt} = -0.162 \sqrt{p} \quad (2.27)$$

At an optimum angle of inclination the distance at which the initially converging beam in the field-free space again assumes its initial radius is equal to

$$2z_{cr} = \frac{2r_0}{|\tan \gamma_{opt}|} \quad (2.28)$$

Using formulas (2.27) and (2.28), we can determine the maximum value of the beam microperveance which can be passed through a cylindrical channel having radius r_0 and length $2z_{cr}$ at an optimum ini-

tial angle of inclination

$$p_{\max} = 38.4 \left(\frac{r_0}{z_{cr}} \right)^2 = 38.4 \left(\frac{d}{L} \right)^2 \quad (2.29)$$

where d is the diameter and L is the length of the channel.

Formula (2.29) indicates that the passage of long intensive beams through narrow cathode-ray tubes without focusing fields (i.e. those compensating for the action of the space charge) is associated with some difficulties. For example, if the length of the cathode-ray

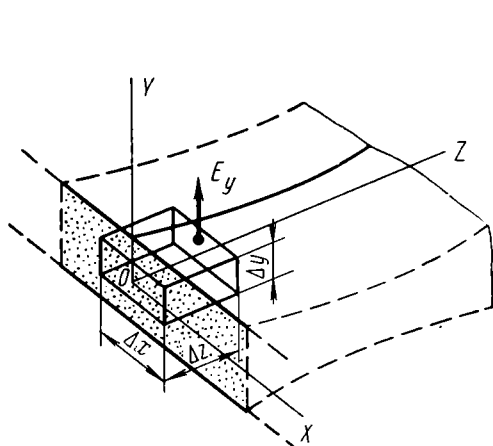


Fig. 2.4. Expansion of strip beam

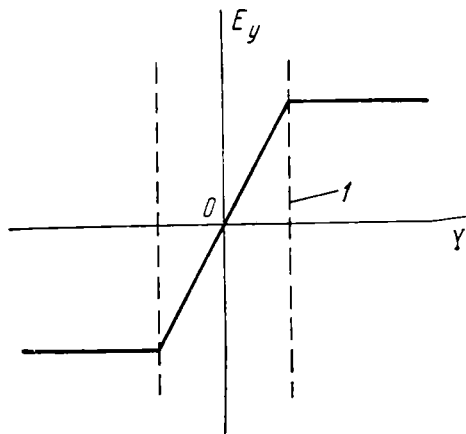


Fig. 2.5. Distribution of transverse component of electric field intensity
1—beam boundary

tube is 20 times its diameter ($d/L = 0.05$), then passed through such a cathode-ray tube is a beam with micropervance of $p \leq 9.6 \times 10^{-2} \mu \text{ A/V}^{3/2}$.

Now consider a beam with a rectangular cross section area (strip beam) spreading through an equipotential space (Fig. 2.4).

Assume that the beam width ($2x$) is much greater than its thickness ($2y$), while the length (along the axis OZ) is much greater than the width. As before, consider that the current density is the same at all points of the cross section area of the beam and restrict yourself to a paraxial approximation, i.e. assume that $v_b = v_z = \sqrt{\frac{2e}{m}U_0}$ where U_0 is the potential of the space into which the beam is inserted.

Inside the beam the potential distribution is described by the Poisson equation. However, in virtue of the accepted assumptions $E_x \ll E_y$ and $E_z \ll E_y$, the change in the potential due to the space charge can also be considered only in the direction OY , i.e. it is considered approximately that $E_x = E_z = 0$. If that is the case the Poisson equation assumes the form

$$\frac{d^2U}{dy^2} = -\frac{\rho}{\epsilon_0} \quad (2.30)$$

i.e. the problem becomes a univariate problem, and for the calculation of the paths of the outer electrons determining the contour of the beam, it is sufficient to find the value E_y .

Separate a parallelepiped with edges Δx , Δy , Δz parallel to the coordinate axes in the beam and use the Ostrogradsky—Gauss theorem

$$\int_s E ds = -\frac{\rho}{\epsilon_0} \Delta x \Delta y \Delta z \quad (2.31)$$

Under the assumptions used the left-hand side of the equation becomes equal to $2E_y \Delta x \Delta y$ since the field intensity vector passes through the two bases of the parallelepiped — upper and lower. Therefore, we have

$$E_y = -\frac{\rho}{2\epsilon_0} \Delta y \quad (2.32)$$

From (2.32) it follows that the intensity of the field produced by the space charge is equal to zero in the centre plane of the beam and rises linearly with a rise of y attaining its maximum at the beam boundary (Fig. 2.5).

Since the longitudinal component of the electron velocity v_z is invariable, the space charge density can be expressed through the density of the current j and the value v_z

$$\rho = \frac{j}{v_z} \quad (2.33)$$

Further calculations are conveniently based on a “linear” current density j_l defined as a current per unit of the beam width in the direction of the axis OX :

$$j = \frac{j_l}{2y} \quad (2.34)$$

where $2y$ is the beam thickness.

By substituting (2.34) into (2.33) and the obtained expression for ρ into (2.32) with simultaneous replacement of v_z by $\sqrt{\frac{2e}{m}} U_0$, then at the beam boundary ($\Delta y = 2y$) the intensity of the field produced by the space charge is equal to

$$E_y = -\frac{j_l}{2\epsilon_0 \sqrt{\frac{2e}{m}} U_0} \quad (2.35)$$

Set up an equation of motion of the outer electron

$$\frac{d^2 y}{dt^2} = -\frac{e}{m} E_y = \frac{j_l}{2\epsilon_0 \sqrt{\frac{2m}{e}} U_0} \quad (2.36)$$

and pass from the differentiation with respect to t to that with respect to z

$$\frac{d^2y}{dz^2} = \frac{1}{4\epsilon_0 \sqrt{\frac{2e}{m}}} \times \frac{j_l}{U_0^{3/2}} \quad (2.37)$$

Designate the first term at the right-hand side of equation (2.37) as a ($a = 4.68 \times 10^4 \text{ V}^{3/2}/\text{A}$) and the other term having the meaning of "linear perveance" as p' ($p' [\text{A}/\text{V}^{3/2}/\text{m}]$). Then equation (2.37) assumes the form

$$\frac{d}{dz} \left(\frac{dy}{dz} \right) = ap' \quad (2.38)$$

Integrate this equation under the following starting conditions: at $z = 0$ $y = y_0$ (starting half-thickness of the beam) and $y' = y'_0 = \tan \gamma_0$ (starting inclination of the outer trajectory of the beam)

$$y' = y'_0 + ap'z \quad (2.39)$$

The integration of equation (2.39) under the same starting conditions results in the expression

$$y = y_0 + y'_0 z + \frac{1}{2} ap' z^2 \quad (2.40)$$

Expression (2.40) indicates that the contour of the strip beam in the plane YOZ is a parabola. If, before entering the equipotential field, the outer trajectories were parallel to the axis OZ ($\tan \gamma_0 = 0$), then the beam will infinitely diverge. An initially converging beam ($\tan \gamma_0 < 0$) first converges in the field-free space and then starts expanding, i.e. like an axisymmetric beam, the converging strip beam will have a minimum cross section area, crossover, at a certain distance from the starting plane.

The position of the crossover is determined by equation (2.39) with $y' = 0$

$$z_{cr} = -\frac{y'_0}{ap'} = -\frac{\tan \gamma_0}{ap'} \quad (2.41)$$

From (2.41) and (2.40) it follows that with an increase in the absolute value of the starting angle of inclination of the outer trajectory the crossover moves away from the starting plane, while the cross section area of the beam decreases. At a certain angle of inclination $\gamma_{0 \text{ opt}}$ the thickness of the beam in the crossover becomes equal to zero, i.e. in contrast to the axisymmetric beam, a strip beam of an unlimited width can theoretically be reduced to a line, to obtain a line focus. It is clear, that when $\gamma_0 = \gamma_{0 \text{ opt}}$ the apex of the parabola determining the beam contour touches the axis OZ . When the angle of inclination is still more increased the line focus appro-

aches the starting plane and the outer trajectories in the focus intersect the beam centre plane at the focus point.

The value of the optimum input angle is found from (2.40) by substituting $y = 0$ and $z = z_{cr}$ [see (2.41)]

$$\tan \gamma_{0 \text{ opt}} = y'_{0 \text{ opt}} = -\sqrt{2ap'y_0} \quad (2.42)$$

The distance from the starting plane ($z = 0$) to the line focus is a focal length equal to

$$f = z_{cr} |_{\gamma_{0 \text{ opt}}} = -\frac{y_{0 \text{ opt}}}{ap'} \quad (2.43)$$

It is evident that without the action of the space charge the outer trajectory of the beam would intersect the axis at a distance $f_0 = -y_0/y'_{0 \text{ opt}}$. Comparing f_0 with f , one can ascertain that $f/f_0 = 2$, i.e. the action of the space charge doubles the focal length.

In the general case the position of the line focus is determined by equation (2.40) with $y = 0$. Solution of this quadratic (with respect to z) equation assumes the form

$$z_f = f = \frac{y_0}{ap'f_0} - \frac{1}{ap'} \sqrt{\left(\frac{y_0}{f_0}\right)^2 - 2ap'y_0} \quad (2.44)$$

where $f_0 = -y_0/y'_0$.

The last equation makes it possible to find the conditions for obtaining a line focus. It is evident that the line focus is obtained, when the radicand in (2.44) is not negative, i.e. when

$$\frac{y_0}{f_0^2} \geq 2ap' \quad (2.45)$$

With the radicand equals zero, the focal length is maximum, i.e. $f = 2f_0$.

From the condition of beam contour symmetry with respect to the crossover plane it follows that the beam attains the starting thickness $2y_0$ at distance of $2z_{cr}$. At an optimal starting angle of inclination the $2z_{cr}$ equals $f_2 = 4f_0$. This condition corresponds to the maximum possible "linear perveance" p' . From (2.45) (when both terms are equal to each other) we obtain

$$p'_{\max} = \frac{j_{l \max}}{U^{3/2}} = \frac{8}{a} \times \frac{y_0}{L^2} = 1.68 \times 10^{-4} \frac{y_0}{L^2} \quad (2.46)$$

where $L = 4f_0$ is the beam length.

Figure 2.6 shows the graphs of strip beam divergence for different ratios $j_l/j_{l \max}$ at a fixed entrance angle of inclination of the outer trajectory.

As can be seen from the figure, with $j_l < j_{l \max}$ the trajectories intersect the axis and there is a line focus, the curves having an inflection point in the focal plane. With $j_l = j_{l \max}$ the apex of the parabolic curve touches the axis OZ. With a further increase in the

current density the cross section area of the beam in the crossover plane is other than zero and no line focus is possible.

The analysis of beams of other types shows that, regardless of the configuration, the intensive electron beams in a field-free space expand due to the repulsive action of the space charge. For example, the calculation of the outer trajectory of a tubular axisymmetric

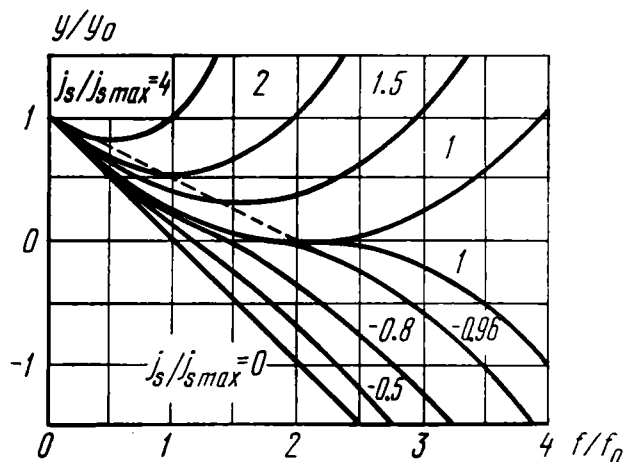


Fig. 2.6. Envelopes of strip beams

beam containing a cylindrical electrode of a potential U_0 shows that such a beam (its outside boundary) expands approximately 7% less than a solid axisymmetric beam at the same current density.

Now consider the other factor, potential drop in the beam due to the space charge. Suppose that an electron beam fills the entire space

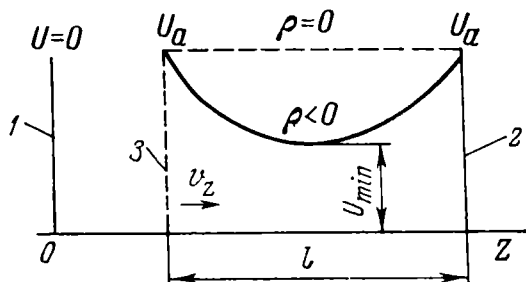


Fig. 2.7. Potential distribution in unlimited flow
1—cathode; 2—anode; 3—accelerating diaphragm

between two flat parallel electrodes at the same potential U_a spaced at a distance l (Fig. 2.7).

Assume that the electrons have only one velocity component v_z , the values of the electron velocities and the current density being the same at all points of any plane perpendicular to the axis OZ . Under these assumptions the problem is univariate and it is sufficient to discuss only the change of the potential along the axis OZ .

The potential distribution in the presence of a space charge is described by the Poisson equation (univariate problem)

$$\frac{d^2U}{dz^2} = -\frac{\rho}{\epsilon_0} = \frac{j}{\epsilon_0 v_z} \quad (2.47)$$

where j is current density.

Expressing v_z through $U(z)$, we obtain an equation describing the potential:

$$\frac{d^2U}{dz^2} = \frac{j}{\epsilon_0 \sqrt{\frac{2e}{m}}} U^{-1/2} \quad (2.48)$$

with limiting condition $U|_{z=0} = U|_{z=l} = U_a$.

The problem is symmetric with respect to the plane $z = l/2$. Therefore, when $z = \frac{l}{2}$ the potential U has its minimum

$$\left. \frac{dU}{dz} \right|_{z=\frac{l}{2}} = 0 \quad (2.49)$$

Multiply both sides of equation (2.48) by $2dU/dz$ and make integration within the range from z to $l/2$ taking into account (2.49). Then

$$\frac{dU}{dz} = 2 \sqrt{\frac{j}{\epsilon_0 \sqrt{\frac{2e}{m}}}} (U^{1/2} - U_{\min}^{1/2}) \quad (2.50)$$

where $U_{\min}$ is the value of potential at its minimum ($U_{\min} = U|_{z=\frac{l}{2}}$).

The other integration within the same limits results in the expression

$$(U^{1/2} - U_{\min}^{1/2})^{1/2} (U^{1/2} + 2U_{\min}^{1/2}) = \pm \frac{3}{2} \sqrt{\frac{j}{\epsilon \sqrt{\frac{2e}{m}}}} \left(z - \frac{l}{2} \right) \quad (2.51)$$

Square the latter equation and divide both its sides by $U^{3/2}$

$$\begin{aligned} [1 - (U_{\min}/U)^{1/2}] [1 + 2(U_{\min}/U)^{1/2}]^2 = \\ = \frac{9}{4} \times \frac{1}{\epsilon_0 \sqrt{\frac{2e}{m}}} \times \frac{j}{U^{3/2}} \left(z - \frac{l}{2} \right)^2 \end{aligned} \quad (2.52)$$

Assume that $z = 0$ (in this case $U = U_a$) and designate $j l^2 / U_a^{3/2} = p'$ (p' has the meaning of space charge coefficient, perveance). Then

$$\left[1 - \left(\frac{U_{\min}}{U_a} \right)^{1/2} \right] \left[1 + 2 \left(\frac{U_{\min}}{U_a} \right)^{1/2} \right]^2 = \frac{9}{16} \times \frac{1}{\epsilon_0 \sqrt{\frac{2e}{m}}} p' \quad (2.53)$$

The obtained equation makes it possible to analyze the potential at its minimum (when $z = l/2$) versus the current density. The dependence of $U_{\min}/U_a$ on p' is presented in the form of a diagram in Fig. 2.8.

A specific feature of this diagram is branching of the curve at the point A with $p' = 9.32 \times 10^{-6} \text{ A/V}^{3/2}$ and $U_{\min}/U_a = 0.75$. Formally this feature stems from the presence of two solutions of equation (2.51), depending on the selection of the sign before the root.

Physically with an increase in the current density the potential at its minimum diminishes monotonically and at $p' = 18.64 \times 10^{-6} \text{ A/V}^{3/2}$ the value of $U_{\min}$ becomes equal to zero (point C), a virtual cathode is formed in the plane $z = l/2$, a part of the electron flow continues to move to the plane $z = l$, and a part returns

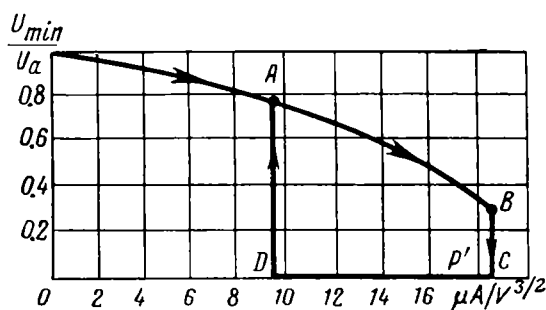


Fig. 2.8. Potential versus current density

back to the accelerating electrode (plane $z = 0$). If that is the case the space between the accelerating electrode and the collector may be considered as two diodes with a cathode located in the plane of minimum potential ($z = l/2$). The current in each diode obeys the law in the degree $3/2$. In case of a decrease in the current density the virtual cathode remains ($U_{\min} = 0$) also at the values of $p' < 18.64 \times 10^{-6} \text{ A/V}^{3/2}$.

If $p' = 9.32 \times 10^{-6} \text{ A/V}^{3/2}$, the potential at its minimum increases up to $0.75 U_a$, the virtual cathode disappears and with a further decrease in the current density the potential at its minimum rises monotonically to the value U_a at $p' = 0$.

Equation (2.53) is obtained under assumptions that the electron flow in the directions of the axes OX and OY is unlimited and that the velocities of all the electrons are laminary and equal in any plane perpendicular to the axis OZ . Since such ideal conditions are practically unrealizable because of the flow limitations and the presence of the starting velocities of the electrons scattered both in magnitude and direction, the obtained relationships are suitable for description of the potential distribution in real flows only with a certain approximation.

The experimental verification shows that when p' approaches its theoretically possible maximum value, the flow becomes unstable, the electrons start oscillating, and the normal current flow becomes disturbed. However, even with $p' > 18.64 \times 10^{-6} \text{ A/V}^{3/2}$ some of the electrons reach the collector. In the same manner, a decrease in p' near the value $9.32 \times 10^{-6} \text{ A/V}^{3/2}$ results in instability of the flow, the virtual cathode disappears gradually and the normal current flow is recovered also gradually.

In the case of an unlimited electron flow the possible maximum value of p' having the meaning of perveance is equal to $18.64 \times 10^{-6} \text{ A/V}^{3/2}$.

In real electron devices with intensive beams the electron flow, as a rule, is introduced into a transit channel (conductive tube) with conducting walls at a constant potential equal to the potential accelerating the electrons before they enter the transit channel. Here we neglect the interaction between the high-frequency field and the electron flow occurring in electron devices used in the super-high frequency range and restrict ourselves to the problems of shaping intensive beams of a definite configuration. In the presence of a channel with conducting walls the variation of the potential in the beam due to the space charge will be lower than in the case of an unlimited flow.

Consider the potential distribution in an axisymmetric beam completely filling a conductive tube with an inner radius r_0 . The tube potential is denoted as U_a . Next make the following assumptions: the beam length L considerably exceeds its radius; the current density is the same at all points of any cross section area of the beam; all the electrons have velocities parallel to the axis OZ (beam axis), i.e. the expansion of the beam is not taken into account. On the basis of the first assumption ($L \gg r_0$) we may consider that the potential drop along the beam is small (at the beam boundary the potential is constant and equal to U_a) and restrict ourselves to the radial potential distribution only. The third assumption can be realized by placing the conductive tube into a longitudinal magnetic field (see Section 2.5).

The potential distribution in a beam is described by the Poisson equation

$$\frac{\partial^2 U}{\partial z^2} + \frac{1}{r} \times \frac{\partial U}{\partial r} + \frac{\partial^2 U}{\partial r^2} + \frac{1}{r^2} \times \frac{\partial^2 U}{\partial \psi^2} = -\frac{\rho}{\epsilon_0} \quad (2.54)$$

On the basis of the first assumption we have $d^2 U/dz^2 = 0$, while the last term in the left-hand side of the equation (2.54) is equal to zero due to the axial symmetry of the beam. Transform the medium terms of the left-hand side of this equation:

$$\frac{1}{r} \times \frac{\partial U}{\partial r} + \frac{\partial^2 U}{\partial r^2} = \frac{1}{r} \times \frac{d}{dr} \left(r \frac{\partial U}{\partial r} \right) = -\frac{\rho}{\epsilon_0} \quad (2.55)$$

and express the density of the space charge ρ through beam current I and the longitudinal component of the velocity v_z

$$\rho = -\frac{j}{v_z} = -\frac{I}{\pi r_0^2 \sqrt{\frac{2e}{m} U(r)}} \quad (2.56)$$

Substituting (2.56) into (2.55), we obtain an equation describing the radial potential distribution:

$$\frac{1}{r} \times \frac{d}{dr} \left(r \frac{dU}{dr} \right) = \frac{I}{\pi \epsilon_0 r_0^2 \sqrt{\frac{2e}{m} U(r)}} \quad (2.57)$$

with boundary conditions

$$U(r) |_{r=r_0} = U_a, \quad \frac{dU}{dr} \Big|_{r=0} = 0$$

The second boundary condition stems from the presence of axial symmetry of the beam — the curve of the radial potential distribution has its minimum on the axis (Fig. 2.9).

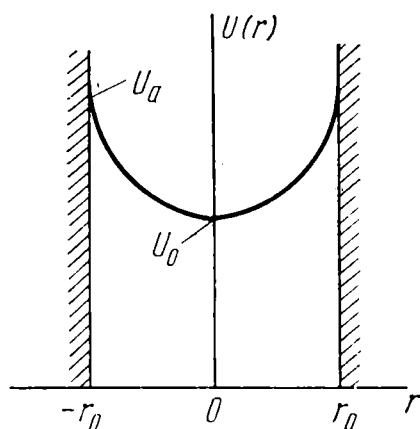


Fig. 2.9. Radial distribution of potential

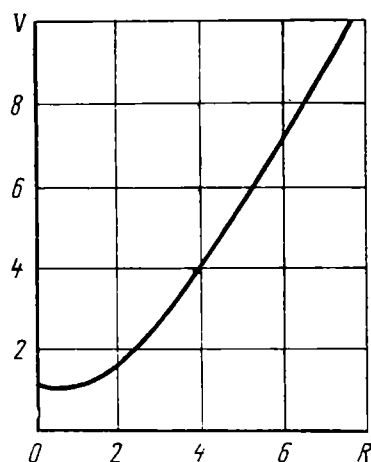


Fig. 2.10. Graph of radial distribution of reduced potential

Let the potential on the beam axis be denoted as U_0 and use the new variables

$$V = \frac{U(r)}{U_0}, \quad R = r \left(\frac{1}{\pi \epsilon_0 r_0^2 \sqrt{\frac{2e}{m}}} \times \frac{I}{U_0^{3/2}} \right)^{1/2}$$

When substituting V and R the equation (2.57) is reduced to the form

$$\frac{1}{R} \times \frac{d}{dR} \left(R \frac{dV}{dR} \right) = V^{-1/2} \quad (2.58)$$

with boundary conditions

$$V|_{R=0} = 1, \quad \left. \frac{dV}{dR} \right|_{R=0} = 0$$

The solution of equation (2.58) in a graphical form is presented in Fig. 2.10.

This graph can be used for calculating the radial potential distribution in an axisymmetric beam filling a conductive tube. The relative change in the potential along the beam axis depending on the perveance is of interest. Figure 2.11 illustrates the ratio of the beam axial potential to the boundary potential *versus* the value of microperveance.

As seen from the drawing, the axial potential decreases with an increase in p and at $p = 32.4 \mu \text{ A/V}^{3/2}$, U_0 attains the value of

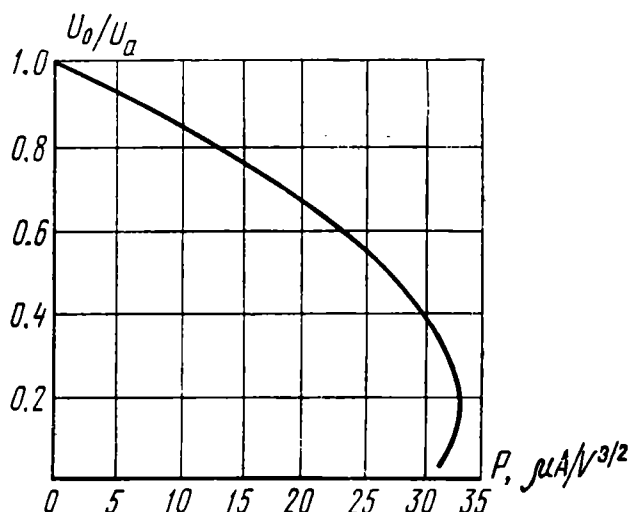


Fig. 2.11. Beam axial potential ratio *versus* microperveance

0.174 U_a . A further increase in the perveance is impossible, the beam becomes unstable, the axial potential drops to zero and a virtual cathode is formed.

If the axisymmetric beam does not completely fill the conductive tube ($r_0 < r_a$, where r_a is the inner radius of the tube), the beam boundary potential becomes lower than the potential of the tube, i.e. a part of the potential drop is applied across the gap between the beam and the internal wall of the tube. Assuming that the field in the gap is described by formula (2.7), the value of the potential drop ΔU across the space between the beam and the tube is as follows:

$$\Delta U = - \int_{r_0}^{r_a} E_r dr = \int_{r_0}^{r_a} \frac{I}{2\pi\epsilon_0 \sqrt{\frac{2e}{m} U}} \times \frac{dr}{r} \quad (2.59)$$

The value $U^{-1/2}$ in formula (2.7) was considered constant (equal to $U^{-1/2}$), since the potential drop across the beam was not taken into account when deriving this formula. Generally $U = U(r)$ and when calculating the integral (2.59), this dependence strictly speaking must be taken into account. However, the calculations show that the potential on the beam boundary with small gaps $[(r_a - r_0) \ll r_0]$ differs, but relatively less, from the potential on the conductive tube and in first approximation we may assume $U = U_a$ in (2.59). Then the potential drop across the gap is described by a simple expression:

$$\Delta U = \frac{I}{2\pi\epsilon_0 \sqrt{\frac{2e}{m} U_a}} \int_{r_0}^{r_a} \frac{dr}{r} = \frac{I}{2\pi\epsilon_0 \sqrt{\frac{2e}{m} U_a}} \ln \frac{r_a}{r_0} \quad (2.60)$$

By dividing all the terms of (2.60) by U_a , we obtain the relative change of the potential across the gap:

$$\frac{\Delta U}{U_a} = \frac{1}{2\pi\epsilon_0 \sqrt{\frac{2e}{m}}} \times \frac{I}{U_a^{3/2}} \ln \frac{r_a}{r_0} = 3.04 \times 10^{-2} p \ln \frac{r_a}{r_0} \quad (2.61)$$

Since the radius of the conductive tube of the most of real devices used in the super-high frequency range is 1.5 to 2 times the medium

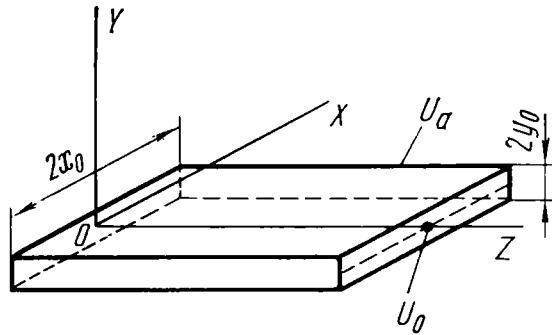


Fig. 2.12. Strip beam

radius of the beam $\ln \frac{r_a}{r_0} < 1$ and the relative "slack" of the potential in the gap at micropervances of a few units $\mu \text{ A/V}^{3/2}$ amounts to only a few per cent of U_a , i.e. we may roughly consider that the potential on the beam boundary equals the potential across the conductive tube.

Now consider a rectangular beam filling the conductive tube having the form of a parallelepiped (Fig. 2.12) $2x_0$ wide and $2y_0$ thick.

Assume that the beam is strip-type ($x_0 \gg y_0$) and long ($z \gg y_0$). In this case the potential drop along the axes OX and OZ may be

neglected as compared with the potential variation by the thickness of the beam (along the axis OY). The potential distribution through the thickness of the beam is described by the univariate Poisson equation:

$$\frac{d^2U}{dy^2} = -\frac{\rho}{\epsilon_0} \quad (2.62)$$

Assuming that the current density is the same in any cross section area of the beam and that the velocities of all electrons are parallel to the axis OZ (the beam does not expand), express ρ through j and $U(y)$ and substitute into (2.62)

$$\frac{d^2U}{dy^2} = aj_l U^{-1/2} \quad (2.63)$$

where $a = \frac{1}{2\epsilon_0 y_0 \sqrt{\frac{2e}{m}}}$ and j_l is the linear current density.

Double integration of equation (2.63) within the limits $U|_{U_0}^U$ (U_0 is the centre-plane potential) and $y|_{y_0}^y$ gives us:

$$\frac{4}{3} (U^{1/2} - U_0^{1/2})^{1/2} (U^{1/2} + 2U_0^{1/2}) = \pm 2 \sqrt{aj_l} y \quad (2.64)$$

which makes it possible to calculate the potential on any plane perpendicular to the axis OY . To determine the relative "slack" of the potential, use the boundary condition $U_{y=y_0} = U_a$, where U_a is the potential across the conducting parallelepiped. After the substitution $U = U_a$ and $y = y_0$ and squaring equation (2.64) we obtain

$$\left[1 - \left(\frac{U_0}{U_a}\right)^{1/2}\right] \left[1 + 2\left(\frac{U_0}{U_a}\right)^{1/2}\right]^2 = \frac{9}{4} ay_0^2 \frac{j_l}{U_a^{3/2}} \quad (2.65)$$

Express j_l in (2.65) through the total current I and the width of the beam $2x_0$: $j_l = I/2x_0$. Then

$$\begin{aligned} \left[1 - \left(\frac{U_0}{U_a}\right)^{1/2}\right] \left[1 + 2\left(\frac{U_0}{U_a}\right)^{1/2}\right]^2 &= \frac{9}{16} \times \\ &\times \frac{1}{\epsilon_0 \sqrt{\frac{2e}{m}}} \left(\frac{y_0}{x_0}\right) \frac{I}{U_a^{3/2}} = 1.07 \times 10.5 \left(\frac{y_0}{x_0}\right) P \end{aligned} \quad (2.66)$$

From expression (2.66) it follows that the relative potential drop across the beam depends on the perveance and the ratio of the thickness ($2y_0$) to the width ($2x_0$) of the beam. The higher the perveance and the ratio y_0/x_0 , the greater is the relative potential drop across the beam. The dependence of U_0/U_a on $P(y_0/x_0)$ is given in Fig. 2.13a. As seen from the drawing, the potential in the centre plane falls with an increase in the perveance and when $P\left(\frac{y_0}{x_0}\right)$ attains the

value $18.64 \times 10^{-6} \text{ A/V}^{3/2}$, the potential U_0 drops down to zero, the beam becomes cut off.

If the strip beam does not completely fill the conductive channel ($y_0 < y_a$, where $2y_a$ is the channel thickness), a part of the potential drop occurs across the gap between the beam and the channel wall. Figure 2.13b shows the ratio of the boundary potential of the beam, U_{y_0} , to the conductive channel potential U_a versus the perveance $P \left(\frac{y_0}{x_0} \right)$ at different values of filling the channel y_0/y_a . From the drawing it is clear that at moderate perveances, the ratio x_0/y_0

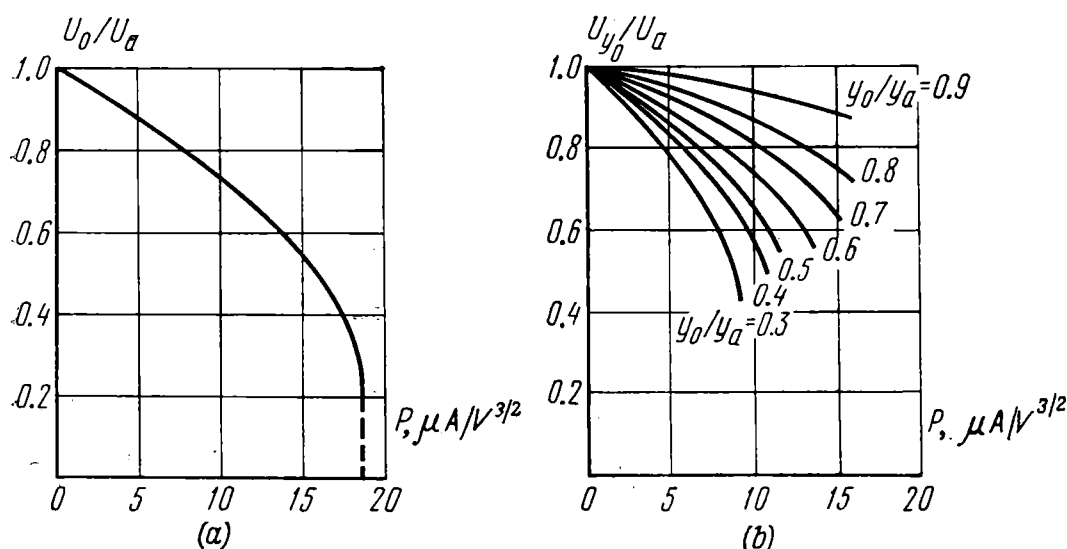


Fig. 2.13. Potential versus microperveance

(a) in the centre-plane of beam which fills conductive channel; (b) at boundary of beam which does not fill conductive channel

is not less than 10 (strip beam) and at the filling ratio $y_0/y_a > 0.5$, the potential of the beam boundary differs from the potential of the conductive channel by not more than a few per cent. The given estimation shows that when a strip beam does not completely fill the conductive channel the potential drop across the gap is small and may be neglected in many practical applications.

The analytical and experimental study of intensive electron flows of different configurations shows that in any case the beam infinitely expands in a space free from external fields. Restriction of the spreading by the external fields reduces the potential in the beam itself and, when the perveance is sufficiently great, the beam becomes unstable, a virtual cathode is formed, and the current is cut off. For any intensive beam there exists a critical value of space charge coefficient the excess of which cuts off the beam.

2.2. Analyses of Electrostatic Fields and Construction of Trajectories in the Presence of Space Charge

Analytic solution of potential distribution problems and finding of the shape of electron trajectories in intensive beams with due account for space charge are far from being attained in all practically important cases. More than that, it is attainable only by introducing a number of simplifying assumptions (see Section 2.1). Therefore, the analysis of electrostatic fields with account for space charge is often based on the methods of simulating such fields in an electrolytic tank and on the methods of grapho-analytic construction of the trajectories.

The method of simulating an electrostatic field with due account for space charge by means of an electrolytic tank furnished with

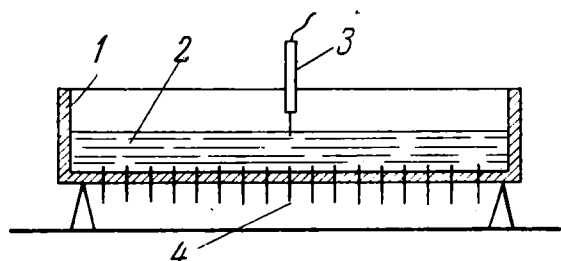


Fig. 2.14. Near-bottom current input elements
1—tank; 2—electrolyte; 3—probe; 4—current-carrying elements

current-input elements proposed by V. Lukoshkov has found the widest use. As an example, consider this method by solving a two dimensional (plane) problem. The potential distribution in a plane field having space charge is described by the Poisson equation

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = - \frac{\rho(x, y)}{\epsilon_0} \quad (2.67)$$

where $\rho(x, y)$ is the space charge density.

The essence of the V. Lukoshkov method consists in simulation of distributed space charge by currents fed into electrolyte by means of bottom current-input elements (Fig. 2.14).

Separate a small area Δs in a thin surface layer of electrolyte and assume that a current $I = j(x, y) \Delta s$ is fed to this area from outside. Using the known expressions for the current

$$I = \int j \, ds$$

and the current density

$$j = \sigma E$$

where σ is the electrolyte conductivity, we can set up an equation

$$\oint_l \sigma E_n dl = j(x, y) ds \quad (2.68)$$

where E_n is the field intensity component normal to the area Δs and the integration is carried out by the outline of the area Δs .

For the plane problem

$$\lim_{\Delta s \rightarrow 0} \frac{\oint \sigma E_n dl}{\Delta s} = \text{div}(\sigma E) \quad (2.69)$$

From (2.68) and (2.69) we obtain

$$\text{div}(\sigma E) = j(x, y)$$

Represent E as $-\text{grad } U$. Then

$$\text{div}(\sigma \text{grad } U) = -j(x, y) \quad (2.70)$$

With a constant conductivity of the electrolyte $\sigma = \sigma_0$ (const) equation (2.70) assumes the form

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = -\frac{j(x, y)}{\sigma_0} \quad (2.71)$$

i.e. coincides with the Poisson equation (2.67) in the case

$$\frac{j(x, y)}{\sigma_0} = \frac{\rho(x, y)}{\epsilon_0}$$

Thus, it shows a principal possibility of simulating the distributed volume charge by a current fed into the electrolyte. However, strictly speaking, the continuous distribution of the volume charge density can be simulated only by continuously distributed current sources and this leads to the necessity of using many and many small current terminals for feeding the current into the electrolyte which is impractical. With many fine (needle) current terminals extending to the electrolyte surface created near each current terminal a sharp rise of the potential corresponding to the logarithmic infinity of the potential in the centre of the infinitely thin needle, i.e. the obtained potential distribution will be very far from the real value available at continuously distributed density of volume charge.

This difficulty can be overcome, as it has been theoretically shown and experimentally confirmed by V. Lukoshkov, by means of short near-bottom current terminals under which there is still a fairly thick layer of electrolyte. The current "spreads" near the bath bottom in the electrolyte layer in all directions producing a "cloud" of continuously distributed sources on the electrolyte surface. In this case it is possible to simulate a continuous distribution of sources composed of "clouds" created by separate current terminals on the electrolyte surface with an accuracy sufficient for many practical problems. Of

course, there are small potential discontinuities on the electrolyte surface caused by the discreteness of the current terminals. However, the resultant error remains within the accuracy limits of the electrolytic bath method.

In practical application of the near-bottom current terminals method it is necessary to determine the correspondence between the density of space charge in the investigated electron flow and the values of the current fed into the electrolyte through the current terminals. Designate the volume of electrolyte in the bath served by the current of the n th terminal through v_n . Since the current terminals are usually located at the corners of a squared grid, v_n is the volume of a straight or inclined (when simulating axisymmetric fields) prism with a square base. Designate a similar volume with distributed density of space charge ρ_n of the simulated electron flow through V_n . It is clear that the volume V_n contains a charge $q_n = \rho_n V_n$. Express the space charge density through the current density and the electron velocity at the corresponding point of the electron flow

$$q_n = V_n \frac{j_n}{\sqrt{\frac{2e}{m} U_n}} \quad (2.72)$$

where U_n is the potential at the point of the electron flow corresponding to the n th current terminal.

Now let us introduce a concept of a "current tube" in an electron flow. Assume that the flow is laminar and separate therein a channel restricted by the paths of electrons emitted from the contour of the area S_c on the cathode. It is this channel that is called current tube. Obviously, the total current is invariable in any cross section area of the tube

$$I = j_c S_c = j_n S_n \quad (2.73)$$

where j_c is the current density of the cathode; S_n is the cross-sectional area of the current tube.

Further calculations will be made on the basis of dimensionless potential $\Phi_n = U_n/U_a$, where U_a is the reference voltage, which in most cases is conveniently selected as the greatest potential difference between the electrodes of the simulated system (usually the anode voltage). The dimensionless potential is the same at the corresponding points of the original and the model.

When simulating electron-optical systems in an electrolytic tank, as has been indicated in Section 1.4, it is expedient to make use of enlarged model electrodes and voltages of the model electrodes usually reduced in proportion to the voltages of the originals. Next, use scale factors k_l for linear dimensions and k_u for voltages. If that is the case, the linear dimensions L of the system being analyzed are

associated with the linear dimensions l of the model through the relationship $L = k_l l$, while the areas and volumes of the original and the model, through the relationships $S = k_s^2 s$, $V = k_v^3 v$. The electric field E of the original is coupled with the field E_M at a corresponding point of the model through the expression $E = (k_U/k_l) E_M$.

Finally, employ the dimension correspondence factor coupling the current I (a) fed into the electrolyte with the interelectrode space charge of the system under analysis according to equation (2.72). Using the known expressions for the current flow in electrolyte through the area Δs : $\Delta I = \sigma_0 E_M \Delta s$ and for vacuum charge $\Delta q = \epsilon_0 E \Delta S$, determine the correspondence factor

$$k = \frac{\Delta I}{\Delta q} = \frac{\sigma_0 E_M \Delta s}{\epsilon_0 E \Delta S}$$

or, taking into account the scale factors for E and ΔS ,

$$k = \frac{\sigma_0}{\epsilon_0 k_U k_l} \quad (2.74)$$

Then the current I_n fed through the n th bottom current terminal simulating the space charge q_n concentrated in the volume V_n of the original is obtained by multiplying expression (2.73) by the correspondence factor (2.74). Using relationship (2.74) and the dimensionless potential Φ , finally gives us

$$\begin{aligned} I_n = k q_n &= \frac{\sigma_0 j_c k^2}{\epsilon_0 \sqrt{\frac{2ek_U}{m} U_a}} \left(\frac{s_c}{s_n} \times \frac{v_n}{\sqrt{\Phi_n}} = \right. \\ &= \sigma_0 u_a \left(1.91 \times 10^5 \frac{j_c k_l^2}{U_a^{3/2}} \right) \left(\frac{s_c}{s_n} \times \frac{v_n}{\sqrt{\Phi_n}} \right) \end{aligned} \quad (2.75)$$

where $u_a = U_a/k_U$ is the reference voltage in the electrolytic tank.

In equation (2.75) the multiplier $\sigma_0 u_a$ remains constant in the process of solution of the whole problem (provided the electrical conductivity σ_0 is constant), the other multiplier (the expression in the brackets) is constant for the given current tube in the given approximation, the third multiplier ($s_c v_n / s_n \sqrt{\Phi_n}$) is calculated for each (n th) current carrying element.

The practical solution of the potential distribution problems in the presence of space charge is carried out by the successive approximations method. The necessity of using the successive approximations method is dictated by the fact that neither the potential distribution, nor the space charge distribution, nor the run of electron trajectories forming the current tubes is known at the beginning. Owing to this, the potential distribution is found without space charge as zero approximation by the usual method. Then, one of the grapho-analytic methods or a trajectory tracer is used for plotting

the electron paths and the whole electron flow is divided into current tubes. After that, the current density of the cathode is calculated.

The known $3/2$ power law may be used for a simple calculation of individual sections of the cathode with the space between the selected cathode section and the found nearest to the cathode equipotential surface, and is approximately parallel to the cathode surface, serving as an interelectrode space of a planar diode. Taking into account the found value of j_c and the graphically determined wide-

ning of the current tube $\left(\frac{S_n}{S_c}\right)$, the current values of the current supply elements used by the selected current tube are calculated by formula (2.75). The currents I_n of the current supply elements used in all the other current tubes, into which the electron flow is divided, are found in a similar way. The calculated currents I_n are applied to the corresponding current supply elements. Then, the equipotential lines are taken again and the electron trajectories are plotted. Thus we obtain the first approximation.

The data of the first approximation with due account for the change in the potential distribution and electron trajectories and, therefore, the current tubes are used for recalculating the currents of the current supply elements. Then the currents I_n are set up according to the calculation, and the equipotentials and electron trajectories of the second approximation are found, etc. The process is continued until the potential distribution obtained in the n th approximation differs from that obtained in the $(n-1)$ th approximation by a value which remains within the total accuracy of the electrolytic bath method. In most cases 4 to 5 approximations are sufficient.

The calculation of the currents flowing in the current supply elements and the grapho-analytic plotting of electron trajectories are the most time-consuming stages of the solution process. Therefore, when using the successive approximations method, the use of a trajectory tracer for automatically plotting the trajectories is preferable. A diagram of the electrolytic tank with current supply elements is given in Fig. 2.15.

To set the specified current values of the current flowing in the current supply elements, each of them is connected in series with a network of three resistors, the two variable resistors (R_1 and R_2) being used for controlling the current flow in the current supply element, while the third calibrating resistor (r_1) being used for measuring the current I_n by the voltage drop across this resistor, as the measurement of low a.c. voltages can be made with a sufficient accuracy, by a vacuum-tube voltmeter for instance.

The electrolytic tank with near-bottom current supply elements can be used also for simulating axisymmetric electric fields with due account for the space charge. In this case the tank bottom is made inclined. The experience of operating the electrolytic tank with

current supply elements shows that when the experiment is carried out fairly well, the accuracy of determining the potential distribution with due account for the space charge lies within 1% which is quite tolerable in many practical cases.

Another method for simulating an electrostatic field with account for space charge was proposed by French scientist Mussone-Jenone and developed in the USSR by V. Breitman. According to this method suitable for solving two-dimensional and axisymmetric problems, the distortion of the field by space charge is imitated by varying the thickness of the electrolyte layer in the interelectrode space of

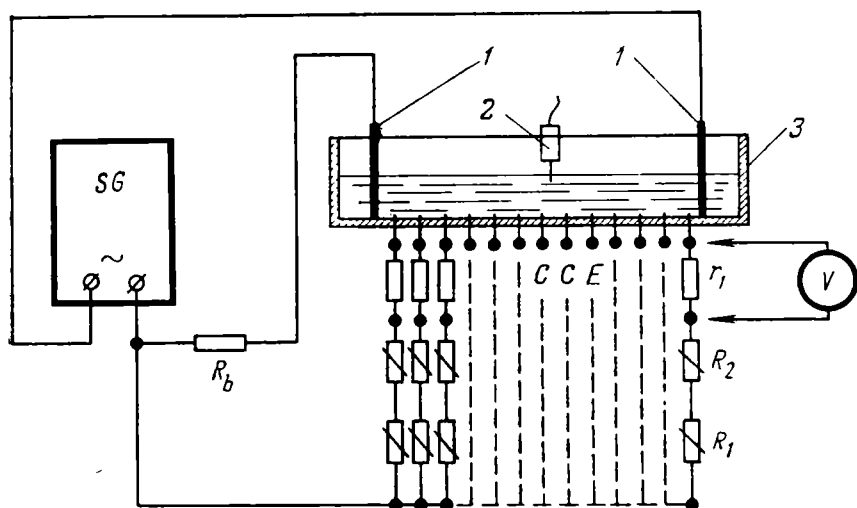


Fig. 2.15. Electrolytic tank with current-carrying elements
1—electrode; 2—probe; 3—tank

the model immersed into the electrolytic tank. The change in the thickness of electrolyte layer in the tank is obtained by deformation of the bottom. Therefore, this method is often referred to as a method of an electrolytic tank with a shaped bottom.

Let the method of the electrolytic tank with shaped bottom be considered by simulating a plane field as an example. As it has been shown in Section 1.4, the potential distribution in the electrolyte of flat electrolytic tank is described by the two-dimensional Laplace equation (1.37). Now assume that the tank bottom is deformed in such a way that the electrolyte layer between the electrodes of the model is not uniform (Fig. 2.16).

In this case the current lines in the electrolyte pass around the bottom irregularities, i.e. there will appear current density components j_z perpendicular to the electrolyte surface. The appearance of these components is accounted for by the presence of the electric field components E_z perpendicular to the plane XOY (electrolyte surface) which follow the bottom profile. Thus, with non-uniform

thickness of the electrolyte layer there is a non-zero derivative in the volume of the tank

$$-\frac{\partial E_z}{\partial z} = \frac{\partial^2 U}{\partial z^2} \neq 0$$

and the potential distribution is described by the three-dimensional Laplace equation

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0 \quad (2.76)$$

The last component in the general case is a function of the coordinates x , y and the thickness (depth) h of the electrolyte layer:

$$\frac{\partial^2 U}{\partial z^2} = -\frac{\partial E_z}{\partial z} = -f(x, y, h)$$

Substituting the last equation into (2.76), we obtain an equation coinciding with the two-dimensional Poisson equation (2.67) when

$$f(x, y, h) = \frac{\rho(x, y)}{\epsilon_0}$$

Thus, if the bottom of the bath is so shaped that at any point on the electrolyte surface the function $f(x, y, h)$ coincides with the function of the distribution of the space charge density $\rho(x, y)/\epsilon_0$, then the potential distribution measured in the bath corresponds to the picture of the field in the system being simulated with account for the space charge. The thickness of the electrolyte layer is associated with the space charge density through the relationship

$$\ln h = - \int_{s_0}^s \frac{\rho ds}{\epsilon_0 |\text{grad } U|} \quad (2.77)$$

where the integration is carried out along the field line of force s .

Since neither the value $|\text{grad } U|$, nor the position of the line of force is known, in practical use of a shaped bottom tank the problem

has to be solved by the successive approximations method, like the case is with the tank furnished with current supply elements. The picture of the field obtained in the tank with a flat bottom, i.e. without taking into account the space charge, is used as zero

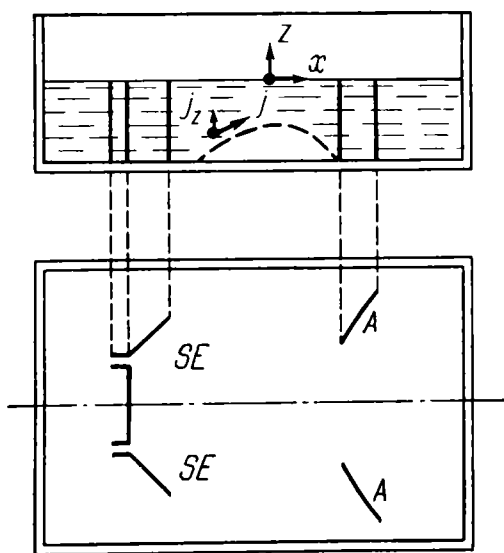


Fig. 2.16. Electrolytic tank with shaped bottom

approximation. Then this field picture is used to determine the grad U , the values of the current and potential are used for determining ρ and equation (2.77) is used for calculating h . The bottom is deformed in accordance with the obtained value of h and the field picture is taken again to obtain the first approximation. Then, the data of the first approximation are used for calculating the new values of h , the shape of the bottom is correspondingly changed and the field picture is taken in the second approximation. Repeating these operations several times, one can obtain a potential distribution which differs, but little from the preceding one and describes the field in the simulated system, allowing for the space charge with an accuracy sufficient for practical purposes. The electrolytic bath with a shaped bottom, when inclined, can also serve for simulating axisymmetric systems.

The method of shaped bottom electrolytic tank has found less use mainly due to practical inconveniences associated with the necessity of removing the electrolyte and model from the tank, changing the bottom shape, refilling it with electrolyte, and setting the model before proceeding to the next approximation. Even with easily deformed bottom materials such as wax, the solution of the problem in the fifth or sixth approximation takes much time. Furthermore, the accuracy of this method is not as good as that of the method using an electrolytic tank with current supply elements.

If the field, taking account for space charge, is calculated or found experimentally in the form of equipotential lines system, the construction of the electron trajectories can be performed by one of the grapho-analytic methods described in Section 1.7. Another technique is possible. Calculate or simulate the field neglecting the space charge (see Section 1.4), the action of this charge being taken into consideration when plotting the trajectories. Let this method be considered, using an axisymmetric electron beam as an example. As shown in Section 2.1, the radial component of the intensity of the electric field produced by space charge at the boundary of the beam is approximately equal to $E_r = -1/2 \pi \epsilon_0 r \sqrt{\frac{2e}{m}} U$ [see equation (2.6)].

This expression can also be used in the first approximation in the presence of an external field, if the value of U determining the electron energy is equal to the real value of the potential in the field under consideration.

The method for plotting electron trajectories with due account for the space charge is based on an assumption that the electron velocity vector at any point of trajectory is equal to the sum of the vectors of: the velocity $\mathbf{v}_e$ determined by the external field and the velocity $\Delta \mathbf{v}$ determined by the action of the space charge

$$\mathbf{v} = \mathbf{v}_e + \Delta \mathbf{v}$$

The value v_e equals $\sqrt{\frac{2e}{m} U}$, and the direction is calculated by the broken line method (see Section 1.7), without taking into account the action of space charge. Find the additional velocity Δv . The force pulse F_r produced by the repulsive action of the space charge is equal to

$$m \Delta v = F_r \Delta t = -e E_r \Delta t \quad (2.78)$$

Since $\Delta v \ll v_e$ (the relative variation of the potential in the beam is small), the value Δt can be estimated as

$$\Delta t = \frac{\Delta l}{v_e} = \frac{\Delta l}{\sqrt{\frac{2e}{m} U}} \quad (2.79)$$

Then on the basis of (2.6), (2.78) and (2.79) for the Δv we obtain an expression

$$\Delta v = \frac{I}{4\pi\epsilon_0 U} \times \frac{\Delta l}{r} \quad (2.80)$$

where Δl is the length of the trajectory section.

The Δv is opposite to E_r , i.e. is directed normally and outside with regard to the beam axis. The practical construction of the trajectory with due account for the action of space charge is carried out as follows. The calculated or simulated field (without taking

into account the space charge) is presented in the form of equipotential lines system located in a meridian plane (Fig. 2.17).

The velocity vector $v_{e1} = \sqrt{\frac{2e}{m} U_1}$ is plotted in a definite scale from the original point O on the equipotential U_1 . A straight line parallel to the axis is drawn through the end of this vector (point A).

Then the value $v_{e2} = \sqrt{\frac{2e}{m} U_2}$ is calculated and the velocity vector v_{e2} is drawn in the same scale from the point O so that its end lies on the straight line parallel to the axis drawn through the end of the vector V_{e1} (point C). The section AC is halved (point B). The value Δv is calculated by formula (2.80); where: Δl is the distance between the equipotentials U_1, U_2 ; r is the mean radius of the beam (the distance of the trajectory from the axis between the equipotentials

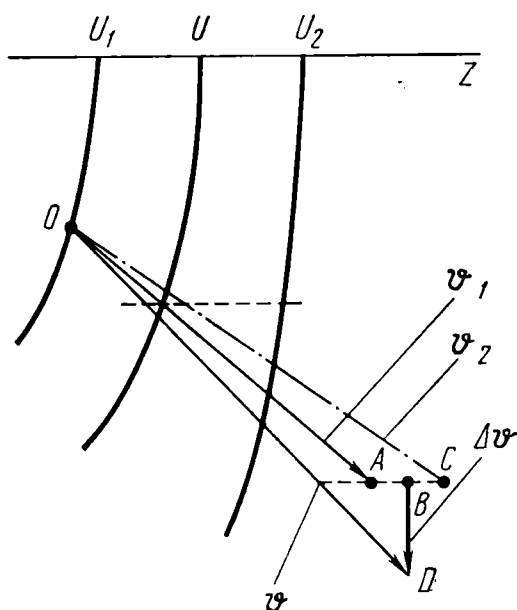


Fig. 2.17. Construction of trajectory with allowance for space charge action

U_1, U_2). Next the vector $v\Delta$ is plotted from the point B (in the same scale). Connecting the point O with the end of the vector Δv , gives us a vector of electron velocity v (section OD) with account for the action of the space charge. A portion of the section OD enclosed between the equipotentials U_1, U_2 is the electron trajectory in this area sought for. Then, the broken line method is used for determining the angle of refraction of the trajectory at the equipotential U_2 . A length of the trajectory is plotted in the interval U_2, U_3 without taking into account the space charge. Next, the values of v_{e3} and the new value of Δv are found, and another length of the trajectory is plotted now with account for the space charge between the equipotentials U_2, U_3 and so on. Finally the whole path of the electron is presented by a broken line closely coinciding with the true trajectory.

The construction of electron trajectories by the above method has shown that with microperveances of up to several units of $\mu \text{ A/V}^{3/2}$, the error does not exceed a few per cent what is quite tolerable in many practical cases. More than that, this method of plotting the trajectories requires no calculation or simulating of the field with account for space charge, which saves much time.

Fairly accurate and rapid finding of electron trajectories with due account for space charge is obtained through combining an electrolytic tank furnished with current supply elements and a trajectory tracer. With a trajectory tracer the available trajectories are found by the successive approximations method. First, the conventional method is used to simulate an electric field in the tank without taking into account the space charge and the trajectory tracer is used for drawing a family of trajectories (zero approximation). Then, the obtained trajectory distribution and the specified current of the beam are used to calculate the currents in the current supply elements. Next, the field is simulated with account for the space charge and the trajectories are drawn again (first approximation). The obtained distribution of trajectories is used for correcting the currents in the current supply elements, the field is resimulated and the trajectories are drawn again (second approximation), and so on. As a rule, the fourth approximation differs but slightly from the third one so that the process of finding the trajectories is completed at this stage. Application of a tank furnished with current supply elements and a trajectory tracer allows electron trajectories to be found in the presence of space charge accurate within 1.5 to 2%. This method is of special convenience when use is made of the automatic technique for calculating currents in the current supply elements. Needless to say that the highest accuracy of calculation of the fields and trajectories with account for space charge is provided by electronic computers.

2.3. General Problems in Forming Intensive Electron Beams

An intensive electron beam, which is not acted on by external fields, expands due to the repulsive force of space charge. Therefore, an intensive beam can be shaped and guided through a channel with a limited cross section only when the space charge forces are compensated for by external electric or magnetic fields.

Analytic or experimental determination of electric or magnetic fields needed to compensate for the space charge forces and, therefore, to provide an electron beam of required configuration is one of the fundamental problems of the intensive beam optics.

It should be noted that the concept of "electron optics", when applied to intensive beams, is to some extent, conventional, since the focusing, i.e. convergence of electron trajectories to a single point and producing electron-optical images by means of beams with perveance not too small has no sense. Theoretically, only an infinitely wide strip beam can be reduced to a line, i.e. a line focus be obtained. The beams of finite section cannot be focused in the strict sense, since the repulsive forces of the space charge grow infinitely, when the beam is compressed materially. Despite this conditionality, the term intensive beams "focusing" is widely used and, when speaking of focusing high-perveance beams, one should remember that by this is meant shaping an intensive beam and holding it within specified boundaries. The accepted terminology includes such terms as focusing and defocusing forces, where the focusing forces are forces directed to the axis of an axisymmetric beam or to the centre-plane of a strip beam, while by the defocusing forces are meant forces of the opposite direction. In this sense the forces widening the beam at the expense of space charge are defocusing and the forces of external fields which prevent the beam widening are focusing. The force caused by the intrinsic magnetic field of beam electrons in motion, which manifests itself only at relativistic velocities, is a focusing force too.

Thus, generation of an intensive electron flow and its confinement within definite limits can be called both as shaping (what is more accurate) or focusing (what is to some extent conventional) of an intensive electron beam.

As stated above, the compensation for the space charge repulsive force at the beam boundary by the force of an external field is a prerequisite of the existence of an intensive beam, i.e. at the beam boundary the focusing force normal to the boundary must equal in magnitude and oppose in direction the defocusing force of space charge. However, in order to provide the beam stability (its existence for a long period of time), the zero sum of the focusing and defocusing forces does not suffice. The beam is stable only when electron displacements from the boundary trajectory in either direction

cause a force returning the electron involved to the beam boundary. It is evident that the beam is the more stable, the greater the absolute value of the returning force and the higher the rate at which it rises in case of electron departure from the boundary trajectory, i.e. the higher the derivative of the force in the direction normal to the beam boundary. The value of this derivative (dF_n/dr for an axisymmetric beam) is one of the parameters of the focusing system which characterize the focusing stability. We may say that in a stable beam any electron at the beam boundary is in a "potential pit": the "deeper" this "pit" and the steeper its "slopes", the better the focusing and the more stable is the beam.

Generally intensive beams can be shaped (focused) by electric fields produced by a system of electrodes, magnetic fields produced by coils (solenoids) or permanent magnets, and by a combination of electrostatic and magnetic fields (focusing by superimposed fields). The fields may be constant, monotonically variable along the beam or have a periodic structure. In accordance with this, there are known electrostatic focusing of intensive beams, magnetic focusing and focusing by periodic electrostatic and magnetic fields. Since the Lorentz magnetic force is proportional to the electron velocity, and the magnetic field cannot be in principle accelerating (varying the energy of the electrons), before entering a magnetic focusing system, the beam electrons must be accelerated by an electric field. The accelerating electric field is usually used for shaping the initial portion of the electron flow. The function performed by the magnetic field is to confine the flow preliminarily shaped by the electric field, i.e. to prevent beam widening. Therefore, alongside the term "magnetic focusing" use is made of such a notion as magnetic field "beam limitation".

Finding the fields shaping intensive electron beams is an "external" problem with respect to the beam in contrast to the "internal" problem dealing with potential distribution inside the beam and on its boundary. The difference between the solution of the internal and external problems stems from the different equations describing the potential distribution inside and outside the beam. The field inside the beam is described by the Poisson equation, while that outside the beam, where there is no space charge, by the Laplace equation. Thus, when developing systems for shaping intensive beams two problems must be solved: internal and external, each of which is fairly complex in many practically important cases.

2.4. Electrostatic Focusing of Intensive Beams

The internal problem for electron flows having straight-line trajectories coinciding with electric lines of force, such as laminar flows between parallel flat electrodes, between two coaxial cylinder

electrodes and two concentric spheres, can be solved analytically with a sufficient accuracy. This problem is solved by the well-known equation of the three halves power law widely used in the current-circuit theory of electron valves. With a flat system of electrodes, for instance, we have

$$j = a \frac{[U(z)]^{3/2}}{z^3} \quad (2.81)$$

where $a = 2.33 \times 10^{-6} \text{ A/V}^{3/2}$; $U(z)$ is the potential in the plane parallel to the electrodes positioned at distance z from the initial electrode (cathode).

When deriving equation (2.81), the cathode potential is taken as zero and it is assumed that the electron velocities in the cathode plane (when $z = 0$) are also zero. It is evident that in a two-dimensional coordinate system the field intensity component $E_z \neq 0$, and the components E_x and E_y are identically equal to zero. In this case all the electrons of the flow have only one velocity component v_z different from zero, i.e. all the electron trajectories are linear and normal to the electrode surface, i.e. parallel to the axis OZ .

In the cylindrical and spherical coordinate systems the electron trajectories are also linear and directed along the radii of cylinder and sphere respectively.

Thus in any system mentioned above the potential distribution along any linear trajectory is known. From the condition of existence of linear trajectories it follows directly that an electric field strength component normal to the trajectory is equal to zero, i.e. there is no force capable of bending the trajectory.

From the known solution of the internal problem a general approach can be formulated to the solution of the problem on shaping an intensive flow of limited cross section by means of an electric field. Assume that cut from an unlimited flow between two flat and parallel electrodes is a parallelepiped with edges parallel to the axis OZ or a cylinder with generating lines parallel to the axis OZ , i.e. lines coinciding with linear electron trajectories. In the first case we obtain a limited beam of a rectilinear cross section (strip beam), in the other case we have an axisymmetric (cylindrical) beam. In a similar manner, from flow between coaxial cylinders a wedge-shaped (converging strip) beam can be obtained while from a flow between concentric spheres converging axisymmetric (conical) beam can be obtained.

However, the portion of the electron flow remaining outside the limited beam cannot be rejected, unless the boundary conditions of the beam are to be changed. In particular, the field intensity component normal to the beam boundary would be other than zero

* Equation (2.81) is obtained from (2.52) by substituting $z = l$, $U_{\text{min}} = 0$.

(see Section 2.1). It is clear that generally an unlimited flow can be obtained, providing the field which existed at the beam boundary due to the rejected portion of the electron flow which surrounded the cut-out beam is produced by a system of electrodes of certain shape at certain potentials, which is located outside the beam. This principle underlies the shaping of intensive electron beams by an electric field.

Thus, in order to confine an intensive beam within specified limits by means of an electric field, the field produced at the beam boundary by the external electrodes must satisfy the following conditions: the potential distribution along the beam boundary must allow describing by a continuous function determined by the three halves power law, and the field intensity component normal to the beam surface must equal zero at all points of the beam surface.

Now, using the formulated general principle as a basis consider the focusing of a rectangular beam by an electric field. From equation (2.81) we directly obtain the function of potential distribution along the beam boundary, $U(z)$:

$$U(z) = \frac{1}{a^{2/3}} j^{2/3} z^{4/3} = Az^{4/3} \quad (2.82)$$

where $A = \frac{1}{a^{2/3}} j^{2/3} = 5.69 \times 10^3 j^{2/3}$.

Align the origin of coordinates with the beam boundary in the initial plane ($z = 0$) and direct the axis OY perpendicularly to the wide side of the beam. In order to find the potential distribution outside the beam, the two-dimensional Laplace equation with boundary conditions $U(z)|_{y=0} = Az^{4/3}$ and $\partial U / \partial y|_{y=0} = 0$ must be generally solved. However, since the $U(0, z)$ (at the beam boundary) is known, we can find $U(y, z)$ without solving the Laplace equation due to the fact that any function of the complex variable, as well as its real and imaginary parts, satisfies the Laplace equation. Let the $U(y, z)$ be presented in the form of the real part of the function of the complex variable $(z + iy)$

$$U(y, z) = \operatorname{Re}[f(z + iy)] = \frac{1}{2}[f(z + iy) + f(z - iy)] \quad (2.83)$$

Function (2.83) introduced in this manner satisfies the Laplace equation and the given boundary conditions. Since according to the first boundary condition, at $y = 0$ the $U(0, z) = Az^{4/3}$, for the region outside the beam ($y > 0$)

$$\begin{aligned} U(y, z) &= \operatorname{Re}[A(z + iy)^{4/3}] = \operatorname{Re}[A(z^2 + y^2)^{2/3} e^{i\frac{4}{3}\Theta}] = \\ &= A(z^2 + y^2)^{2/3} \cos \frac{4}{3}\Theta \end{aligned} \quad (2.84)$$

where $\Theta = \arctan \frac{y}{z}$.

Equation (2.84) describes the potential distribution outside the beam ($y > 0$) providing for the existence of a strip beam. Designate the potential of the accelerating electrode (anode) as U_a , the beam length, as l and use the dimensionless coordinates y/l and z/l . Then, using equation (2.84), we may conveniently represent the potential distribution of the beam as a family of equipotential lines $U/U_a = \text{const}$ in the plane YOZ (Fig. 2.18).

From the figure it is clear that all the equipotential surfaces $U/U_a > 0$ are square with the beam boundary. Only the "cathode"

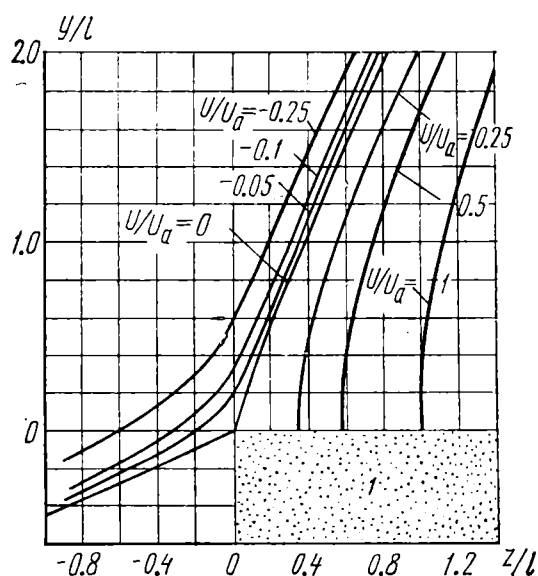


Fig. 2.18. Potential distribution near strip beam
1—beam boundary

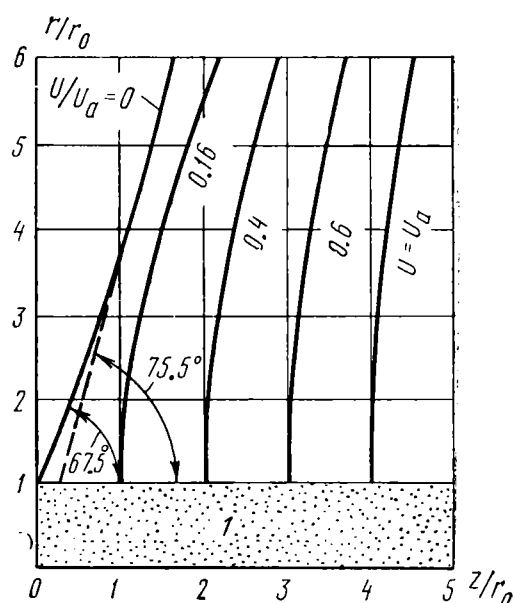


Fig. 2.19. Potential distribution near cylindrical beam
1—beam boundary

equipotential surface ($U = 0$) is a plane forming an angle Θ_1 with the cathode. This angle is determined from the condition

$$Ay^{4/3} \cos \frac{4}{3} \Theta_1 = 0 \quad (2.85)$$

from which

$$\Theta_1 = \frac{\pi}{2} / \frac{4}{3} = \frac{3}{8} \pi = 67.5^\circ \quad (2.86)$$

Practically, a strip beam is shaped by placing the cathode between two planes adjoining its edges and forming an angle of 67.5° with regard to the beam axis. The anode should be an electrode having the shape of equipotential surface $U = U_a$ with a slot equal to the thickness of the beam (cathode).

An axisymmetric (cylindrical) beam can be obtained in a similar way. To this end, a cylinder is cut out of the solid flow filling the space between a flat cathode and anode which are perpendicular to

the axis OZ , the cylinder generating lines being parallel to the axis OZ . The problem is reduced to a solution of the Laplace equation for the region $r \geq r_0$ (r_0 is the beam radius). It is obvious that in this case the first boundary condition $U(z, r)|_{r=r_0} = Az^{4/3}$ remains valid, while the other one assumes the form $\partial U/\partial z|_{r=r_0} = 0$. Such a problem has no strict analytic solution. The potential distribution outside the beam can be found either by means of an approximate analytic solution or experimentally — by simulating it in an electrolytic tank.

An adequate accuracy is obtained when the potential distribution near the beam is approximated by the series

$$U(z, r) = Az^{4/3} \left[F_0\left(\frac{r}{r_0}\right) + \left(\frac{r_0}{z}\right) F_1\left(\frac{r}{r_0}\right) + \left(\frac{r_0}{z}\right)^2 F_2\left(\frac{r}{r_0}\right) + \dots \right] \quad (2.87)$$

where $F_0, F_1, \dots, F_n$ are cylindrical functions.

However, near the cathode ($z < r_0$) the solution in cylindrical functions is less suitable, and with $z = 0$ (on the cathode) it is not suitable at all. Therefore, when dealing with low values of z , one has to seek another approximate solution or to simulate the field in an electrolytic tank.

The possibility of simulating the field in an electrolytic tank at the presence of an electron beam stems from the existence of the boundary condition, i.e. when the field intensity component normal to the beam surface ($\partial U/\partial r$ in the base of an axisymmetric beam) is equal to zero. This condition is automatically satisfied at the boundary of any dielectric immersed into the tank in which the field is being simulated. So it is, the current does not enter the dielectric, the current lines envelope the dielectric so that they are parallel to the dielectric surface near it, while the equipotential surfaces (to which the current lines are always perpendicular) are normal to the dielectric surface. Therefore, if a dielectric plate is placed into an electrolytic tank with an inclined bottom (see Section 1.4) in parallel to the "shore line" at a distance r_0 (in the simulating scale) from this line, this plate simulates the beam boundaries at which one of the boundary conditions $\partial U/\partial r|_{r=r_0} = 0$ is always satisfied. The electrodes serving as models of the cathode and anode are installed near the ends of the dielectric plate. The tank supply voltage is applied to the electrodes. The second boundary condition $U(r_0, z) = Az^{4/3}$ is satisfied by varying the shape and position of the electrodes (their inclination to the boundary of the beam). It is evident that the found system of electrodes produces an axisymmetric (cylindrical) beam.

Figure 2.19 illustrates a family of equipotential lines $U/U_a = \text{const}$ in the meridian plane of an axisymmetric field producing a cylindrical beam.

As shown in the figure, the zero (cathode) equipotential surface near the beam boundary has the form of a cone with a generating line inclined at an angle of 67.5 degrees to the axis. The aperture angle of the cone gradually increases with the distance from the beam and at $r > 2r_0$ the cathode electrode assumes the shape of a cone with the generating line inclined at about 75 degrees to the axis. The anode electrode (equipotential surface $U = U_a$) is shaped as a lightly concave body of revolution having a hole equal to the beam diameter, the electrode being perpendicular to the beam boundary near the edge of the hole.

A converging (wedge-shaped) strip beam is obtained by cutting out a portion of the flow filling the space between two coaxial cylinders. The potential distribution along the beam boundary by the

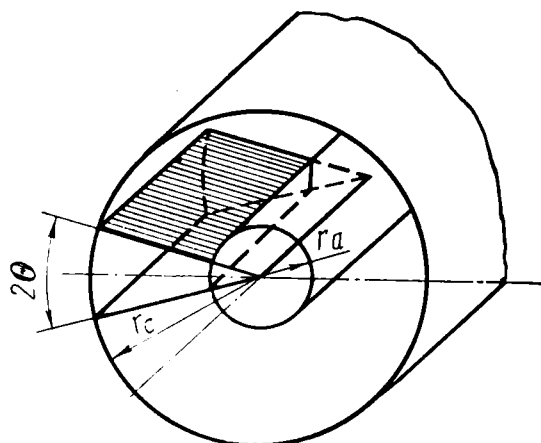


Fig. 2.20. Formation of converging strip beam

cylinder radius is described by the three halves power law for a cylindrical system

$$I_a = 14.68 \times 10^{-6} \frac{l_a}{r_a \beta^2} U_a^{3/2} \quad (2.88)$$

where l_a and r_a are the length and radius of the anode; $\beta^2 = r \left(\frac{r_a}{r_c} \right)$ is the tabulated Langmuir function known from the electron valve theory (r_c is the cathode radius).

The converging strip beam is produced when $r_c > r_a$, i.e. when a cylindrical anode is placed inside a cylindrical cathode. For such an "inverted" cylindrical diode expression (2.88) holds true at $\beta^2 = f \left(\frac{r_c}{r_a} \right)$. Assume that the beam convergence angle is $\psi = 2\theta$ (Fig. 2.20). Then, the current per unit of the beam width j_l (linear current density), according to (2.88), is equal to

$$j_l = a \frac{R_a}{r_c \beta^2 (R_a)} \left(\frac{\theta}{\pi} \right) U_a^{3/2} \quad (2.89)$$

where $a = 14.68 \times 10^{-6} \text{ A/V}^{3/2}$ and $R_a = r_c/r_a$.

Since the current lines run along the radii, the total current I and, therefore, the current density j_l , are the same in any cylindrical surface coaxial to the cathode and anode and located between them. Therefore, for the current in the section of a wedge-shaped beam with an angle of convergence of 2Θ by any cylindrical surface coaxial with the cathode and anode and located between them ($r_a < r < r_c$) it can be written

$$j_l = a \frac{R}{r_c \beta^2(R)} \left(\frac{\Theta}{\pi} \right) [U(r)]^{3/2} \quad (2.90)$$

where $R = r_c/r$.

By setting a ratio from expressions (2.89) and (2.90) we find the potential distribution along the radius (along the beam boundary)

$$U(r) = \left[\frac{R_a \beta^2(R)}{R \beta^2(R_a)} \right]^{2/3} U_a = k U_a \left[\beta^2 \left(\frac{r_c}{r} \right) r \right]^{2/3} \quad (2.91)$$

$$k = \left[\frac{1}{r_a \beta^2 \left(\frac{r_c}{r_a} \right)} \right]$$

Expression (2.91) is the first boundary condition for an external problem. The other boundary condition, as before, stems from the fact that the field component normal to the beam boundary is equal to zero:

$$\left. \frac{\partial U}{\partial \psi} \right|_{\psi=\Theta} = 0 \quad (2.92)$$

where ψ is the azimuthal angle.

Thus, this problem is reduced to the solution of the Laplace equation for the region $\psi \geq \Theta$ with the boundary conditions (2.91), (2.92). In the general form this problem has no analytic solution. Approximate calculations and simulating in an electrolytic tank show that a converging strip beam can be formed by means of cathode electrodes which form angles of 67.5° with the beam boundary and adjoin a cathode which is a cylindrical surface from both sides. The anode electrode may be made in the form of a part of a cylinder having a radius lower than r_a with a slot equal to the beam opening. Figure 2.21 shows a family of equipotential lines of a field forming a wedge-shaped beam with a convergence angle $\Theta = 20^\circ$.

An axisymmetric converging (conical) beam results from cutting out (with a conical surface) a portion of an electron flow filling the space between two concentric spheres. The potential distribution along the boundary of the conical beam (along the radii of the spheres) can be obtained from the three halves power law for a spherical diode

$$I_a = 29.35 \times 10^{-6} \frac{1}{(-\alpha)^2} U^{3/2} \quad (2.93)$$

where $(-\alpha)^2 = f\left(\frac{r_a}{r_c}\right)$ is the function of the ratio between the radii of the cathode and anode spheres.

The graph of this function is presented in Fig. 2.22.

The total current of the spherical anode is as many times the current I_a^0 of the anode portion cut out by the cone with an apex angle (angle of convergence) of 2Θ as the area of the anode sphere is larger than the cut-off portion of the anode

$$I'_a = I_a \left(\frac{1 - \cos \Theta}{2} \right) = 2a \frac{1}{[-\alpha(R_a)]^2} \left(\frac{1 - \cos \Theta}{2} \right) U_a^{3/2} \quad (2.94)$$

where $a = 14.68 \times 10^{-6} \text{ A/V}^{3/2}$, $R_a = r_c/r_a$.

As the current lines coincide with the radii of the spheres, the current will be the same in the section of the conical beam by any spherical surface located between the anode and cathode and concentric with the anode sphere. Therefore, the current in the section of the beam by any spherical surface with a radius r ($r_a < r < r_c$) may be as follows

$$I' = 2a \frac{1}{[-\alpha(R)]^2} \left(\frac{1 - \cos \Theta}{2} \right) [U(r)]^{3/2} \quad (2.95)$$

where $R = r_c/r$.

By composing the ratio of expressions (2.94) and (2.95) define the potential distribu-

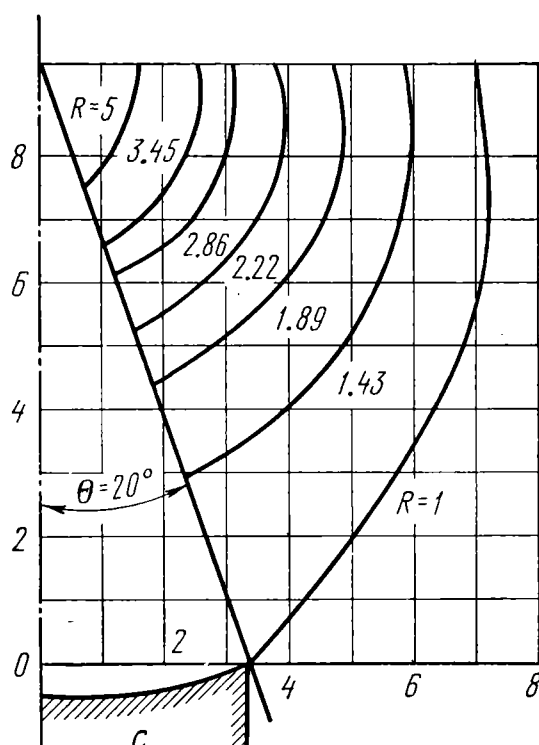


Fig. 2.21. Potential distribution near wedge-shaped beam
C—cathode

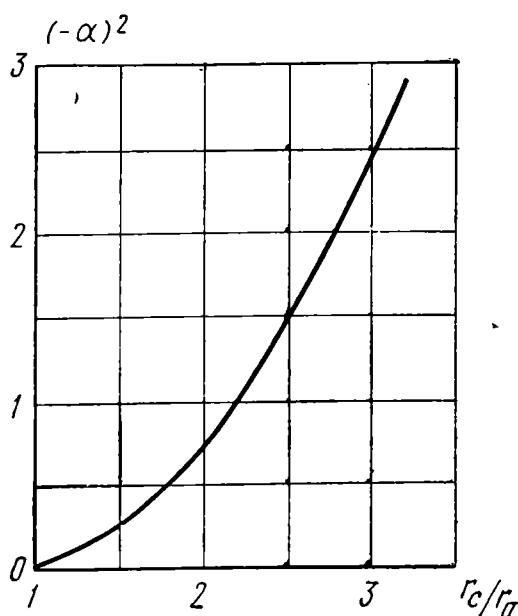


Fig. 2.22. Graph of function $(-\alpha)^2$

tion along the radius (along the beam boundary)

$$U(r) = U_a \left[\frac{-\alpha(R)}{-\alpha(R_a)} \right]^{4/3} \quad (2.96)$$

i.e. the first boundary condition.

The other boundary condition is expressed by

$$\frac{\partial U}{\partial \Psi} = 0 \quad (2.97)$$

where Ψ is the polar angle of the spherical coordinate system.

The Laplace equation describing the potential distribution outside a conical beam with the boundary conditions (2.96), (2.97) can

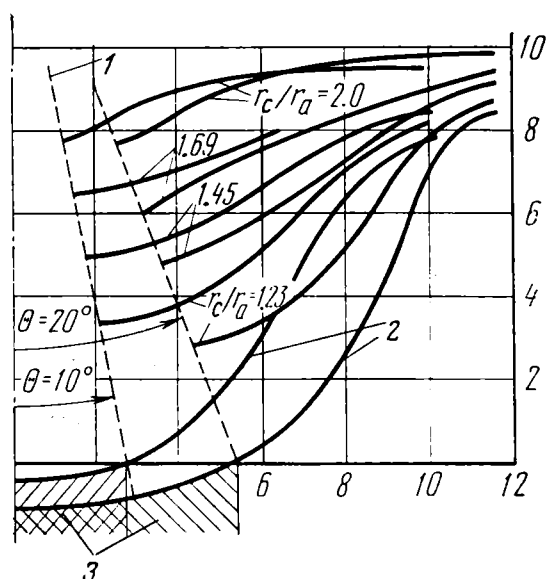


Fig. 2.23. Potential distribution near conical beams
1—beam boundary; 2—cathode electrode; 3—cathode

be roughly solved analytically. The system of equipotential lines of the field forming a conical beam can be obtained experimentally by simulating in an electrolytic tank. Figure 2.23 shows a family of equipotential lines of fields forming conical beams with angles of convergence $\Theta = 10^\circ$ and $\Theta = 20^\circ$.

As is seen from the figure, an electric field for forming an axisymmetric converging (conical) beam can be built up by two electrodes, one of which is a cathode electrode of cup-shape, at a zero potential. It forms an angle of 67.5 degrees with regard to the beam boundary. The shape of the anode electrode is determined by any equipotential surface. The anode electrode approaches the beam boundary along a line normal to it and has a hole whose radius equals that of the beam.

The above-considered systems forming intensive beams are known as the *Pierce systems* by the name of American physicist J. Pierce who proposed to form electron beams by using portions of electron flows filling the space between the cathode and anode in diodes of a simple geometric shape.

The Pierce systems provide the existence of beams of required shape, axisymmetric (cylindrical and conical) and strip-like (converging or constant-section) in the space between the cathode and anode electrodes. In the above considerations it was assumed that the equipotential anode surface in the space occupied by the beam coincides with the original anode, i.e. is either a plane or a cylindrical

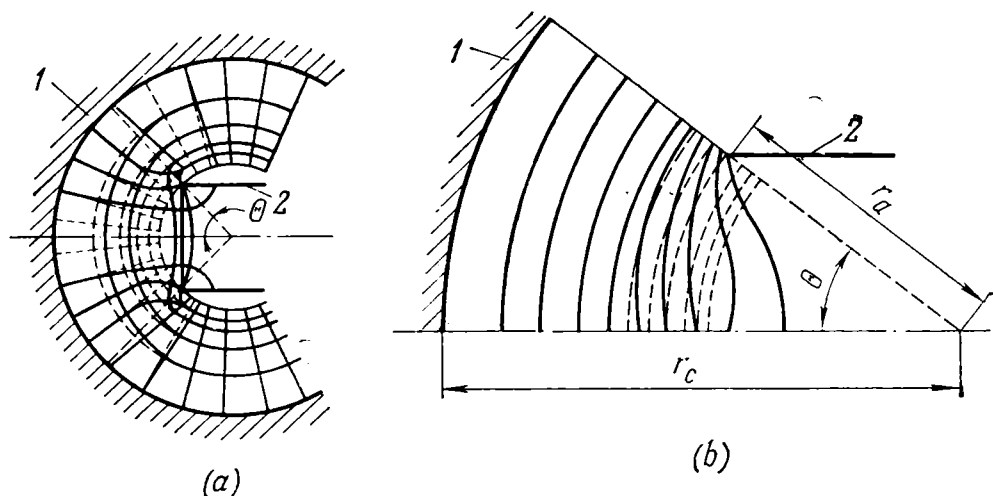


Fig. 2.24. "Sagging" of field into anode aperture
(a) cross section; (b) longitudinal section; 1—cathode; 2—anode;

(or spherical) surface. In real systems forming intensive beams the anode electrode is always provided with an aperture for passing the shaped beam into the space behind the anode. The presence of the anode aperture results in distortion of the field near the anode. If there is no field behind the anode, i.e. the beam passes through the equipotential space, then the field "sags" into the anode aperture (Fig. 2.24) from the space between the cathode and anode (the beam-forming space).

This "sagging" results in formation of an electron lens in the region of the anode aperture. This lens is axisymmetric in the case of axisymmetric beams (circular anode aperture) or cylindrical in the case of a strip beam (the anode aperture is slit-type).

The lens produced is always diverging, since the equipotential surfaces of the field "sagging" into the anode aperture are convex towards the behind-anode space. Consequently, this field has a component E_r , directed to the axis (centre-plane) of the beam. This com-

ponent produces a force acting on the electron in the direction from the axis (centre-plane) (Fig. 2.25).

Thus the electron is acted on by a defocusing force and the anode lens is a diverging one. The focal length of this lens can be estimated in a first approximation by the formula for the optical strength of a thin aperture lens [see (1.164)] as

$$f_{axis} = \frac{4U_a}{|E_2| - |E_1|} \quad (2.98)$$

in the case of an axisymmetric beam, and

$$f_{cnt} = \frac{2U_a}{|E_2| - |E_1|} \quad (2.99)$$

in the case of a strip beam [see (1.225)].

In formulas (2.98) and (2.99) E_1 is the field intensity near the anode aperture from the cathode side and E_2 is the field intensity

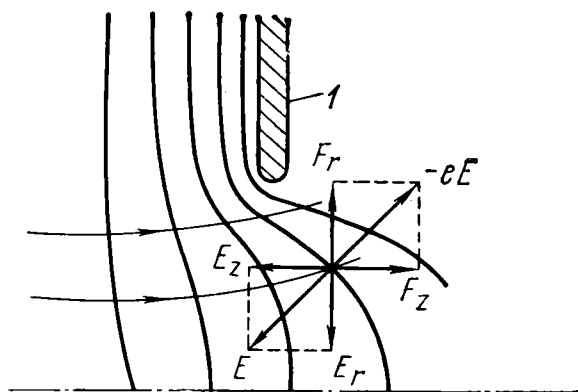


Fig. 2.25. Beam defocusing in anode aperture
1—anode

in the space behind the anode. If the beam is admitted into the equipotential space, $E_2 = 0$. The intensity E_1 is determined by differentiating the function of the potential distribution on the axis or in the centre-plane of the beam along the beam. Since in the systems considered above the potential distribution along the axis of axisymmetric beams and in the centre-plane of a strip beam coincides with the potential distribution on the beam boundary, the expressions for the boundary potential are used for finding the value E_1 .

For axisymmetric cylindrical beams and strip beams of a constant cross section the potential on the boundary and, therefore, on the axis or in the centre-plane is proportional to $z^{4/3}$. In this case the value E_1 is obtained by differentiating expression (2.82) with respect to z while substituting $U = U_a$ and $z = d$ (the distance bet-

ween the cathode and the anode)

$$E_1 = - \frac{dU(z)}{dz} \Big|_{\substack{z=d \\ U=U_a}} = - \frac{4}{3} \times \frac{U_a}{d} \quad (2.100)$$

In this case the focal lengths of the axisymmetric and cylindrical aperture lenses are described by the following expressions

$$f_{axis} = -3d \quad \text{and} \quad f_{cnt} = -\frac{3}{2}d \quad (2.101)$$

The minus indicates that the lens is diverging and the slit lens defocusing the strip beam is twice as strong as the axisymmetric lens. The use of an anode lens results in expanding the beam in the space behind the anode. The beam formed in the space between the

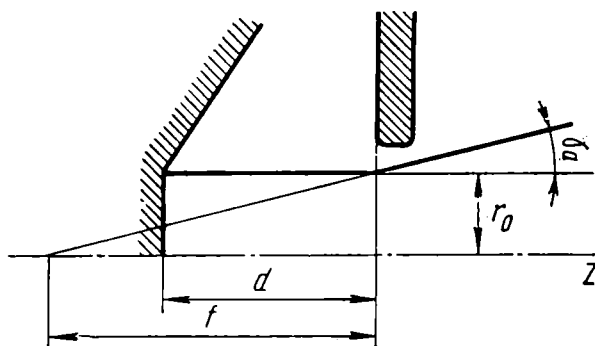


Fig. 2.26. On calculation of anode lens

cathode and anode as a cylinder or parallelepiped with boundaries parallel to the axis OZ becomes diverging after the outlet of the anode aperture. Find the angle γ_a formed by the boundary trajectories of the beams with the axis (centre-plane) in the space behind the anode (Fig. 2.26).

Since the anode lens is diverging, its focal (imaginary) point is spaced to the left from the anode aperture plane. According to the geometrical construction in Fig. 2.26, the outlet angle is determined by the expressions:

for the axisymmetric beam

$$\tan \gamma_a = \frac{r_0}{|f_{axis}|} = \frac{1}{3} \times \frac{r_0}{d} \quad (2.102)$$

for the strip beam

$$\tan \gamma_a = \frac{y_0}{|f_{cnt}|} = \frac{2}{3} \times \frac{y_0}{d} \quad (2.103)$$

where r_0 and y_0 are the radius of the axisymmetric beam and the half-thickness of the strip beam respectively.

Expressions (2.102) and (2.103) show that the outlet angle is uniquely determined by the geometric relationships of the beam forming

system, i.e. the distance between the cathode and anode and the size of the anode aperture. However, one can easily see that an increase in the perveance results in an increase in the outlet angle. Substitute $z = d$, $U = U_a$ into expression (2.81) and find the perveance for the beam section πr_0^2

$$P = \frac{I}{U_a^{3/2}} = \frac{\pi r_0^2 j}{U_a^{3/2}} = a\pi \frac{r_0^2}{d^2} \quad (2.104)$$

Find d from (2.104) and substitute it into (2.102). In this case we have the following expression for the outlet angle

$$\tan \gamma_a \frac{1}{3} \sqrt{\frac{P}{a\pi}} = 1.26 \times 10^2 \sqrt{P} \quad (2.105)$$

i.e. the divergence angle of the cylindrical beam at the anode outlet is proportional to the perveance in $1/2$ power.

In the equipotential space behind the anode the beam expands due to the action of the space charge; the beam contour can be determined by formulas (2.24) and (2.40), the action of the diverging anode lens being taken into account by substituting the initial angle of inclination determined by formulas (2.103) and (2.102), i.e. $r'_0 = \tan \gamma_a$ and $y'_0 = \tan \gamma_a$ into these expressions. Since the expansion due to the space charge is determined by the perveance value, the divergence angle of an axisymmetric beam can be roughly evaluated by using an additional term in equation (2.102) depending on the perveance. The formula making allowance for the additional angle of divergence due to the action of the space charge is written as follows

$$\tan \gamma_a \approx \frac{1}{3} \times \frac{r_0}{d} (1 + 2.5 \times 10^3 \sqrt{P}) \quad (2.106)$$

In the systems producing converging beams—conical and wedge-shaped—the action of the lens in the region of the anode aperture reduces the convergence of the beam, and with a sufficiently large absolute value of the optical strength of the diverging lens the extreme trajectories emerge from the anode aperture parallel to the axis (centre-plane), i.e. the beam is converted from converging axisymmetric or wedge-shaped into cylindrical or strip one.

Consider the action of the anode lens on an axisymmetric converging (conical) beam having a convergence angle of 2Θ and emerging from the anode aperture. It is clear that due to the action of the anode lens in the space behind the anode the angle of convergence changes (reduces) and becomes equal to γ_a . This change in the angle of convergence can be estimated by the refraction coefficient of the outer trajectory of the beam in the anode aperture

$$n = \frac{\sin \gamma_a}{\sin \Theta} \quad (2.107)$$

The anode lens can approximately be considered thin and geometrical optics relation (1.142) can be used, which couples the distance from the lens to the object (a) and the image (b) with the focal length (Fig. 2.27)

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f} \quad (2.108)$$

From the geometrical constructions in Fig. 2.27 it follows that $r_0/b = \tan \Theta$, $r_0/a = \tan \gamma_a$ where r_0 is the radius of the anode aperture. In paraxial approximation (for small values of Θ and γ_a) we

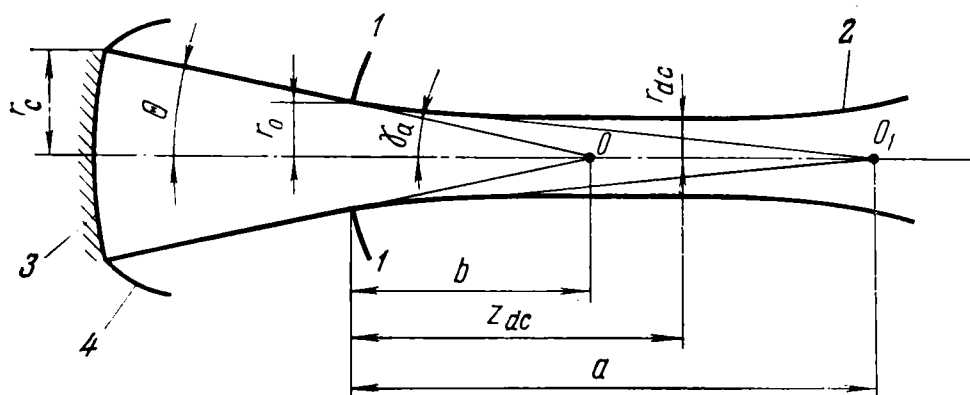


Fig. 2.27. Expansion of formed conical beam
1—anode; 2—beam boundary; 3—cathode; 4—focusing electrode

may assume that $\tan \Theta \approx \sin \Theta$ and $\tan \gamma_a \approx \sin \gamma_a$. Then with due allowance for (2.108) the refraction coefficient is

$$n = \frac{\sin \gamma_a}{\sin \Theta} = \frac{b}{a} = 1 - \frac{b}{f} \approx 1 - \frac{r_a}{f} \quad (2.109)$$

The focal length of the anode lens is determined by formula (2.98) with $E_2 = 0$. The value E_1 is calculated as $\left. \frac{dU}{dr} \right|_{r=r_a}$ by differentiating equation (2.96). Substituting the found expression into (2.109), we finally obtain an expression for the refraction coefficient

$$n = \frac{\sin \gamma_a}{\sin \Theta} = 1 - \frac{R_a}{4 [-\alpha(R_a)]^{4/3}} \times \frac{d}{dR} [-\alpha(R)]^{4/3} \Big|_{R=R_a} \quad (2.110)$$

where $R_a = \frac{r_c}{r_a}$ and $R = \frac{r_c}{r}$.

Expression (2.110) shows that the refraction coefficient is determined by the geometrical relations of the system of electrodes, namely: the ratio of the radii of the cathode and anode spheres of the original spherical diode. At the same time, since the beam perveance is determined by the value $[-\alpha(R_a)]^2$ and the convergence angle Θ [see (2.95)], the refraction coefficient must depend on the value

of P . Expressing $P = I/U_a^{3/2}$ from formula (2.95) with $R = R_a$ and replacing $(1 - \cos \Theta)$ by $2 \sin^2 \frac{\Theta}{2}$, we obtain

$$P = 2a \frac{\sin^2 \frac{\Theta}{2}}{[-\alpha(R_a)]^2} \quad (2.111)$$

In paraxial approximation it can be assumed that $\sin \frac{\Theta}{2} \approx \frac{\Theta}{2}$ and $\sin \gamma_a \approx \gamma_a$. Then, with due account for $\gamma_a = n\Theta$, formula (2.111) is transformed into

$$\gamma_a = \sqrt{\frac{2}{a}} [-\alpha(R_a)] [n(R_a)] \sqrt{P} = 3.69 \times 10^2 [-\alpha(R_a)] [n(R_a)] \sqrt{P} \quad (2.112)$$

i.e. the decrease in the convergence angle of conical beam (as well as cylindrical beam) is proportional to $\sqrt{P}$.

Figure 2.28 illustrates the refraction coefficient n versus the ratio of the radii of the cathode and anode spheres, R_a .

From this drawing it is clear that when $R_a \approx 1.46$ the value n is zero, i.e. at the outlet of the anode aperture an axisymmetric converging beam becomes cylindrical.

In the case of a strip converging (wedge-shaped) beam the similar calculation of the action of anode (slit) lens gives the following expression for the refraction coefficient

$$n = \frac{\sin \gamma_a}{\sin \Theta} = \frac{4}{3} - \frac{R_a}{3\beta^2(R_a)} \left(\frac{d\beta}{dR} \right)_{R=R_a}^{2/3} \quad (2.113)$$

where $R_a = \frac{r_c}{r_a}$ and $R = \frac{r_c}{r}$.

The reduction of the convergence angle of the wedge-shaped beam is also proportional to $\sqrt{P}$. The graph of the function $n(R_a)$ is given in Fig. 2.29.

From this figure it is clear that with $R_a \approx 1.9$ the value n is zero, i.e. the wedge-shaped beam, having passed the anode aperture, becomes a strip beam with boundaries parallel to the axis OZ .

The calculation of the contours of axisymmetric and converging strip beams in the space behind the anode with allowance for the action of space charge can be made by formulas (2.24) and (2.40) when substituting $r'_0 = \tan \gamma_a$ and $y'_0 = \tan \gamma_a$, where γ_a is determined by formulas (2.102) and (2.103) both for the axisymmetric and strip beams respectively.

An electric field can be used for shaping an axisymmetric hollow (tubular) beam. It is evident that the conditions at the external and internal boundaries of the tubular beam do not differ from the conditions for a solid cylindrical beam. Consequently, the Pierce system consisting of a cathode electrode inclined at an angle of 67.5° to the

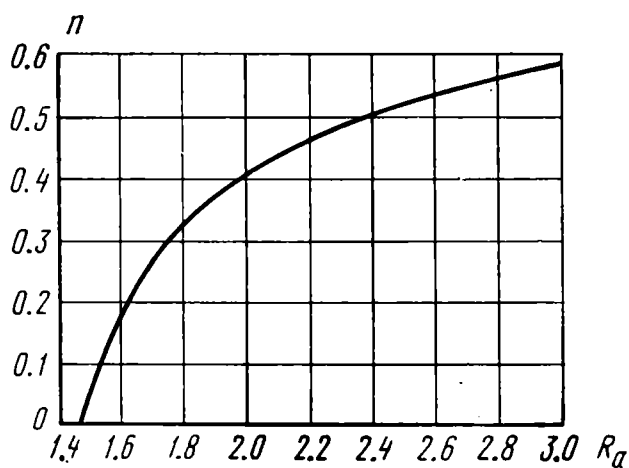


Fig. 2.28. Refraction coefficient *versus* ratio of curvature radii of cathode and anode

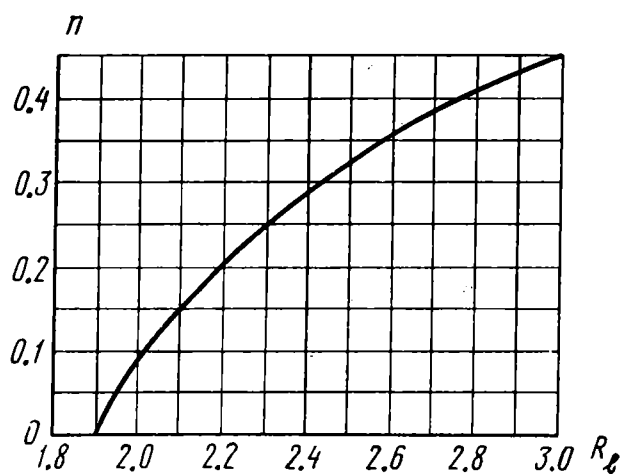


Fig. 2.29. Refraction coefficient *versus* ratio of radii of cathode and anode cylinders

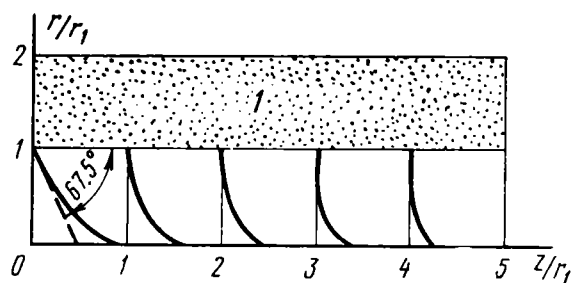


Fig. 2 30. Potential distribution inside tubular beam
1—beam boundary

generating line of the beam near its boundary and an anode, having an aperture with a diameter equal to that of the beam, curved to follow the shape of one of the equipotential surfaces shown in Fig. 2.19 provides for the external contour of the beam.

To meet the boundary conditions along the internal generating line of a hollow cylinder, the tubular beam must be controlled, besides the cathode electrode, with some more electrodes at a gradually (proportional to $z^{4/3}$) increasing potential. The shape of the electrodes can be calculated or simulated in an electrolytic tank.

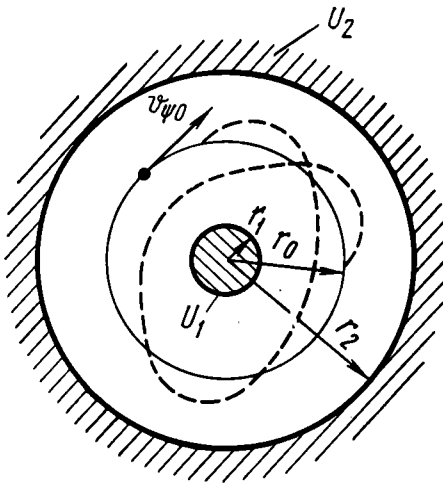


Fig. 2.31. Centrifugal-electrostatic focusing

U_1 and U_2 ($U_1 > U_2$) at a distance r_0 from the axis. If the radial component of the electric field intensity F_{r0} at a distance r_0 from the axis satisfies the equation

$$eE_{r0} = \frac{mv_{\psi 0}^2}{r_0} = m\omega_0^2 r_0 \quad (2.114)$$

where ω_0 is the circular frequency, then the electron moves along an equilibrium circular trajectory having radius r_0 . Let us prove that this orbit is stable, i.e. in case of small deviations of the electron from the trajectory described by equation (2.114) a force will appear to return the electron to the equilibrium orbit. If the electron is displaced from the equilibrium orbit in the radial direction for a small value of Δr , then, obviously a radial force acting on the electron arises and equals

$$m \frac{d^2 r}{dt^2} = mr\omega^2 - eE_r \quad (2.115)$$

where ω and E_r are the circular frequency and the radial component of the field intensity on the nonequilibrium orbit $r = r_0 + \Delta r$.

Figure 2.30 illustrates a family of equipotential lines in a meridian plane inside a tubular beam.

As seen from the figure, the zero (cathode) equipotential line is inclined at an angle of 67.5° to the axis near the beam boundary. All other equipotential surfaces ($U > 0$) are square with the beam boundary.

An axisymmetric tubular beam can also be shaped by a centrifugal-electrostatic focusing system. Let us consider the motion of an electron in the field of cylindrical capacitor. Assume that an electron with azimuthal velocity $v_{\psi 0}$ is admitted into the space between two coaxial cylinders with radii r_1 and r_2 (Fig. 2.31) and potentials

For small values of Δr we may roughly assume that

$$E_r \approx E_{r0} \frac{r_0}{r} = E_{r0} \frac{r_0}{r_0 + \Delta r} \approx E_{r0} \left(1 - \frac{\Delta r}{r_0}\right) \quad (2.116)$$

Using the law of preservation of the angular momentum

$$mr^2\omega = mr_0^2\omega_0 \quad (2.117)$$

known in mechanics, present ω as $r_0^2\omega_0/(r_0 + \Delta r)^2$. Substitute the obtained expression ω , E_r from (2.116) and $r = r_0 + \Delta r$ into (2.115). Since Δr is a small magnitude, the terms of the second order with respect to Δr may be neglected.

In this case equation (2.115) is transformed into

$$\frac{d^2}{dt^2}(\Delta r) = r_0\omega_0^2 - 3\omega_0^2\Delta r - \frac{e}{m}E_{r0} + \frac{e}{m}E_{r0}\frac{\Delta r}{r_0} \quad (2.118)$$

If now E_{r0} from (2.114) is substituted into (2.118), the last equation is reduced to

$$\frac{d^2}{dt^2}(\Delta r) + 2\omega_0^2\Delta r = 0 \quad (2.119)$$

The solution of this equation with boundary conditions $t = t_0$, $\Delta r = 0$ takes the form

$$\Delta r = A \sin [\sqrt{2}\omega_0(t - t_0)] \quad (2.120)$$

Consequently, the electron displaced from the equilibrium orbit will oscillate about this circular trajectory intersecting it through each $\pi/\sqrt{2}$ radian—127 degrees.

The stability of the electrons motion in a cylindrical capacitor can be also explained as follows. It is known that the potential distribution in a cylindrical capacitor is described logarithmically as follows

$$eE_{r0} = \frac{e(U_1 - U_2)}{\ln\left(\frac{r_2}{r_1}\right)} \times \frac{1}{r_0} = \frac{c_1}{r_0} \quad (2.121)$$

Therefore, the last term of equation (2.114) can be presented as

$$m\omega_0^2 r_0 = \frac{mP_\phi^2}{r_0^3} = \frac{c_2}{r_0^3} \quad (2.122)$$

where $P_\phi = r_0 v_\phi$ is the torque.

Taking into account (2.121) and (2.122), equation (2.114) can be written as follows

$$\frac{c_1}{r_0} = \frac{c_2}{r_0^3} \quad (2.123)$$

When the electron is displaced from the equilibrium orbit to the axis, equation (2.123) is disturbed, in which case its right-hand side

becomes greater than the left hand side. This means that the electric (focusing) force is insufficient for holding the electron in the orbit with a radius $r < r_0$, i.e. the electron will be displaced from the axis to the equilibrium orbit. On the contrary, shifting the electron from the equilibrium orbit aside from the axis ($r > r_0$), the left-hand side of equation (2.123) becomes greater than the right-hand side, i.e. an excessive electric force appears which is directed to the axis and returns the electron back to the equilibrium orbit.

Thus, in the field of a cylindrical capacitor a stable electron flow can exist in the form of a cylinder coaxial with the electrodes. If the electrons entering the field of a cylindrical capacitor have a non-zero

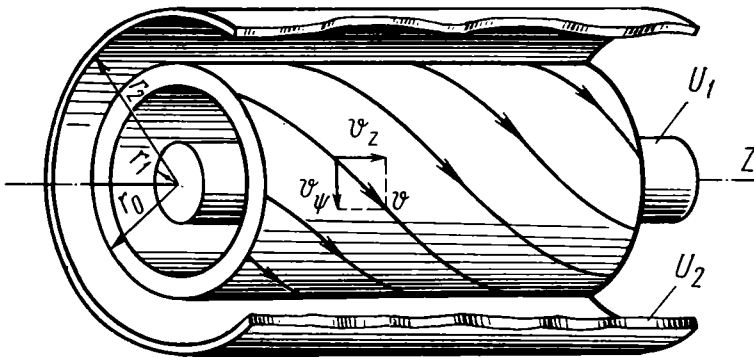


Fig. 2.32. Beam formed by centrifugal-electrostatic focusing system

axial velocity v_z , in addition to the azimuthal velocity v_ϕ , it is evident that the component v_z remains unchanged, since $E_z = 0$ between the plates of the capacitor, and the electron trajectories have the shape of spirals wound on a cylinder having the radius r_0 , while the beam itself has the form of a thin-wall tube (Fig. 2.32).

In the discussion of the centrifugal focusing we did not consider the action of space charge. With perveances not too small, the presence of a space charge changes the potential distribution between the electrodes of a cylindrical capacitor. With perveances of a certain magnitude, the effect of the space charge may be compensated for by increasing the radial component of the electric field intensity. When the perveance overcomes this limit, the rigidity of centrifugal-electrostatic focusing becomes insufficient, the beam becomes unstable, and the current flow is disturbed. An analysis of the stability of a centrifugally focused tubular beam shows that the maximum value of perveance at which a stable electron flow can exist in the field of a cylindrical capacitor is as follows

$$P_{\max} = \frac{I}{U_z^{3/2}} = 33 \times 10^{-6} a \frac{\Delta U}{U_z} \quad (2.124)$$

where U_z is the accelerating voltage determining the axial velocity of the electrons; $\Delta U = U_1 - U_2$.

$$a = \frac{1}{\ln\left(\frac{r_b}{r_a}\right) - \frac{r_a^2}{r_b^2 - r_a^2} \ln\left(\frac{r_b}{r_a}\right) + \frac{r_b^2}{r_b^2 - r_a^2} \ln\left(\frac{r_b}{r_a}\right) - \frac{1}{2}} \quad 159$$

(r_a and r_b are the radii of the internal and external boundaries of the beam).

The given examples of focusing of intensive electron beams indicate that electrostatic fields can be used for shaping electron flows practically to any configuration. However, each configuration of the beam is characterized by a maximum value of the space charge coefficient $P_{\max}$, the excess of which results in disturbance of the electron flow stability.

2.5. Magnetic Focusing of Intensive Beams

It has been shown that the repulsive action of the space charge of an intensive electron beam can be compensated for by an external magnetic field. It is obvious that if at the beam boundary the Lorentz magnetic force is equal in magnitude and opposite in direction to the Coulomb repulsion electric force, the beam preserves its original shape. In other words, an external magnetic field can be used for maintaining a stable electron flow in a space restricted by a surface at any point of which the given boundary condition is satisfied.

The Lorentz force is proportional to the electron velocity [see (1.6)]. Therefore, when using magnetic focusing, the beam electrons must have a required velocity prior to entering the magnetic field, i.e. must be preliminarily accelerated by an electric field. It may be said that in many practical cases a magnetic field is used for restricting the expansion of the beam preliminarily shaped by an electrostatic system. In view of this particularity of shaping electron beams by magnetic fields the general term "magnetic focusing" is often replaced by more specific concept of "limiting" the beam by a magnetic field.

From Section (1.6) it also follows that in magnetic focusing systems the beam electrons must have velocity components perpendicular to the direction of the magnetic induction, since with $\mathbf{v} \parallel \mathbf{B}$ the focusing force is equal to zero. Magnetic fields can be used for shaping (collimating) of electron flows practically to any configuration.

Let us consider an axisymmetric electron beam. As shown in Section 2.1, the presence of space charge results in a force acting on the electrons in the radial (from the axis) direction [see (2.8)]. In order to compensate for this defocusing force, it is necessary to have a radial magnetic focusing force directed to the axis. When a longitudinal magnetic field is used for focusing (collimating) the beam, a radial focusing force appears only if the electrons of the beam have

an azimuthal velocity $v_\psi = r\dot{\psi}$ [this follows directly from the definition of the Lorentz force, see (1.6)].

The azimuthal motion of electrons, as is shown in Section 1.6, is described by the third equation of system (1.92)

$$\frac{d}{dt} (r^2\dot{\psi}) = \frac{e}{m} \times \frac{d}{dt} (rA_\psi) \quad (2.125)$$

Integrate this equation with due account for the initial conditions at $t = t_0$ $r = r_0$ $\dot{\psi} = \dot{\psi}_0$, $A_\psi = A_{\psi_0}$

$$r^2\dot{\psi} = r_0^2\dot{\psi}_0 = \frac{e}{m} (rA_\psi - r_0A_{\psi_0}) \quad (2.126)$$

For an axisymmetric magnetic field $A = A_\psi$ ($A_z = A_r = 0$) and the vector potential can be expressed through the magnetic flux Ψ piercing a part of the plane perpendicular to the symmetry axis (OZ) limited by the circle with the radius r

$$A = \frac{\Psi}{2\pi r} \quad (2.127)$$

$$\Psi = 2\pi \int_0^r B_z r dr \quad (2.128)$$

Substitution of (2.127) into (2.126) gives the expression

$$r^2\dot{\psi} - r_0^2\dot{\psi}_0 = \frac{e}{2\pi m} (\Psi - \Psi_0) \quad (2.129)$$

This ratio known as the *Bush theorem* shows that the angular momentum acquired by an electron displaced from the orbit with the radius r_0 to a circular trajectory with the radius r is proportional to the difference of the magnetic fluxes penetrating through the corresponding cross sections restricted by the circles of the radii r_0 and r . The Bush theorem is widely used in calculations of magnetic systems producing axisymmetric electron beams.

In some cases expression (2.129) can be simplified. Thus, for example, when considering an electron trajectory emerging from the edge of a circular cathode ($r_0 = r_c$, the cathode radius) pierced by the magnetic flux Ψ_c and neglecting the initial velocities of the electrons ($\dot{\psi}_c = 0$), the Bush theorem assumes the form

$$r^2\dot{\psi} = \frac{e}{2\pi m} (\Psi - \Psi_c) \quad (2.130)$$

For the paraxial region we may roughly assume that $B_z(z, r) \approx B_z(z, 0) = B_0(z)$. In this case

$$\Psi_c = \pi r_c^2 B_c, \quad \Psi = \pi r^2 B_0(z) \quad (2.131)$$

where B_c is the magnitude of magnetic induction on the axis in the cathode plane.

Substituting the expression of the magnetic fluxes from (2.131) into (2.130), we obtain the Bush theorem for the paraxial region

$$\dot{\psi} = \frac{e}{2m} B_0(z) \left[1 - \frac{B_c}{B_0(z)} \left(\frac{r_c}{r} \right)^2 \right] \quad (2.132)$$

If the total electron flow emitted by the cathode and the cathode itself are in a uniform longitudinal magnetic field, $B_0(z) = B_c = \text{const}$, then

$$\dot{\psi} = \frac{e}{2m} B_c \left[1 - \left(\frac{r_c}{r} \right)^2 \right] \quad (2.133)$$

i.e. the azimuthal velocity of the electrons is determined by the value of magnetic induction and the ratio r_c/r . For the electron trajectories spaced from the axis by a distance equal to the cathode radius ($r = r_c$) the azimuthal velocity vanishes and the trajectories are not "wound" around the cylindrical surface with the radius r_c .

If the magnetic induction in the cathode plane is equal to zero ($B_c = 0$), i.e. the cathode is completely separated from the magnetic field or is a point source of electrons ($r_c = 0$), then all the electrons of this flow have the same azimuthal velocity uniquely determined by the value of magnetic induction [compare with (1.94)]

$$\dot{\psi} = \frac{e}{2m} B_0(z) \quad (2.133a)$$

Set up an equation of radial motion of an axisymmetric beam outer electron in the paraxial region. Assume that the electron flow spreads in an equipotential space. The action of space charge, as it was shown in Section 2.1, results in a radial defocusing force, i.e. the Coulomb repulsive force [see (2.8)]

$$F_{re} = \frac{eI}{2\pi\epsilon_0 \sqrt{\frac{2e}{m} U_0} r}$$

This force should be added to the right-hand side of the second equation of system (1.91) describing the radial motion of an electron in the paraxial region of an axisymmetric magnetic field. Thus, the equation of electron motion in the radial direction with due account for the action of space charge takes the form

$$m (\ddot{r} - r\dot{\psi}^2) = -er\dot{\psi}B_0(z) + \frac{eI}{2\pi\epsilon_0 \sqrt{\frac{2e}{m} U_0} r} \quad (2.134)$$

Substituting $\dot{\psi}$ from (2.132) into this equation and making some simple transformations, we obtain

$$m\ddot{r} + \frac{e^2}{4m} B_0^2 \left[1 - \frac{B_c^2}{B_0^2} \left(\frac{r_c}{r} \right)^4 \right] r - \frac{eI}{2\pi\epsilon_0 \sqrt{\frac{2e}{m} U_0} r} = 0 \quad (2.135)$$

The beam contour is determined by the trajectory of the outer electron. The trajectory equation can be obtained when proceeding in (2.135) from differentiation with respect to t to differentiation with respect to z ; in this case we should use the relationship (see Section 1.6)

$$\frac{d^2r}{dt^2} = 2 \frac{e}{m} U_0 \frac{d^2r}{dz^2}$$

Thus equation (2.135) is transformed into

$$\frac{d^2r}{dz^2} + \frac{e}{8mU_0} B_0^2 \left[1 - \frac{B_c^2}{B_0^2} \left(\frac{r_c}{r} \right)^4 \right] r - \frac{1}{4\pi\epsilon_0 \sqrt{\frac{2e}{m}}} \times \frac{I}{U_0^{3/2} r} = 0 \quad (2.136)$$

Equation (2.136) is the trajectory equation of the outer electron of the intensive axisymmetric beam in the paraxial region of the axisymmetric magnetic field in the projection on a meridian plane rotating about the axis at an azimuthal velocity determined by the Bush theorem (2.132).

In the general case the solution of equation (2.136) is very difficult. Assuming that in the area of propagation of the electron flow the magnetic field is uniform, we arrive at an important conclusion that there can exist an equilibrium radius of the beam at which the total radial force at the beam boundary is equal to zero. Actually, with certain magnitudes of magnetic induction and perveance of the beam, it is possible to find such a value of $r = r_0$ at which the second and third terms of equation (2.136) are equal in the absolute magnitude. It is clear that in this case $d^2r/dz^2 = 0$ and, therefore, $d^2r/dt^2 = 0$, i.e. the electrons are not subjected to radial acceleration at the beam boundary. Thus, in a longitudinal uniform magnetic field a stable intensive axisymmetric beam having the constant radius r_0 , i.e. a cylindrical beam can exist. From (2.136) it also follows that the magnetic induction magnitude which provides for the existence of a stable beam is at its minimum when $B_c = 0$, i.e. when the cathode is completely shielded from the magnetic field.

The last case ($B_c = 0$, $B_0 = \text{const}$) is of special interest. It is called the *Brillouin focusing*, and a stable beam in a uniform longitudinal magnetic field is called the *Brillouin flow*. Consider this case in more detail.

The Brillouin flow is described by equation (2.136) with $B_c = 0$, $B_0 = \text{const}$

$$\frac{e}{m} B_0^2 r_0^2 - \frac{I}{\pi\epsilon_0 \sqrt{\frac{2e}{m}} U_0} = 0 \quad (2.137)$$

where r_0 is the equilibrium radius of the beam.

Besides, according to (1.94), the azimuthal velocity of the electrons does not depend on r , i.e. the total flow rotates about the axis as an integer at an azimuthal velocity determined by equation (1.94).

In equation (2.136) the value of U_0 determining the longitudinal velocity of the electrons is identified with the constant potential of the space where the electron flow spreads. Such a substitution is possible only at small perveances, since, as it has been shown in Section 2.1, [see (2.57)], the action of space charge in an intensive beam causes a drop of the beam potential, i.e. we have the radial distribution of the potential. The potential distribution along the beam radius can be calculated by presenting the last term of equation (2.136) as $-\frac{1}{2U_0} E_r = \frac{1}{2U_0} \times \frac{dU}{dr}$ [see (2.7)]. In this case, with $B_c = 0$, $B_0 = \text{const}$, the equation assumes the form

$$\frac{dU}{dr} = \frac{e}{4m} B_0^2 r \quad (2.138)$$

The integration of this equation with the boundary condition $U|_{r=0} = U_0$ (the axis potential) gives the following expression for the radial potential distribution

$$U(r) = U_0 + \frac{e}{8m} B_0^2 r^2 \quad (2.139)$$

Thus, the potential is at its minimum ($U = U_0$) on the axis and rises up by the square law with departure from the axis. Now let us show that in the Brillouin flow the longitudinal component of the velocity of all electrons is the same and is determined by the potential on the axis. According to the law of preservation of energy, the total velocity of any beam electron is uniquely determined by the potential of the space point at which the electron is located.

$$v_{total} = \sqrt{v_z^2 + (r\dot{\psi})^2} = \sqrt{\frac{2e}{m} U} \quad (2.140)$$

Substituting $\dot{\psi}$ from (1.94) and U from (2.139) into (2.140) we obtain the longitudinal velocity component

$$v_z = \sqrt{\frac{2e}{m} U_0} \quad (2.141)$$

Physically this result shows that some energy of the longitudinal motion of electrons is converted into the energy of rotation about the axis, the magnitude of this energy being such that the longitudinal component of the electron velocity becomes independent of the distance between the electron and the axis.

The space charge density in the Brillouin flow is constant too. Substitute the expression of potential distribution (2.139) into the

Poisson equation (2.55)

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r} \times \frac{\partial U}{\partial r} = \frac{e}{4m} B_0^2 + \frac{e}{4m} B_0^2 = -\frac{\rho}{\varepsilon_0} \quad (2.142)$$

hence the space charge density is

$$\rho = -\varepsilon_0 \frac{e}{2m} B_0^2 \quad (2.143)$$

i.e. it is constant and uniquely determined by the magnetic induction magnitude.

The constancy of the longitudinal velocity component and the space charge density result in a constant value of the current density

$$i = -\rho v_z = \frac{\varepsilon_0}{\sqrt{2}} \left(\frac{e}{m} \right)^{3/2} B_0^2 U_0^{1/2} \quad (2.144)$$

Now determine the value of the equilibrium (Brillouin) radius r_0 . According to (2.137) the Brillouin radius is uniquely determined by three quantities: the beam current, the axis potential and the magnetic induction

$$r_0^2 = \frac{\sqrt{2}}{\pi \varepsilon_0} \left(\frac{m}{e} \right)^{3/2} \frac{I}{B_0^2 \sqrt{U_0}} = 69 \frac{I}{B_0^2 \sqrt{U_0}} \quad (2.145)$$

In a similar way, for the values of current and magnetic induction we obtain the following expressions

$$I = \frac{\pi \varepsilon_0}{\sqrt{2}} \left(\frac{e}{m} \right)^{3/2} B_0^2 U_0^{1/2} r_0^2 = 1.45 \times 10^{-2} B_0^2 U_0^{1/2} r_0^2 \quad (2.146)$$

$$B_0^2 = \frac{\sqrt{2}}{\pi \varepsilon_0} \left(\frac{m}{e} \right)^{3/2} \frac{I}{\sqrt{U_0} r_0^2} = 69 \frac{I}{\sqrt{U_0} r_0^2} \quad (2.147)$$

In formulas (2.145) to (2.147) the quantities are given as follows: I in a, U_0 in V, B_0 in Gs, and r_0 in m.

Expression (2.139) shows that the potential at the boundary of the Brillouin flow should be higher than the axis potential

$$U|_{r=r_0} = U_0 + \frac{e}{8m} B_0^2 r_0^2 \quad (2.148)$$

Find the beam current through the potential at the beam boundary, $U|_{r=r} = U_a$, which coincides with the potential of the conducting tube filled by the beam

$$I = \frac{\pi \varepsilon_0}{\sqrt{2}} \left(\frac{e}{m} \right)^{3/2} B_0^2 r_0^2 \left(U_a - \frac{e}{8m} B_0^2 r_0^2 \right)^{1/2} \quad (2.149)$$

and find the maximum possible value of the current. By differentiating (2.149) with respect to B and making the derivative equal to

zero, we obtain the condition for the maximum current

$$B_0^2 r_0^2 = \frac{16}{3} \times \frac{m}{e} U_a \quad (2.150)$$

Substituting the obtained expression into (2.149), we obtain the maximum value of the current

$$I_{\max} = \frac{16}{3 \sqrt{6}} \pi \epsilon_0 \left(\frac{e}{m} \right)^{1/2} U_a^{3/2} \quad (2.151)$$

or the maximum perveance

$$P_{\max} = \frac{I_{\max}}{U_a^{3/2}} = \frac{16}{3 \sqrt{6}} \pi \epsilon_0 \left(\frac{e}{m} \right)^{1/2} = 25.4 \times 10^{-6} \text{ A/V}^{3/2} \quad (2.152)$$

The optimum value of magnetic induction providing for the maximum current of the beam is

$$B_{opt} = \frac{4}{\sqrt{3}} \left(\frac{m}{e} \right)^{1/2} \frac{U_a^{1/2}}{r_0} = 5.5 \times 10^{-2} \frac{U_a^{1/2}}{r_0} \quad (2.153)$$

(here the quantities are given as follows: B in Gs, U_a in V, and r_0 in m).

It is interesting to note that under the maximum perveance conditions the potential on the beam axis substantially differs from the potential U_a of the conducting tube filled by the beam. Substituting the value $B_0^2 r_0^2$, corresponding to the maximum current, into (2.139) from (2.150), we obtain

$$U_0 = \frac{1}{3} U_a \quad (2.154)$$

i.e. the axis potential determining the longitudinal component of the electron velocity is three times less the potential at the beam boundary. This result shows that the radial potential drop can be neglected only at small values of microperveance ($p < 1 \mu \text{ A/V}^{3/2}$). Generally the axis potential U_0 determined by formula (2.139) should be substituted into the design formulas for the Brillouin flow.

In the Brillouin axisymmetric flow the Coulomb repulsive force is completely compensated for by the Lorentz radial force. As it has been already pointed out a radial force appears in a longitudinal magnetic field when the electrons feature an azimuthal velocity. From the physical point of view the appearance of azimuthal velocity in the electrons of a beam inserted into a uniform magnetic field is explained as follows. As the magnetic field near the cathode is equal to zero and at some distance from the cathode it reaches a constant value B_0 , there must be a region of nonuniform magnetic field where the magnetic lines of force are directed along the beam radius, i.e. enter the flow or escape therefrom depending on the direction of the field. In this case when entering a uniform field the

electrons intersect the radial lines of force with resultant tangential component of the Lorentz force which swirls the electrons about the axis. Since the magnetic field cannot change the energy of the electrons (see Section 1.2), the total velocity remains unchanged and the appearance of a tangential component of the velocity correspondingly decreases its longitudinal component.

The generation of the Brillouin flow dictates that some vigorous initial conditions be satisfied: (1) the cathode must be reliably shielded from the magnetic field (condition $B_c = 0$); (2) prior to entering the uniform magnetic field, the beam must be shaped as a circular cylinder, the trajectories of all electrons being parallel to the beam axis the (laminarity condition of the flow); (3) the initial beam radius, the current, the axial potential and the magnetic induction must satisfy the equation (2.145). A failure to satisfy even one of these conditions causes non-uniformity of the beam, a pulsating beam boundary, in particular, or interrupts the current flow completely.

Consider the motion of beam electrons in a uniform longitudinal magnetic field, when the initial conditions for producing the Brillouin flow are not satisfied. Assume that the outer electron of the beam inserted into the magnetic field is spaced from the axis at a distance which somewhat differs from the Brillouin radius r_0 . For a small deviation from the equilibrium radius we may assume that

$$r = r_0 (1 + \delta) \quad (2.155)$$

With $\delta \ll 1$ and the approximate equations

$$\frac{1}{r} \approx \frac{1}{r_0} (1 - \delta), \quad \frac{1}{r^2} \approx \frac{1}{r_0^2} (1 - 2\delta) \quad (2.156)$$

Besides, assume that the magnetic field on the cathode differs from zero. In order to evaluate the penetration of the field in the direction of the cathode, introduce a parameter of the cathode conditions, k

$$k = \left(\frac{B_c}{B_0} \right) \left(\frac{r_c}{r_0} \right)^2 \quad (2.157)$$

It is clear that in the case of the Brillouin focusing $k = 0$. Now let us return to general equation (2.136). The analysis of this equation shows that an equilibrium radius can exist when the magnetic field reaches the cathode, i.e. with $k \neq 0$. Assuming that $d^2r/dz^2 = 0$ in (2.136) and taking

$$\kappa = \frac{\sqrt{2} \left(\frac{m}{e} \right)^{3/2} I}{\pi \epsilon_0 \sqrt{U_0} B_0^2} \quad (2.158)$$

we obtain a biquadratic equation determining the equilibrium radius r_0

$$r_0^4 - \kappa^2 r_0^2 - \left(\frac{B_c}{B_0}\right)^2 r_0^4 = 0 \quad (2.159)$$

The solution of this equation has the form

$$r_0^2 = \left[\frac{1}{2} + \frac{1}{2} \sqrt{1 + 4 \left(\frac{B_c}{B_0}\right)^2 \left(\frac{r_c}{\kappa}\right)^4} \right] \kappa^2 \quad (2.160)$$

and with $B_c = 0$ results in the previously calculated value of the Brillouin radius (2.145).

On the basis of (2.179) express κ^2 through the parameter of the cathode conditions

$$\kappa^2 = (1 - k^2) r_0^2 \quad (2.161)$$

and employ the designation

$$\sqrt{\frac{e}{8mU_0}} B_0 = \chi \quad (2.162)$$

Now replace r in (2.136) with $r_0 (1 + \delta)$ according to (2.155). Then, with due account for (2.156), (2.157) and (2.161), we obtain an equation for small deviation of δ

$$\frac{d^2 \delta}{dz^2} + 2\chi^2 (1 + k^2) \delta = 0 \quad (2.163)$$

with the initial conditions

$$\delta|_{z=0} = \delta_0, \quad \frac{d\delta}{dz} \Big|_{z=0} = \delta'_0 \quad (2.164)$$

The solution of equation (2.163) has the form

$$\delta(z) = A \cos [\sqrt{2\chi^2 (1 + k^2)} z + \varphi_0] \quad (2.165)$$

where the constants A and φ_0 are determined by the initial conditions (2.164)

$$A = \sqrt{\delta_0^2 + \frac{(\delta'_0)^2}{2\chi^2 (1 + k^2)}} \\ \varphi_0 = -\arctan \left[\frac{\delta'_0}{\delta_0 \sqrt{2\chi^2 (1 + k^2)}} \right] \quad (2.166)$$

Now let us proceed from δ to r in accordance with (2.155). In this case, at low deviations from the equilibrium radius, the motion of the outer electron of intensive beam in a uniform longitudinal magnetic field in paraxial approximation is described by the equation

$$r(z) = r_0 + A_r \cos \left(\frac{2\pi}{\lambda_r} z + \varphi_i \right) \quad (2.167)$$

where r_0 is the equilibrium radius of the beam determined by equation (2.160); A_r is the ripple (pulsation) amplitude

$$A_r = r_i \sqrt{\left(1 + \frac{r_0}{r_i}\right)^2 + \frac{4}{1+k^2} \times \frac{m}{e} \frac{U_0}{B_0^2 r_i} \tan \gamma_i} \quad (2.168)$$

r_i is the initial radius of the beam [$r_i = r(z) |_{z=0}$]; γ_i is the initial angle of inclination of the outer electron trajectory to the axis ($\tan \gamma_i = \frac{dr}{dz} |_{z=0}$); λ_z is the ripple wavelength

$$\lambda_r = \frac{4\pi}{\sqrt{\frac{2e}{m}}} \times \frac{\sqrt{U_0}}{B_0} \sqrt{\frac{2}{1+k^2}} \quad (2.169)$$

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φ_i is the initial phase of the ripple

$$\varphi_i = -\arctan \frac{2 \sqrt{\frac{m}{e} U_0 (1+k^2)}}{(r_i - r_0) B_0} \quad (2.170)$$

The analysis of equation (2.167) shows that generally the intensive beam in a uniform longitudinal magnetic field has a wavy boundary, i.e. the beam radius periodically changes or pulsates about the equilibrium value of r_0 . The pulsation amplitude the higher, the greater is the difference between the initial radius of the beam and the equilibrium radius and the higher the initial angle of inclination of the outer electron trajectory to the axis. At small deviations of r_i from the equilibrium radius r_0 and γ_i from zero the beam is stable, the greater the magnetic induction, the lower being the pulsation amplitude.

Physically, the pulsation (ripple) appears due to disturbance of the equilibrium of the beam boundary forces. Assume that the initial radius of the beam is greater than the equilibrium radius ($r_i > r_0$). In this case the magnetic force radially directed to the axis at the beam boundary exceeds the quantity necessary to compensate for the Coulomb repulsive force. The total radial force at the beam boundary is nonzero and directed towards the axis, i.e. the electron acquires radial acceleration in the direction of the axis, namely, the radius of the beam starts reducing. On the contrary, with $r_i < r_0$, the Coulomb repulsive force is not first completely compensated for by the magnetic force and the beam starts expanding. However, the contraction of the beam in the first case and the expansion of it in the other case continue until the relationship of the forces at the beam boundary is reversed. It is clear that the contracting beam starts expanding again, at $r < r_0$ and the expanding beam starts contracting at $r > r_0$, i.e. the beam boundary ripples (pulsates).

Similar phenomena appear with the initial angle γ_i other than zero. If $\gamma_i < 0$ (converging beam) the beam radius is first reduced but when $r < r_0$, there appears a force, directed outward from the axis, which results in expansion of the beam. With $\gamma_i > 0$ (diverging beam) the beam is first expanded, but with $r > r_0$ the total force is directed towards the axis and the beam starts contracting. Thus, in this case, too, the beam boundary ripples.

As follows from (2.168) a smooth flow in the presence of whatever small initial radial component of the velocity (i.e. $\gamma_i \neq 0$) can be obtained only when $B \rightarrow \infty$. Since, practically the electrons have

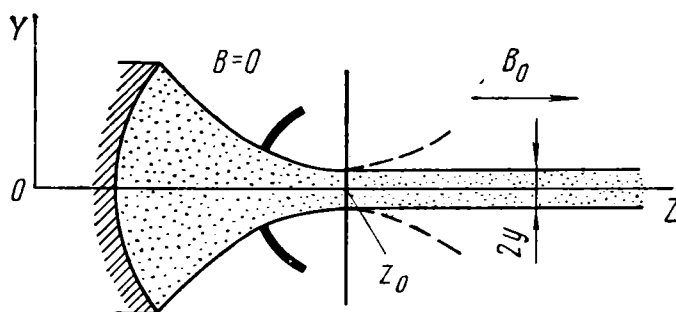


Fig. 2.33. Formation of strip beam in longitudinal magnetic field

an initial radial component of the velocity due to the heat velocities, the action of the anode diverging lens, etc., and the value of magnetic induction cannot be infinitely large, a real beam always has a more or less rippling boundary.

When the radius of an electron beam entering a magnetic field largely differs from the equilibrium radius or when γ_i is of a material value equation (2.136) cannot be linearized and is solved by numerical integration or with the aid of computers. The solutions obtained for large disturbances indicate that in this case, too, the flow is rippling but, in contrast to small disturbances, the flow boundary is asymmetrical relative to the cylindrical surface with the equilibrium radius, i.e. inward deviations of the beam boundary are smaller than outward ones.

Now consider the process of collimating a beam having a rectangular cross section (strip beam) by means of a longitudinal magnetic field. Let a wedge-like beam shaped, say, by an electrostatic field, be inserted into a uniform longitudinal magnetic field B_0 , the initial plane ($z = 0$) being aligned with the crossover plane, i.e. the beam electrons entering the magnetic field have only a longitudinal velocity component $v_z \neq 0$ (Fig. 2.33).

Assume that the beam width $2x$ is much greater than its thickness $2y$. This assumption allows us to consider that $E_x = 0$ (see Section 2.4). A strip beam with boundary trajectories parallel to the axis OZ can be obtained by compensating for the repulsive force

of space charge with the Lorentz magnetic force, i.e. by satisfying the condition

$$-E_y + v_x B_0 = 0 \quad (2.171)$$

at the boundary of the flow.

Relation (2.171) shows that the focusing magnetic force directed along the axis OY to the centre-plane appears only when the beam electrons have a transverse velocity component $v_x \neq 0$. As before the introduction into the magnetic field $v_x = 0$, the transverse velocity component appears due to intersection of the lines of force of the magnetic field by the electrons when entering the magnetic field. Actually, in the plane where $z = 0$ the magnetic flux either enters the region of propagation of the electron beam or leaves it, i.e. with $z = 0$ there is a component of the magnetic induction B_y other than zero. The presence of $B_y \neq 0$ causes a Lorentz force parallel to the axis OX . The latter force makes the electrons "drift" along the wide side of the beam.

To find the "drift" rate, set up an equation of electrons motion in the direction towards the axis OX

$$m \frac{d^2 x}{dt^2} = -\frac{e}{m} E_x = \frac{e}{m} v_y B_0 \quad (2.172)$$

Integrating equation (2.172) between 0 and y with $E_x = 0$ (as specified) gives an expression for the "drift" rate

$$v_x = -\frac{e}{m} B_0 y \quad (2.173)$$

From (2.173) it is clear that in the symmetry plane ($y = 0$) no "drift" occurs but with departure from the centre-plane the velocity v_x rises in proportion to the amount of departure y . Physically this dependence is explained by an increase in the number of the magnetic field force lines intersected by the beam electrons with departure from the centre-plane.

Let us express the values in (2.171) through E_y from (2.35) and v_x from (2.173)

$$\frac{j_l}{2\varepsilon_0 \sqrt{\frac{2e}{m} U_0}} = \frac{e}{m} B_0^2 y \quad (2.174)$$

The half-thickness of the equilibrium strip flow $y = y_0$ in (2.174) is as follows

$$y_0 = \frac{1}{2 \sqrt{2} \varepsilon_0 \left(\frac{e}{m}\right)^{3/2}} \times \frac{j_l}{\sqrt{U_0} B_0^2} = 54.1 \frac{j_l}{\sqrt{U_0} B_0^2} \quad (2.175)$$

where the values are given as follows: j_l in A/m, U_0 in V, B_0 in Gs and y_0 in m.

An equilibrium strip flow in a uniform longitudinal magnetic field with a completely shielded cathode is often called the *Brillouin strip flow* by analogy with an axisymmetric flow, and the values y_0 and B_0 are respectively referred to as the Brillouin half-thickness and the Brillouin magnetic field. The magnetic induction of a Brillouin field according to (2.175) is

$$B_0^2 = 54.1 \frac{j_l}{y_0 \sqrt{U_0}} \quad (2.176)$$

The Brillouin field is a minimum field where an equilibrium strip flow with specified parameters can exist. Since a transverse velocity component v_x is necessary for maintaining an equilibrium strip flow, the vector of the total velocity $v = \sqrt{v_x^2 + v_z^2}$ is turned through a certain "drift" angle with respect to the axis OZ . The "drift" angle is determined by the formula

$$\tan \chi = \frac{v_x}{v_z} = \frac{1}{4\epsilon_0 \frac{e}{m}} \times \frac{j_l}{U_0 B_0} = 1.6 \times 10^3 \frac{j_l}{U_0 B_0} \quad (2.177)$$

where the quantities are given as follows: j_l in A/m, U_0 in V and B_0 in Gs.

The electron velocity is longitudinal only in the centre-plane ($\chi = 0$). The calculations similar to those given for the axisymmetric flow show that all the electrons of Brillouin strip flow have the same longitudinal velocity component v_z determined by the centre-plane potential U_0 .

To consider the problem more generally assume that the magnetic field penetrates to the cathode and the induction $B_0 = B(0, z)$ can vary along the axis OZ . Designate the unit magnetic fluxes piercing rectangular areas having a width equal to unit and a height of $2y$ and $2y_c$ through ψ and ψ_c respectively (y_c is the cathode height). Then, neglecting the initial velocities of the cathode electrons, equation (2.173) can be written as

$$\frac{dx}{dt} = \frac{e}{2m} (\psi - \psi_c) \quad (2.178)$$

This relation may be called the Bush theorem for a strip flow. In the paraxial approximation

$$\left. \begin{aligned} \psi &= 2B_0 y \\ \psi_c &= 2B_c y_c \end{aligned} \right\} \quad (2.179)$$

where B_c is the magnetic induction in the cathode plane.

Substituting (2.179) into (2.178), we obtain another form of the Bush theorem for the paraxial region of a strip flow

$$\frac{dx}{dt} = \frac{e}{m} B_0 \left(y - \frac{B_c}{B_0} y_c \right) \quad (2.180)$$

Let us set up an equation of the motion of beam outer electron using equations (1.10), (2.35) and (2.180)

$$\frac{d^2 y}{dt^2} = \frac{j_l}{2\epsilon_0 \sqrt{\frac{2m}{e} U_0}} - \frac{e^2}{m^2} B_0^2 \left(y - \frac{B_c}{B_0} y_c \right) \quad (2.181)$$

Express the longitudinal component of the electron velocity through U_0 ($v_z = \frac{dz}{dt} = \sqrt{\frac{2e}{m} U_0}$) and proceed from the differentiation with respect to t to the differentiation with respect to z

$$\frac{d^2 y}{dz^2} - \frac{j_l}{4\epsilon_0 \sqrt{\frac{2e}{m} U_0^{3/2}}} + \frac{e}{2m} \times \frac{B_0^2}{U_0} \left(y - \frac{B_c}{B_0} y_c \right) = 0 \quad (2.182)$$

The analysis of equation (2.182) shows that an equilibrium strip flow can be also obtained in the presence of a magnetic field in the cathode plane, but if that is the case the value of magnetic induction necessary for maintaining a stable flow exceeds the Brillouin value. It is obvious that the equation of the Brillouin flow (2.174) can be obtained from general equation (2.182) with $d^2 y/dz^2 = 0$ and $B_c = 0$. Making the first term in (2.182) equal to zero and using the expression for the Brillouin half-thickness y_0 (2.175), we get the value for the half-thickness of the equilibrium strip flow y_e

$$y_e = y_0 + \frac{B_c}{B_0} y_c \quad (2.183)$$

A non-rippling (smooth) equilibrium strip flow is obtained on satisfying the following initial conditions: $y_i = y|_{z=0} = y_e$, $\tan \gamma_i - \frac{dy}{dz} \Big|_{z=0} = 0$. In the general case (when $y_i \neq y_e$ and $\tan \gamma_i \neq 0$) the solution of equation (2.182) has the form

$$y(z) = y_e + A_r \sin \left(\frac{2\pi}{\lambda_r} z + \varphi_0 \right) \quad (2.184)$$

where y_e is the half-thickness of the equilibrium strip flow determined by expression (2.183); A_r is the ripple amplitude

$$A_r = y_i \sqrt{\left(1 - \frac{y_e}{y_i} \right)^2 + 2 \frac{m}{e} \times \frac{U_0}{B_0^2 y_i^2} \tan^2 \gamma_i} \quad (2.185)$$

λ_r is the ripple wavelength

$$\lambda_r = \frac{4\pi}{\sqrt{\frac{2e}{m}}} \times \frac{\sqrt{U_0}}{B_0} \quad (2.186)$$

φ_0 is the initial phase:

$$\varphi_0 = -\arctan \frac{\left(1 - \frac{y_e}{y_i}\right)}{\sqrt{\frac{2m}{e}U_0}} \times \frac{B_0 y_i}{\tan \gamma_i} \quad (2.187)$$

As strictly satisfying the initial conditions providing for the existence of a smooth strip flow is hardly realizable in practice, the real flows always have more or less wavy boundaries. From the physical point of view the presence of ripple, as the case is with an axisymmetric beam, is accounted for by the fact that the condition of complete compensation for the Coulomb repulsive force by the Lorentz focusing magnetic force is not satisfied at the boundary of the beam. With $y_i > y_e$ the magnetic force is predominant, the beam is contracted, while with $y_i < y_e$, on the contrary, the Coulomb repulsive force is predominant and the beam expands. The same phenomena occur with $\gamma_i \neq 0$, i.e. when the beam electrons have a velocity component $v_y \neq 0$.

A longitudinal magnetic field can also be used for collimating a tubular intensive beam. However, the focusing (collimating) of the tubular beam by a longitudinal magnetic field substantially differs from the focusing of a solid axisymmetric beam. If the tubular beam is hollow, i.e. does not contain internal electrons, the field intensity at the internal boundary of the flow is equal to zero according to the Ostrogradsky—Gauss theorem. When such a beam is inserted into a uniform magnetic field, it is possible to satisfy the conditions of compensation for the Coulomb repulsive force by the focusing magnetic force only at the external boundary of the beam. The focusing magnetic force cannot be compensated for at the internal boundary of the beam due to the absence of a radial electrostatic force. The internal boundary will be pulled to the axis and the tubular beam will finally be converted into an intensively rippling turbulent solid flow.

Principally a stable tubular flow similar to the Brillouin flow can be formed by placing inside the beam a cylindrical electrode at a potential which is negative with respect to the external boundary. The value of the potential of the internal electrode is selected so that radial component of the electric field at the internal boundary of the beam is equal to the intensity at the external boundary of a solid Brillouin axisymmetric flow with a radius equal to that of the internal boundary of the tubular beam. In this case the internal electrode simulates the field of the absent internal part of the solid flow. Practical realization of the tubular Brillouin flow is difficult due to the complexity associated with attachment of the internal electrode.

When considering the magnetic focusing of solid axisymmetric and strip beams, it was assumed that all the electrons of the flow

entering the magnetic field intersect the magnetic lines of force. This intersection produces an azimuthal or transverse component of the electron velocity. This component generates a magnetic focusing force compensating for the Coulomb repulsive force. As stated above, there is no radial electrostatic force at the internal boundary of the hollow tubular beam. Therefore, a stable tubular flow can exist only when the Lorentz magnetic force at the internal boundary of the beam is equal to zero. It is clear that this condition can be met, when the electrons at the internal boundary of the beam have no azimuthal velocity component.

The absence of such a component can be attained by introducing the beam into the magnetic field in such a manner that the paths

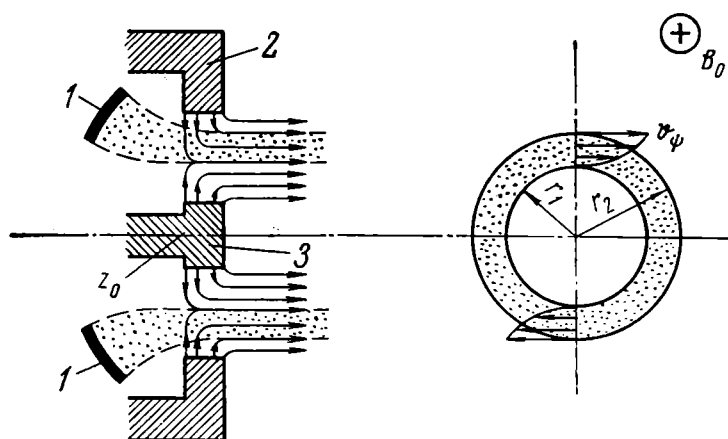


Fig. 2.34. Formation of tubular beam in longitudinal magnetic field
1—cathode; 2—external magnetic circuit; 3—internal magnetic circuit

of the electrons moving along the internal boundary of the beam do not intersect magnetic lines of force.

Consider a magnetic focusing system for limitation of a hollow tubular flow, in which the above condition is satisfied (Fig. 2.34).

The system has an internal magnetic circuit which provides for branching of a magnetic flux in an initial plane z_0 . The tubular electron flow is introduced into the magnetic field so that the section of its internal boundary by the initial plane coincides with the magnetic flux branching circle. It is clear that in this case the internal trajectories do not intersect the magnetic lines of force and, therefore, the electrons at the internal boundary of the beam attain no azimuthal velocity component.

Designate the radii of the internal and external boundaries of the beam as r_1 and r_2 respectively. For the electrons removed from the axis at a distance r_1 the azimuthal velocity $\dot{\psi} = 0$. For the electrons removed from the axis at distances r other than r_1 ($r_1 < r < r_2$) the azimuthal velocity is different from zero and can be determined

by means of the Bush theorem (2.129)

$$r^2\dot{\psi} - r_1^2\dot{\psi}_1 = \frac{e}{2\pi m}(\Psi - \Psi_1) \quad (2.188)$$

where $\dot{\psi}_1 = 0$ (the azimuthal velocity of the electrons at the internal boundary of the beam).

Expressing the magnetic fluxes through the magnetic induction B_0 of a uniform field

$$\Psi = \pi r^2 B_0, \quad \Psi_1 = \pi r_1^2 B_0 \quad (2.189)$$

we obtain the azimuthal velocity

$$\dot{\psi} = \frac{e}{2m} B_0 \left[1 - \left(\frac{r_1}{r} \right)^2 \right] \quad (2.190)$$

From (2.190) it directly follows that the electrons spaced from the axis at various distances have different azimuthal velocities. With $r = r_1$ (the internal boundary of the beam) $\dot{\psi}_1 = 0$, the electron trajectories are parallel to the axis. As r rises up, the azimuthal velocity increases and the trajectories are twisted about the axis more and more. The azimuthal velocity $v_\psi = r\dot{\psi}$ results in a radial (focusing) Lorentz force. Substitute $\dot{\psi}$ from (2.190) into the equation of electron motion in a radial direction (2.134)

$$m\ddot{r} = \frac{e^2}{4m} B_0^2 r \left[\left(\frac{r_1}{r} \right)^4 - 1 \right] - eE_r \quad (2.191)$$

The last term of the obtained equation taking into account the action of space charge cannot be, as in (2.134), presented by expression (2.8), since in this case, as will be shown later, the current density cannot be regarded uniform through the beam section.

On the basis of (2.191) a zero radial velocity of the electrons at all the points of the beam section ($r_1 \leq r \leq r_2$) is a prerequisite of existence of an equilibrium tubular beam. Equating the left-hand side of equation (2.191) to zero, we obtain the condition of the equilibrium motion of electrons

$$\frac{\partial U}{\partial r} = \frac{e}{4m} B_0^2 r \left[1 - \left(\frac{r_1}{r} \right)^4 \right] \quad (2.192)$$

Equation (2.192) shows that an equilibrium tubular flow can be produced with a radial variation of the electric field in the region occupied by the electrons. Such a variation of the component E_r , in turn, is possible only with a variation of the space charge density and, therefore, the current density through the beam section. Substitute (2.192) into the Poisson equation

$$\frac{\partial^2 U}{\partial z^2} + \frac{1}{r} \times \frac{\partial U}{\partial r} + \frac{\partial^2 U}{\partial r^2} = - \frac{\rho}{\varepsilon_0} \quad (2.193)$$

Assuming that U is independent of z , we obtain an expression for the distribution of the space charge density

$$\rho(r) = \frac{e_0 e}{2m} B_0^2 \left[1 + \left(\frac{r_1}{r} \right)^4 \right] \quad (2.194)$$

Thus, satisfying the conditions $\dot{\psi}|_{r=r_1} = 0$ and (2.194), it is theoretically possible to produce an equilibrium non-rippling tubular flow in a uniform longitudinal magnetic field with a completely shielded cathode ($B_c = 0$). When the above conditions are satisfied the flow stability is provided by compensating for the Coulomb repulsive forces by the magnetic focusing force at any point of the beam cross section ($r_1 < r < r_2$). This equilibrium tubular flow is sometimes called the *Samuel flow*.

Generation of a Samuel flow is practically rather difficult since it is impossible to provide a space charge density distribution in an electron flow strictly corresponding to equation (2.194). However, if the electron tube is thin-walled (r_1 is close to r_2) and the conditions of equilibrium motion are fulfilled at the internal and external boundaries of the flow, a quazistationary tubular flow with $\rho = \text{const}$ is practically realizable. In this case electron layers ripple occurs inside the beam, but its internal and external boundaries remain linear. When the conditions for introducing a beam into a uniform field are not satisfied (for example, the presence of radial velocity components at $z = 0$ or when the radii of the internal and external boundaries of the beam do not correspond to the equilibrium values), the tubular beam will be rippling like an axisymmetric or strip beam. The ripples of the external boundary of a tubular beam are described by equations (2.167) to (2.169), as the case is with a solid axisymmetric beam. The analysis of the internal boundary ripple shows that, depending on the initial conditions, this can be both in-phase or out-of-phase with respect to the external boundary ripple and when the initial conditions are badly violated, the ripple amplitude can exceed the value r_1 , in which case the result may be a broken beam.

At the present time systems with crossed magnetic and electric fields known as *magnetron* or *injector guns* are widely used for shaping tubular electron flows. The action of such a system is based on pulling a cloud of space charge formed in a simple magnetron into the drift space. As is known a diode with a cylindrical cathode and anode placed into a uniform magnetic field B_0 directed along the axis of the electrode system is the simplest static magnetron. Due to the action of the magnetic field the trajectories of the electrons flowing from the cathode and accelerated by the anode voltage U_a start curving and at a certain critical value of the magnetic induction B_{cr} the electrons fail to reach the anode and form a circular space charge around the cathode. The magnitude of the critical magnetic induc-

tion is defined by the formula which is obtained from (1.13) when substituting $R = r_a - r_c$

$$B_{cr} = \frac{4}{\sqrt{\frac{2e}{m}}} \times \frac{\sqrt{U_a}}{r_a \left(1 - \frac{r_c}{r_a}\right)} \quad (2.195)$$

where r_a and r_c are the radii of the anode and cathode respectively.

Now, if a longitudinal electric field is used to impart a longitudinal velocity to the electrons which form the space charge in the

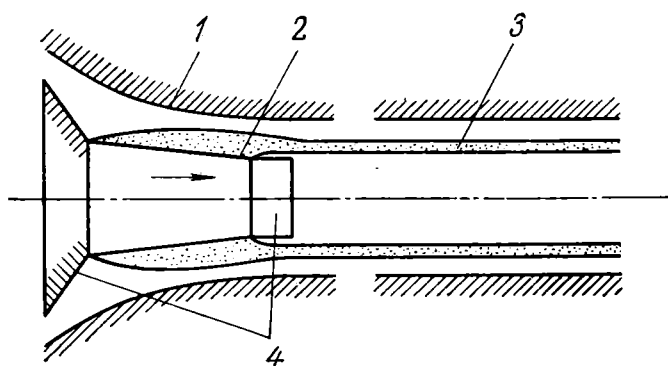


Fig. 2.35. Magnetron gun

1—anode; 2—cathode; 3—beam; 4—focusing electrodes

interelectrode space of the magnetron, the circular cloud of the space charge starts to move along the axis thus forming a tubular electron flow. The longitudinal intensity component of the electric field

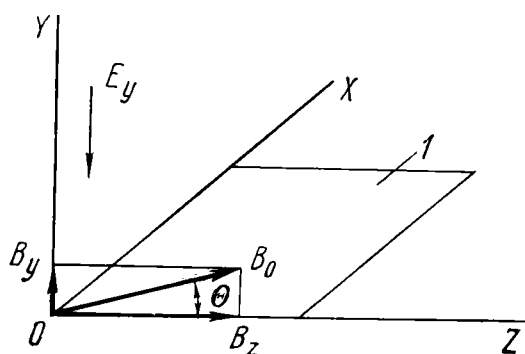


Fig. 2.36. On calculation of magnetron gun

1—cathode

required to pull the electrons from the magnetron can be obtained by making the cathode and anode of the magnetron in the form of a truncated cone (Fig. 2.35).

Let us consider the motion of electrons in a magnetron gun. Let the origin of coordinates be positioned on the cathode surface and the axis OZ be directed along the generatrix of the conical cathode,

OX be directed along the tangent, and OY be directed along the normal to the cathode surface. With such directions of the coordinate axes the longitudinal magnetic field and the axis OZ form an angle Θ which is half the angle at the apex of the cathode cone (Fig. 2.36).

If the width of the circular gap between the cathode and the anode is small as compared to the cathode radius $[(r_a - r_c)] \ll r_c$, the given areas of the cathode and anode may in a first approximation be regarded as planes, thus a problem for a planar magnetron will be solved. It is obvious that in the selected system of coordinates the uniform longitudinal magnetic field is determined by two components parallel to the axes OZ and OY

$$B_z = B_0 \cos \Theta, \quad B_y = B_0 \sin \Theta \quad (2.196)$$

The electric field in a planar diode has only one component $E_y \neq 0$. Let us set up an equation of the electron motion in a planar magnetron [see (1.7) and (1.10)]

$$\left. \begin{aligned} \ddot{x} &= -\frac{e}{m} B_z \dot{y} + \frac{e}{m} B_y \dot{z} \\ \ddot{y} &= -\frac{e}{m} E_y + \frac{e}{m} B_z \dot{x} \\ \ddot{z} &= -\frac{e}{m} B_y \dot{x} \end{aligned} \right\} \quad (2.197)$$

Since in the planar diode $E_x = E_z = 0$, the Poisson equation assumes the form

$$\frac{dU^2}{dy^2} = -\frac{dE_y}{dy} = \frac{\rho}{\epsilon_0} \quad (2.198)$$

Furthermore, the current density components j_x and j_z are independent of x and z , and the continuity equation is written as follows

$$dj_y/dy = 0 \quad (2.199)$$

hence

$$j_y = \rho v_y = j_0 = \text{const} \quad (2.200)$$

From (2.198) and (2.200) we obtain

$$\frac{dE_y}{dt} = \frac{dE_y}{dy} \times \frac{dy}{dt} = \frac{\rho v_y}{\epsilon_0} = -\frac{j_0}{\epsilon_0} \quad (2.201)$$

Integration of equation (2.201) with the starting condition $E_y|_{t=0} = 0$ results in an expression for E_y

$$E_y = -\frac{j_0}{\epsilon_0} t \quad (2.202)$$

where j_0 is the current density on the cathode.

Substituting (2.202) into (2.197), we obtain

$$\left. \begin{aligned} \ddot{x} &= -\frac{e}{m} B_z \dot{y} + \frac{e}{m} B_y \dot{z} \\ \ddot{y} &= \frac{e}{m \epsilon_0} j_0 t + \frac{e}{m} B_z \dot{x} \\ \ddot{z} &= -\frac{e}{m} B_y \dot{x} \end{aligned} \right\} \quad (2.203)$$

The solution of equations (2.203) with the starting conditions at $t = 0$; $x = y = z = 0$, $\dot{x} = \dot{y} = \dot{z} = 0$ in the parametric form is as follows

$$\left. \begin{aligned} x &= -\frac{j_0 \cos \Theta}{2 \epsilon_0 B_0} t^2 + \frac{j_0 \cos \Theta}{\epsilon_0 \left(\frac{e}{m} \right)^2 B_0^3} \left[1 - \cos \left(\frac{e}{m} B_0 t \right) \right] \\ &= \frac{\frac{e}{m} j_0 \sin^2 \Theta}{6 \epsilon_0} t^3 + \frac{j_0 \cos^2 \Theta}{\epsilon_0 \frac{e}{m} B_0^2} t - \frac{j_0 \cos^2 \Theta}{\epsilon_0 \left(\frac{e}{m} \right)^2 B_0^3} \sin \left(\frac{e}{m} B_0 t \right) \\ z &= \frac{\frac{e}{m} j_0 \sin \Theta \cos \Theta}{6 \epsilon_0} t^3 - \frac{j_0 \sin \Theta \cos \Theta}{\epsilon_0 \frac{e}{m} B_0^2} t + \frac{j_0 \sin \Theta \cos \Theta}{\epsilon_0 \left(\frac{e}{m} \right)^2 B_0^3} \sin \left(\frac{e}{m} B_0 t \right) \end{aligned} \right\} \quad (2.204)$$

The given solution shows that in the magnetron gun the electrons have all the three velocity components. These are a component directed along the cathode (v_z), one normal to the cathode surface (v_y) and one parallel to the cathode surface (v_x). The latter component has the meaning of a "drift" rate of the electrons across the magnetic field. Figure 2.37 illustrates the projection of the electron trajectory onto the plane YOZ (dash line) calculated by equations (2.204).

The trajectory can be regarded as a smooth curve determined by the first two terms of the second and third equations (2.204), on which superimposed are periodic disturbances determined by the last terms of these equations including

$$\sin \left(\frac{e}{m} B_0 t \right)$$

Now assume that the electrons emerging from the cathode have other than zero velocity $v_y = \dot{y}_0$ perpendicular to the cathode sur-

face. In this case the solution of equation (2.203) assumes the form

$$\left. \begin{aligned}
 x &= -\frac{j_0 \cos \Theta}{2\varepsilon_0 B_0} t^2 + \frac{\left[1 - \cos\left(\frac{e}{m} B_0 t\right)\right] \cos \Theta}{\frac{e}{m} B_0} \left(\frac{j_0}{\varepsilon_0 \frac{e}{m} B_0^2} - \dot{y}_0\right) \\
 y &= \frac{\frac{e}{m} j_0 \sin \Theta \cos \Theta}{6\varepsilon_0} t^3 + \dot{y}_0 t + \frac{\frac{e}{m} B_0 t - \sin\left(\frac{e}{m} B_0 t\right)}{\frac{e}{m} B_0} \times \\
 &\quad \times \cos^2 \Theta \left[\frac{j_0}{\varepsilon_0 \frac{e}{m} B_0^2} - \dot{y}_0\right] \\
 z &= \frac{\frac{e}{m} j_0 \sin \Theta \cos \Theta}{6\varepsilon_0} t^3 - \frac{\frac{e}{m} B_0 t - \sin\left(\frac{e}{m} B_0 t\right)}{\frac{e}{m} B_0} \times \\
 &\quad \times \sin \Theta \cos \Theta \left[\frac{j_0}{\varepsilon_0 \frac{e}{m} B_0^2} - \dot{y}_0\right]
 \end{aligned} \right\} \quad (2.205)$$

The first terms of equation system (2.205) describe a trajectory in the form of a smooth curve, while the last terms describe the

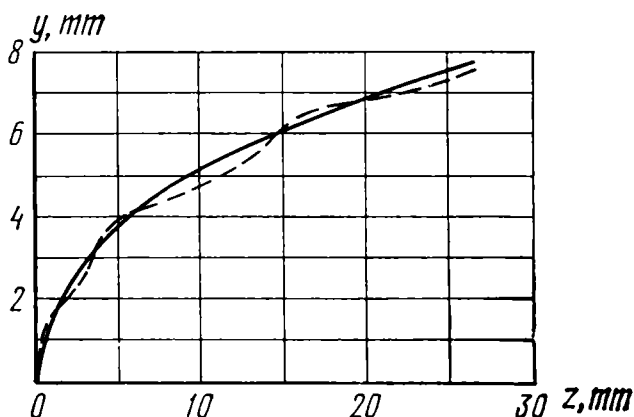


Fig. 2.37. Trajectory of outer electron in magnetron gun (projection on plane YOZ)

disturbances superimposed onto this trajectory. The amplitude of these disturbances is the lower, the higher is the value of magnetic induction B_0 of the longitudinal field. With a sufficiently high value of B_0 we may roughly consider that at some distance from the cathode the trajectory becomes smooth, i.e. the last terms of the system of equations (2.205) may be neglected in the calculations. It is evident that the rejection of the last terms is equivalent to the

assumption that the initial velocity is equal to

$$j_0/\varepsilon_0 \frac{e}{m} B_0$$

Under such an assumption the smooth trajectory will be determined by the following equations

$$\left. \begin{aligned} x &= -\frac{j_0 \cos \Theta}{2\varepsilon_0 B_0} t^2 \\ y &= \frac{\frac{e}{m} j_0 \sin^2 \Theta}{6\varepsilon_0} t^3 + \frac{j_0}{\varepsilon_0 \frac{e}{m} B_0^2} t \\ z &= \frac{\frac{e}{m} j_0 \sin \Theta \cos \Theta}{6\varepsilon_0} t^3 \end{aligned} \right\} \quad (2.206)$$

The trajectory described by equations (2.206) is shown in Fig. 2.37 by a continuous line. Express t from the last equation of system (2.206) and substitute it into the second equation. As a result, we obtain an equation of the projection of the trajectory $y = f(z)$ onto the plane YOZ

$$y = z \tan \Theta + \sqrt[3]{\frac{6}{\sin \Theta \cos \Theta}} \frac{j_0^{2/3}}{\varepsilon_0^{2/3} \left(\frac{e}{m}\right)^{4/3} B_0^2} z^{1/3} \quad (2.207)$$

This equation shows that at a low current density and adequate magnetic induction the trajectory approximates the line of force of the longitudinal magnetic field. In a similar manner it is possible to calculate the paths of the electrons leaving the cathode at any point $(x_0, 0, z_0)$. Thus, an equilibrium strip flow with slightly wavy boundaries can exist in a planar magnetron. Proceeding to the magnetron gun formed by axisymmetric electrodes, we may assert on the basis of the given analysis that such a system is applicable for producing a tubular beam. It is only necessary to select the electrodes of such a shape that the electric field in the near-cathode region remain unchanged, when changing over from a planar system to an axisymmetric system. When changing over from the planar to the axisymmetric system the quantity $\dot{x} = v_x$ will have the sense of an azimuthal velocity determining the revolution of the tubular flow about the axis.

2.6. Focusing Intensive Beams by Periodic Fields

Nowadays systems with electrostatic or magnetic fields periodically varying along the beam are widely used for limiting the expansion of intensive electron beams of a considerable length. The system

of periodic focusing can in principle be presented as a series of converging electrostatic or magnetic lenses through which passes an electron beam.

Assume that an electron lens is placed in the path of an axisymmetric beam spreading in a field-free space. The lens optical power is selected so that the beam diverging due to the action of the space charge becomes converging after passing the lens (Fig. 2.38).

Having passed the crossover plane, the beam diverges again at some distance from the lens. Another similar electron lens is placed in the plane where the radius of the beam reaches the value which it had at the inlet of the first lens. The beam becomes converging again, passes a crossover and starts diverging once more. The diverging beam is directed to a third lens, etc. Thus, a series of electron

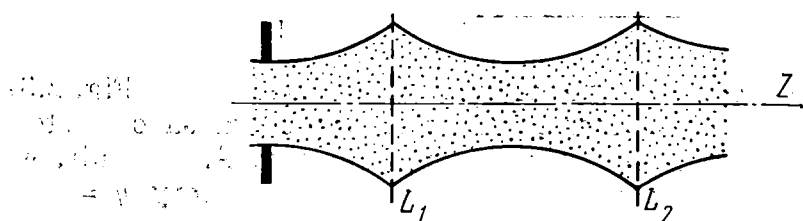


Fig. 2.38. Schematic diagram of periodic focusing system

lenses can be used to form a long intensive beam with a wavy boundary. The undulation of the boundary can be reduced to minimum by means of proper selection of the optical power of the lenses and their arrangement, i.e. the system of periodic focusing is suitable for producing a stable and generally pulsating intensive beam. Since the amount of identical lenses can be sufficiently great, the periodic focusing system is particularly convenient for producing long beams. The wavy boundary of such a beam does not impede practical application of such systems, since the production of ideally smooth beams (for example, Brillouin type) requires strict observance of the initial conditions under which the beam is inserted into a magnetic field and real beams are always pulsating to some extent.

One of the advantages of periodic focusing systems is in their high economy as compared to the systems with a uniform magnetic field. Replacement of an elongated solenoid producing a uniform field by a series of short coils allows the dimensions and weight of the focusing system to be reduced considerably and also to reduce its power consumption. The magnetic system becomes particularly economical when use is made of permanent magnets to produce a magnetic field periodically varying along the length of the beam.

The periodic focusing system can be regarded as a sequence of electron lenses. However, in practice the focusing action of individual lens included into a system for periodic focusing cannot be

considered without taking into account the action of the adjacent lenses. This is associated with the fact that the period of the system, i.e. the distance between the centre-planes of the adjacent lenses cannot be too long, since a high-perveance beam would be excessively diverged due to the Coulomb repulsive force. When the lenses are positioned near one another (a small period of the system), the fields of the adjacent lenses unavoidably interfere with each other. Furthermore, the formulas of geometric electron optics become unsuitable for calculation of the parameters of electron lenses in the case

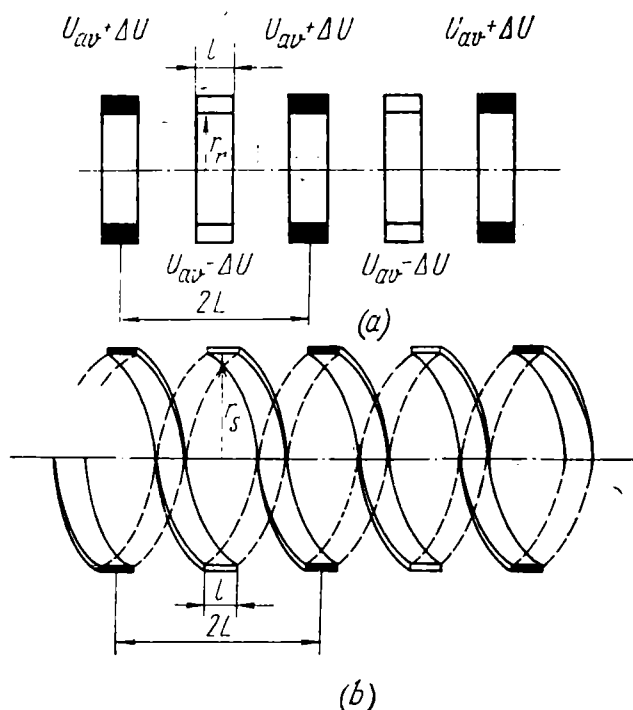


Fig. 2.39. Electrostatic periodic focusing systems
(a) series of rings; (b) double helix

of intensive beams. Therefore, in the analysis of the periodic focusing systems the real potential distribution or magnetic induction is usually approximated by a suitable periodic function and the electron trajectory in a periodic field is calculated with due account for the action of space charge.

Consider an electrostatic system for periodic focusing formed of a series of electrically conductive rings located along the axis (Fig. 2.39a). Designate the potentials of the rings as $U_{av} \pm \Delta U$, where U_{av} is the average potential determining the averaged axial velocity of the electrons of the beam; ΔU is the focusing potential determining the radial (focusing) component of the electric field strength. The potential in such a system can be described with an

adequate practical accuracy by the expression

$$U(z, r) = U_{av} + U_1(r) \cos\left(\frac{\pi}{L} z\right) \quad (2.208)$$

where $2L$ is the period of the system (the distance between the rings with similar potential).

$$U_1(r) = \Delta U F = \frac{J_0\left(\frac{\pi}{L} r\right)}{J_0\left(\frac{\pi}{L} r_r\right)} \quad (2.209)$$

In its turn

$$F = \frac{4}{\pi} \times \frac{\sin \frac{\pi}{2} \left(1 - \frac{l}{L}\right)}{\frac{\pi}{2} \left(1 - \frac{l}{L}\right)} \quad (2.210)$$

where J_0 is modified Bessel's function of the first kind; l is the width (axial length) of the rings; r_r is the radius of the rings.

A similar potential distribution is obtained in practically employed focusing system formed by a double helix (Fig. 2.39b) with the turn potentials $U_{av} + \Delta U$ and $U_{av} - \Delta U$. Some difference lies only in the azimuthal dependence of U

$$U(z, r, \psi) = U_{av} + U_1(r) \cos\left(\frac{\pi}{L} z - \psi\right) \quad (2.211)$$

As before, U_1 is determined by formula (2.209).

Let us set up an equation of motion of the outer electron of axisymmetric beam in a field determined by the expression (2.208) or (2.211) with due account for the action of space charge. Use the following simplifying assumptions: the current density is the same at any point of the cross section of the beam; the beam is laminar, i.e. the outer electron always remains at the beam boundary: $\Delta v_z \ll v_{z, av}$ ($v_{z, av}$ is the average axial velocity of the electron determined by the value of U_{av}). Designate the average radius of the beam as r_0 .

The trajectory of the outer electron can be presented in the form of a periodic curve oscillating near the generating line of a cylinder with the radius r_0

$$r(z) = r_0 + r_1(z) \quad (2.212)$$

Under the assumptions taken above the equation of the radial motion of the outer electron assumes the form

$$\ddot{r}_1 = \frac{e}{m} \times \frac{\partial U}{\partial r} + \frac{e}{m} \times \frac{I}{2\pi\epsilon_0 \sqrt{\frac{2e}{m} U_{av} r_0}} \quad (2.213)$$

the last term in (2.123) takes account for the action of space charge [see (2.7)].

Expand $U(r)$ into a Taylor series near $r = r_0$ with power r_1 :

$$U(r) = U(r_0) + U'(r_0)r_1 + \frac{1}{2}U''(r_0)r_1^2 + \dots \quad (2.214)$$

where the primes stand for differentiation with respect to r .

Assuming that $r_1 \ll r_0$, we may neglect the terms with r_1 having powers higher than second in (2.214). Then, using equation (2.208), we obtain

$$\frac{\partial}{\partial r} U(z, r) = [U'(r_0) + U''(r_0)r_1] \cos \frac{\pi}{L} z \quad (2.215)$$

Substituting (2.215) into (2.213) and proceeding from differentiation with respect to t to the differentiation with respect to z we have

$$\begin{aligned} & \frac{d^2 r_1}{dz^2} - \frac{U''(r_0)}{2U_{av}} \left(\cos \frac{\pi}{L} z \right) r_1 = \\ & = \frac{U'(r_0)}{2U_{av}} \cos \frac{\pi}{L} z + \frac{I}{4\sqrt{2}\pi\epsilon_0 \left(\frac{e}{m}\right)^{1/2} U_{av}^{3/2} r_0} \end{aligned} \quad (2.216)$$

The obtained nonlinear equation with periodic coefficients relates to the differential equations of the Mathieu type. In a canonical form the homogeneous Mathieu equation is written as

$$\frac{d^2 y}{dx^2} + (a - 2q \cos 2x)y = 0 \quad (2.217)$$

The Mathieu equation is well studied. Its solutions can be found in the form of series. A specific feature of the Mathieu equation is the presence of stable (converging) and unstable (diverging) solutions depending on the ratio of the parameters a and q . The regions of stable and unstable solutions are described by the so-called *Mathieu stability diagram* (Fig. 2.40) in which the digits 1, 2, 3 in circles stand for the stability regions.

Considering the left-hand side of equation (2.216), it is clear that it coincides with the canonical Mathieu equation when

$$x = \frac{\pi}{2L} z, \quad q = \left(\frac{L}{\pi}\right)^2 \frac{U''(r_0)}{U_{av}}, \quad a = 0 \quad (2.218)$$

Since in our case $a = 0$, a stable solution is possible only at $q < 0.8$ (Fig. 2.40), i.e. the focusing can be effected when the con-

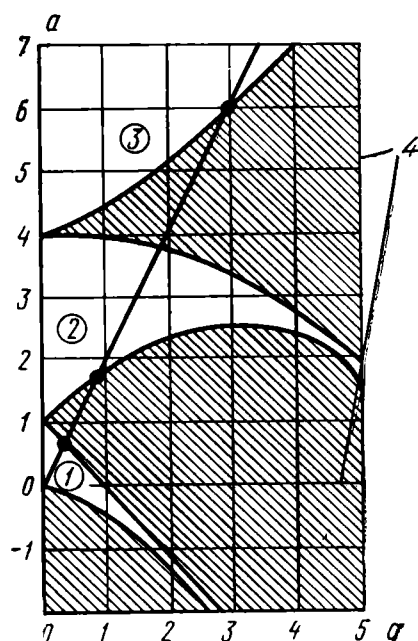


Fig. 2.40. Mathieu stability diagram

1, 2 and 3—stability region; 4—instability region

dition

$$U''(r_0) < 0.8 \left(\frac{\pi}{L} \right)^2 U_{av} \quad (2.219)$$

is satisfied.

The condition (2.219) shows that when the magnitude of the second derivative U'' at the beam boundary is large enough, the optical power of the lens increases to such an extent that the beam is re-focused, the electrons start settling onto the ring of the focusing system and the current flow stops. The presence of a cut-off region of the beam corresponding to the transfer to the instability area in the Mathieu diagram is characteristic of periodic focusing systems.

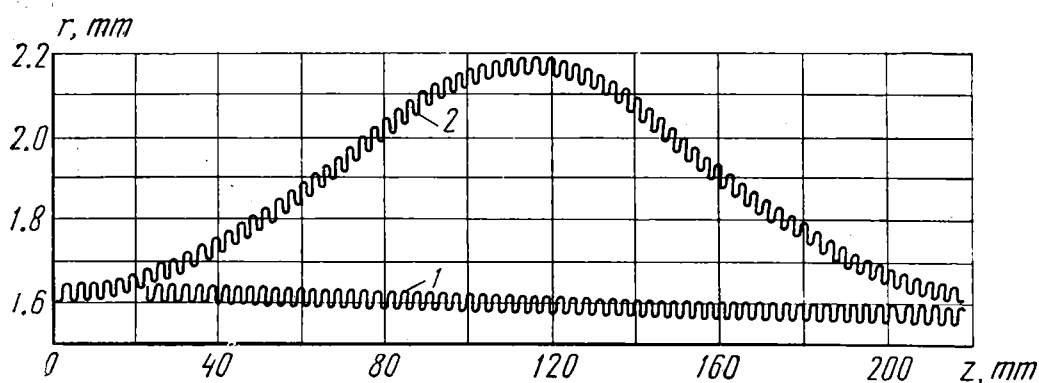


Fig. 2.41. Trajectory of beam outer electron in periodic electrostatic field

Since the value $U''(r_0)$ is determined by the focusing potential ΔU , it follows from (2.219) that a stable flow can be obtained only when $\Delta U \ll U_{av}$.

Now let us consider the solution of nonhomogeneous equation (2.216). It is known, the general solution of a nonhomogeneous differential equation can be presented in the form of the sum of the general solution of homogeneous equation and partial solution of nonhomogeneous equation

$$r_1(z) = C_1 f_1(z) + C_2 f_2(z) + \sum_{m=1}^{\infty} a_m \cos \frac{m\pi}{L} z \quad (2.220)$$

where $f_1(z)$ and $f_2(z)$ are the Mathieu functions (elliptical cosine and sine of the real fractional order) calculated by means of corresponding series; C_1 and C_2 are integration constants determined by the initial conditions. The third term is the partial solution of equation (2.216).

The analysis of the solution of equation (2.220) shows that the first two terms in the right-hand side describe the pulsations of the flow with a period considerably exceeding $2L$, and the last term describes the waviness with a period equal to $2L$ (Fig. 2.41).

The pulsations can be made as small as desired by the appropriate selection of the initial conditions. For example, if the radius of a beam introduced into a periodic focusing system is equal to the average value of r_0 , the constant C_2 vanishes; when $dr_1/dz|_0 = 0$, the constant C_1 vanishes. In this case the trajectory of the outer electron is a wavy line oscillating near the generatrix of a cylinder with the radius r_0 .

When the given initial conditions $C_1 = C_2 = 0$ are satisfied, the outer electron trajectory is described by the series

$$r(z) = r_0 + r_1(z) = r_0 + \sum_{m=1}^{\infty} a_m \cos \frac{m\pi}{L} z \quad (2.221)$$

In order to determine the coefficient a in expansion (2.221), let us differentiate the series with respect to z two times

$$\frac{\partial^2 r}{\partial z^2} = \frac{d^2 r_1}{dz^2} = -a_1 \left(\frac{\pi}{L} \right)^2 \cos \frac{\pi}{L} z - a_2 \left(\frac{2\pi}{L} \right)^2 \cos \frac{2\pi}{L} z - \dots \quad (2.222)$$

and substitute r_1 and $d^2 r_1/dz^2$ into equation (2.216).

Since series (2.221) is a solution of equation (2.216), this substitution transfers equation (2.216) to an identity which holds for any value of z . Obviously, it is possible, when the coefficients at $\cos \frac{m\pi}{L} z$ (the values for any m) and the free term go to zero.

Substituting $\cos^2 \frac{m\pi}{L} z$ in the form $\frac{1}{2} \left(1 + \cos \frac{2m\pi}{L} z \right)$ and equating the corresponding terms to zero we obtain:

for the free term

$$a_1 \frac{U''(r_0)}{4U_{av}} + \frac{I}{4\sqrt{2}\pi\epsilon_0 \left(\frac{e}{m} \right)^{1/2} U_{av}^{3/2} r_0} = 0 \quad (2.223)$$

for the term with $\cos z \frac{\pi}{L}$

$$\frac{U'(r_0)}{2U_{av}} + a_1 \left(\frac{\pi}{L} \right)^2 + a_1 \frac{U''(r_0)}{4U_{av}} = 0 \quad (2.224)$$

for the common term with $\cos \frac{m\pi}{L} z$ at $m \geq 2$

$$m^2 a_m + \frac{U''(r_0)}{4U_{av}} \left(\frac{L}{\pi} \right)^2 (a_{m-1} + a_{m+1}) = 0 \quad (2.225)$$

Taking into account that the value $\left(\frac{L}{\pi} \right)^2 \frac{U''(r_0)}{4U_{av}}$ is small as compared to unity [see (2.219)], the coefficient a_m from (2.225) may approximately be expressed, neglecting a_{m+1} in comparison with a_{m-1}

$$a_m \approx \frac{U''(r_0)}{4U_{av}} \left(\frac{L}{\pi} \right)^2 \frac{a_{m-1}}{m^2}, \quad (m \geq 2) \quad (2.226)$$

Therefore, the series quickly converges and for rough calculations we may restrict ourselves to its first term determining a_1 from (2.223).

Thus, under the given initial condition ($r_{\text{initial}} = r_0$ and $dr_1/dz_{\text{initial}} = 0$) the radial motion of the outer electron of the beam is described by the approximate equation

$$r(z) \approx r_0 - \frac{I}{\sqrt{2} \pi \epsilon_0 \left(\frac{e}{m}\right)^{1/2} U_{av}^{1/2} U''(r_0) r_0} \cos \frac{\pi}{L} z \quad (2.227)$$

or according to (2.224)

$$r(z) \approx r_0 - \left(\frac{L}{\pi}\right)^2 \frac{U'(r_0)}{2U_{av}} \cos \frac{\pi}{L} z \quad (2.228)$$

The last equation shows that the waviness (ripple) of the beam is reduced and the beam becomes more smooth with a decrease in the period of the system and U' , i.e. at small values of ΔU as compared to U_{av} . It should be noted that more accurate solutions taking into account the last terms of the series (2.221) differ but slightly from the approximate solutions (2.227) and (2.228).

The value of focusing potential can be approximately found from the formula

$$\frac{\Delta U}{U_{av}} \approx 3.5 \times 10^2 \frac{\sqrt{P}}{F \sqrt{\varphi\left(\frac{\pi r_0}{L}\right)}} \quad (2.229)$$

where P is the beam perveance, $A/V^{3/2}$; F is determined by formula (2.210)

$$\varphi\left(\frac{\pi r_0}{L}\right) = \frac{J'_0\left(\frac{\pi r_0}{L}\right) J_0\left(\frac{\pi r_0}{L}\right) + J'_0\left(\frac{\pi r_0}{L}\right) J''_0\left(\frac{\pi r_0}{L}\right)}{J_0^2\left(\frac{\pi r_0}{L}\right)} \times \frac{\pi r_0}{L} \quad (2.230)$$

J_0 , J'_0 , J''_0 are modified Bessel's function and its first and second derivatives.

The above-given solutions for the periodic focusing system formed by a series of rings also hold for a focusing system shaped as a double helix (see Fig. 2.39b). The difference is in that the "humps" and "gaps" of the wavy surface of the beam do not lie in planes perpendicular to the beam axis but form helical lines on the beam surface in accordance with the positions of the double-helix turns.

The periodic electrostatic focusing can be also effected by means of a system of apertured discs (diaphragms) arranged along the axis at potentials $U_{av} + \Delta U$ (Fig. 2.42).

The potential distribution along the axis of such a system can approximately be presented as

$$U_0(z) \approx U(z, 0) = U_{av} + U_1 \left(1 - \cos \frac{\pi}{L} z\right) \quad (2.231)$$

where U_{av} is the averaged potential on the axis determining the average value of the axial velocity of the beam electrons; U_1 is the amplitude value of the periodic component of the axis potential; $2L$ is the period of the system.

Let us set up an equation of the motion of outer electron of axisymmetric laminar flow in the focusing system under discussion, restricting ourselves to the paraxial region. Use the basic equation of paraxial geometrical optics (1.8) and take into account the action of space charge according to (2.7)

$$\frac{d^2r}{dz^2} + \frac{U'_0(z)}{2U_0(z)} \times \frac{dr}{dz} + \frac{U''_0(z)}{4U_0(z)} r - \frac{I}{4 \sqrt{2\pi\epsilon_0} \left(\frac{e}{m}\right)^{1/2} U_{av}^{3/2} r} = 0 \quad (2.232)$$

Assume that the amplitude value of the periodic potential is associated with the beam parameters through the relation

$$U_1 = \frac{4L}{\sqrt{3} \pi^{3/2} \epsilon_0^{1/2} \left(\frac{2e}{m}\right)^{3/4} r_0} \sqrt{P} U_{av} \quad (2.233)$$

where r_0 is the mean radius of the beam; P is the beam perveance.

When condition (2.233) and the initial condition $dr/dz|_0 = 0$ are satisfied, i.e. when inserted into the system of periodic focusing

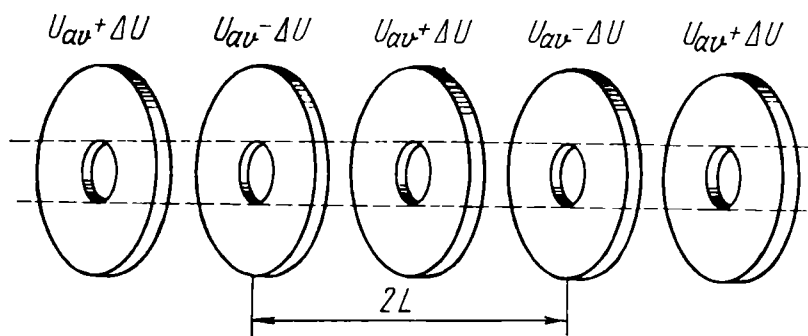


Fig. 2.42. Periodic electrostatic focusing system formed by aperture discs is a beam with boundaries parallel to the axis, equation (2.232) has an approximate solution

$$r(z) \approx r_0 - \frac{U_1}{4U_{av}} \left(1 - \cos \frac{\pi}{L} z\right) r_0 \quad (2.234)$$

Thus, in the case of a focusing system consisting of a series of discs the waviness will be the lower, the lower the amplitude of variations in the axial potential U_1 as compared to U_{av} . However, this is feasible only with a corresponding decrease in the period of the system according to condition (2.233).

There is another approach to the development of the periodic electrostatic focusing systems. Assume that along the boundary of

an intensive electron beam there are arranged a number of electrodes whose shape and potential are selected so that at each point on the beam boundary the repulsive action of space charge is balanced by the force of the external electric field formed by the system of electrodes. Obviously, such formulation of the problem is similar to the Pierce

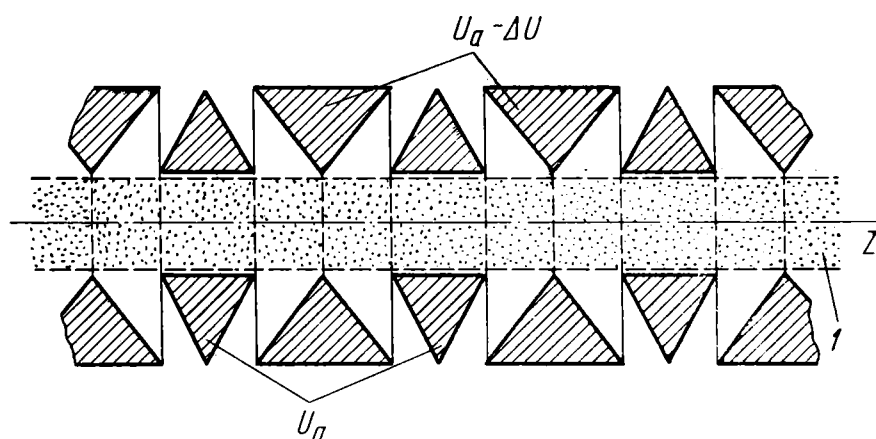


Fig. 2.43. Simplified shape of electrodes of periodic focusing system
1—beam

method employed in the solution of the problems on forming intensive beams (see Sec. 2.4).

The shape and potentials of the electrodes of the periodic focusing system of the type in question can be calculated either approximately by solving the Laplace equation (an external problem) with boundary conditions determined by the parameters of the beam (an internal problem) or found experimentally by simulating in an electrolytic tank. By the way of example, Fig. 2.43 shows the cross section of the electrode system for periodic electrostatic focusing which ensures an approximately parallel electron flow.

Let us consider the system of magnetic periodic focusing. Such a system consists of a series of magnetic lenses arranged along the axis and formed by short coils or circular permanent magnets with pole pieces (Fig. 2.44). The magnetic fields in the adjacent lenses are opposite, i.e. the lenses feature opposite polarity.

In this case an alternating periodic magnetic field is built up in the near-axial region. Generally with an axisymmetric system the magnetic induction varies both along the axis and along the radius: $B = B(z, r)$. However, if the beam radius is substantially less than the inner radius of the pole pieces of the lenses, we get adequate practical accuracy, though neglecting the dependence of B on r , i.e. by restricting ourselves to the paraxial approximation and by approximating the near-axial distribution of the magnetic field

by the harmonic function

$$B_0(z) = B(z, 0) = B_m \cos \frac{\pi}{L} z \quad (2.235)$$

where B_m is the amplitude magnitude of the magnetic induction along the axis; $2L$ is the period of the system.

In order to find the shape of the axisymmetric beam in the system of periodic magnetic focusing, employ the equation for the motion

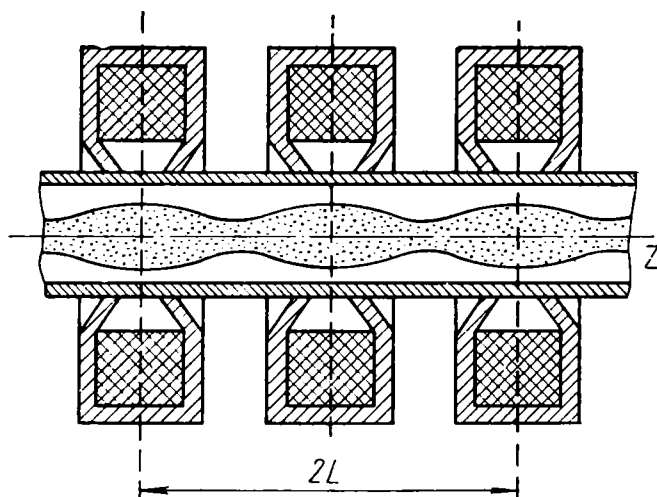


Fig. 2.44. Magnetic periodic focusing system

of the outer electron in a longitudinal magnetic field (2.136). Next, introduce the dimensionless coordinates

$$R = \frac{r}{r_{av}}, \quad Z = \frac{\pi}{L} z \quad (2.236)$$

(r_{av} is the average radius of the beam.)

The parameter of the magnetic field is as follows

$$\alpha = \frac{e}{64\pi^2 m} \times \frac{B_m^2 L^2}{U_0} = 2.8 \frac{B_m^2 L^2}{U_0} [\text{Gs, m, V}] \quad (2.237)$$

The parameter of space charge of the beam is

$$\beta = \frac{P}{4\pi^3 \epsilon_0 \sqrt{\frac{2e}{m}}} \left(\frac{L}{2r_{av}} \right)^2 = 1.53 \times 10^3 P \left(\frac{L}{2r_{av}} \right)^2 [\text{A, V, m}] \quad (2.238)$$

The parameter of the cathode conditions

$$k = \left(\frac{B_c}{B_m / \sqrt{2}} \right) \left(\frac{r_c}{r_{av}} \right)^2 [\text{see (2.157)}] \quad (2.239)$$

In this case, when substituting $B_0(z)$ from (2.235) into (2.136) with the use of (2.236), (2.237), (2.238) and (2.239), we obtain the

equation of the motion of outer electron of the axisymmetric beam in a periodic magnetic field

$$\frac{d^2 R}{dZ^2} + \alpha (1 + \cos 2Z) R - \alpha \frac{k^2}{R^3} - \frac{\beta}{R} = 0 \quad (2.240)$$

The obtained nonlinear differential equation with a periodic coefficient is an equation of the Mathieu type [compare with (2.216)], but, in contrast to (2.216), in the case considered a from the canonical Mathieu equation (2.217) is not equal to zero ($a = \alpha \neq 0$), while a and q of the canonical equation are connected through the ratio

$$a = 2q \quad (2.241)$$

This connection is presented in the Mathieu stability diagram (see Fig. 2.40) by a straight line passing through the origin of coordinates. Thus, a stable beam can be obtained, as in the case of periodic electrostatic focusing, when the parameter α corresponds to the stability regions in the Mathieu diagram. As shown in Fig. 2.40, the first region of stable solutions corresponds to the value of the parameter of the magnetic field, α , from 0 to 0.66, while the second region corresponds to the above parameter from 1.72 to 3.76, etc. In many practical cases the value of α lies within the range of 10^{-2} to 10^{-1} , i.e. it is possible to obtain a stable flow in the first region of stability with a comparatively low magnitude of magnetic induction.

The rough solution of equation (2.240) can be obtained under an assumption of a low waviness of the flow, i.e. with the substitution

$$r = r_{av} (1 + \delta), \quad R = 1 + \delta \quad (2.242)$$

where $\delta \ll 1$ [compare with (2.155)].

In this case equation (2.240) can be linearized on the basis of the approximate relations

$$\frac{1}{R^3} \approx 1 - 3\delta, \quad \frac{1}{R} \approx 1 - \delta \quad (2.243)$$

Then for low deviations of δ we obtain

$$\frac{d^2 \delta}{dZ^2} + \alpha \delta \left[\cos 2Z + 1 + 3k^2 + \frac{\beta}{\alpha} \right] + \alpha \left(\cos 2Z + 1 - k^2 - \frac{\beta}{\alpha} \right) = 0 \quad (2.244)$$

The analysis of equation (2.244) shows that in the general case the boundary of an intensive beam in a periodic magnetic field is represented by a wavy line. Here the deviations of the radius of the beam from the average value depend not only on the relationship of I , U_0 , B_m , k [α , β , k in equation (2.244)], as for the focusing in a uniform field [compare with (2.167)], but also on the character

of the variation in the magnetic field along the axis taken into account by the terms with $\cos 2Z$. In contrast to the limitation of the intensive beam by a uniform longitudinal magnetic field, the repulsive action of beam space charge is compensated for by the magnetic field in the case of limiting by a periodic field not at all times but on the average per period. Since the focusing force in the axisymmetric field is proportional to B_0^2 [see (1.95)], in a periodic field it reaches its maximum twice and goes to zero two times during each period (Fig. 2.45).

Close to the maximums of B_0^2 the focusing force exceeds the Coulomb repulsive force and the beam is confined, but when close to $B_0^2 = 0$ the acting force of space charge is predominant and the beam expands. Only at four points per period — with $|B_0(z)| = B_m/\sqrt{2}$ — the focusing magnetic force compensates for the defocusing force of space charge — eEr .

Thus, owing to the fact that the compensation for the action of the Coulomb repulsive force by the magnetic field takes place only on the average per period, it is in principle impossible to obtain a smooth boundary of the beam (parallel beam) when using periodic magnetic focusing. Therefore, the concept of an "equilibrium" radius of a beam in a periodic field loses its sense and the concept of "average radius r_{av} " is used for estimation of the beam.

Conventionally, the variation of the beam radius may be considered as superposition of the ripple determined, as in the case of a uniform field, by the beam parameters (including the initial conditions) and the root-mean-square value of the magnetic induction and waviness determined by the periodic change in the magnetic field. The amplitude and period of the ripples (pulsations) depend on the relation between the beam parameters and the magnetic field. The period of waviness is half the period of the magnetic field variation, which physically follows from the above stated arguments and formally follows from the presence of the terms with $\cos 2Z$ in equation (2.240).

The pulsations can be reduced to minimum by means of properly selecting the relationships between the beam parameters, the root-mean-square value of the magnetic induction, and the initial conditions, as in the case of a uniform field. From the analysis of equation (2.244) it follows that the minimum pulsations correspond to the

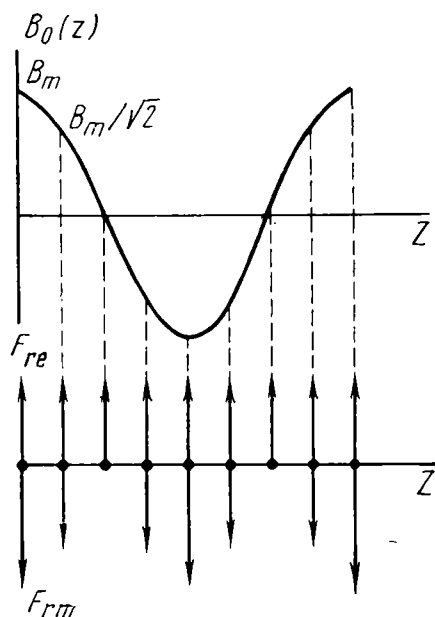


Fig. 2.45. Balance of forces in magnetic periodic focusing system

condition

$$1 - k^2 - \frac{\beta}{\alpha} = 0 \quad (2.245)$$

Express the relation

$$\frac{\beta}{\alpha} r_{av}^2 = \frac{\sqrt{2}I}{\pi\epsilon_0 \left(\frac{e}{m}\right)^{3/2} \left(\frac{B_m}{\sqrt{2}}\right)^2 \sqrt{U_0}} \quad (2.246)$$

from (2.237) and (2.238).

Denote the relation (2.246) as κ_0^2

$$\kappa_0^2 = \frac{\beta}{\alpha} r_{av}^2 \quad (2.247)$$

Substitution of (2.247) with account for (2.239) into (2.245) results in the equation

$$r_{av}^4 - \kappa_0^2 r_{av}^2 \left(\frac{B_c}{B_m/\sqrt{2}}\right)^2 r_c^4 = 0 \quad (2.248)$$

coinciding with (2.159) when $B_m/\sqrt{2} = B_0$. The solution of equation (2.248) is expressed as follows [compare with (2.160)]

$$r_{av}^2 = \frac{1}{2} + \frac{1}{2} \sqrt{1 + 4 \left(\frac{B_c}{B_m/\sqrt{2}}\right)^2 \left(\frac{r_c}{\kappa_0}\right)^4 \kappa_0^2} \quad (2.249)$$

When the cathode is completely screened ($B_c = 0$)

$$r_{av}^2 = \kappa_0^2 = \frac{\sqrt{2}I}{\pi\epsilon_0 \left(\frac{e}{m}\right)^{3/2} \sqrt{U_0} \left(\frac{B}{\sqrt{2}}\right)^2} \quad (2.250)$$

The expression for r_{av} coincides with the Brillouin radius (2.145) in a uniform magnetic field having a value of induction equal to $B_m/\sqrt{2}$. Therefore, a flow similar to the Brillouin flow can exist in a periodic magnetic field, but the role of the Brillouin field is played not by the amplitude but by the root-mean-square (effective) value of the magnetic induction. Of course, in this case, too, the waviness of the beam boundary determined by the periodic variation of the field remains and the average value of the beam radius per period has the sense of a Brillouin flow.

Assume now that condition (2.245) is satisfied. Then equation (2.244) is written as follows

$$\frac{d^2\delta}{dZ^2} + 2\alpha(1+k^2)\delta + \alpha\cos 2Z = -\alpha\delta\cos 2Z \quad (2.251)$$

The solution of this equation can be found by the approximation method. Let us use the solution of "shortened" (without the right-

hand side) equation (2.251) which has the form

$$\delta = C_1 \cos \omega Z + C_2 \sin \omega Z + \frac{\alpha \cos 2Z}{4 - \omega^2} \quad (2.252)$$

where $\omega^2 = 2\alpha(1 + k^2)$.

Assume that $C_1 = C_2 = 0$ and present $\cos 2Z$ in the form of $\frac{1}{2}(1 + \cos 4Z)$ and substitute δ from (2.252) into the right-hand side of equation (2.251). Then the second approximation is expressed as a solution of the equation obtained

$$\frac{d^2\delta}{dZ^2} + \omega^2\delta + \alpha \cos 2Z = -\frac{1}{2} \times \frac{\alpha^2(1 + \cos 4Z)}{4 - \omega^2} \quad (2.253)$$

Write the general solution of this equation

$$\delta(Z) = C_1 \cos \omega Z + C_2 \sin \omega Z + \frac{\alpha}{4 - \omega^2} \left[\cos 2Z - \frac{\alpha}{2\omega^2} + \frac{\alpha \cos 4Z}{2(16 - \omega^2)} \right] \quad (2.254)$$

The subsequent approximations can be found in a similar way. It is clear that they contain decreasing terms with $\cos 6Z$, $\cos 8Z$, etc. Since in many practical cases $\alpha \ll 1$, we may restrict ourselves to second approximation (2.254). In order to determine the coefficients C_1 and C_2 , let us introduce the initial conditions: the initial deviation of the beam radius from the average value $\delta(Z)|_{z=0} = \delta_0$ and the initial inclination of the outer electron trajectory $\left. \frac{d\delta}{dz} \right|_{z=0} = \delta'_0$.

Then

$$\delta_0 = C_1 + \xi, \quad \delta'_0 = C_2\omega \quad (2.255)$$

where

$$\xi = \frac{\alpha}{4 - \omega^2} \left[1 - \frac{\alpha}{2\omega^2} + \frac{\alpha}{2(16 - \omega^2)} \right] \quad (2.256)$$

Substituting (2.255) into (2.254) with account for (2.236) and (2.242), we obtain the equation of motion of the outer electron of the beam in a periodic magnetic field

$$\begin{aligned} r(z) = r_{av} + r_{av} \left[(\delta_0 - \xi) \cos \frac{2\pi\omega}{L} z + \right. \\ \left. + \frac{\delta'_0}{\omega} \sin \frac{2\pi\omega}{L} z + \frac{\alpha}{4 - \omega^2} \left(\cos \frac{4\pi}{L} z - \frac{\alpha}{2\omega^2} + \frac{\alpha}{2(16 - \omega^2)} \cos \frac{8\pi}{L} z \right) \right] \end{aligned} \quad (2.257)$$

In the given equation the terms with $\sin \frac{2\pi\omega}{L} z$ and $\cos \frac{2\pi\omega}{L} z$ determine the ripple of the beam, the terms with $\cos \frac{4\pi}{L} z$ and $\cos \frac{8\pi}{L} z$, the waviness. This equation adequately describes the contour of the beam in a periodic magnetic field under the following conditions:

α and β are less than unity and the deviation of the initial radius of the beam from the average value (small δ_0) and the initial inclination of the trajectories of the outer electrons (small δ'_0) are small. If these conditions are not satisfied, the beam contour can be found with the aid of electronic computers. The results of these computations show that the nature of variation of the beam contour remains the same at higher values of α , β and δ_0 as well, i.e. the pulsation

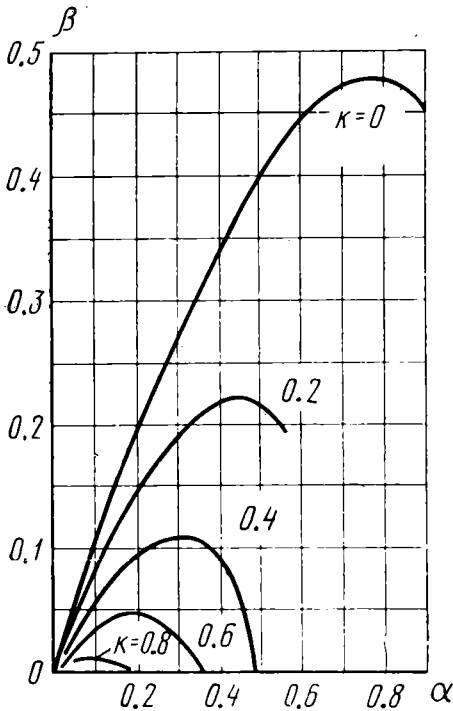


Fig. 2.46. Optimum ratio of parameters α and β

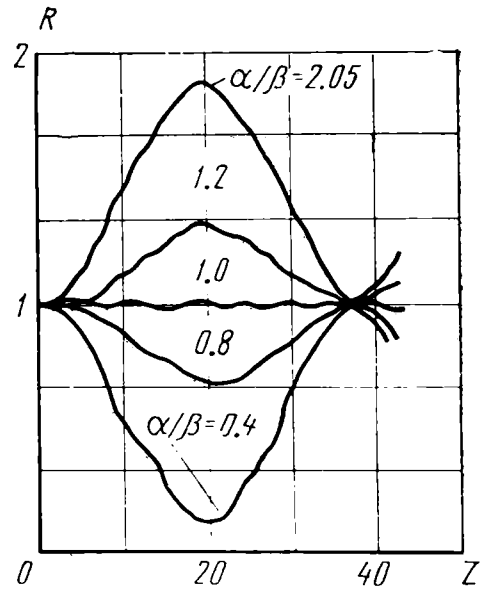


Fig. 2.47. Beam contour versus β/α ratio

and the waviness are preserved but the amplitude of the ripple (pulsations) and waviness remain as such, but their amplitude may rise to an intolerable value.

It has been shown that the minimal pulsations are obtained when condition (2.245) is satisfied. With a completely screened cathode ($k = 0$) this condition results in an equality $\beta = \alpha$. Equating α and β , one can determine the optimum value of the magnetic induction providing for a beam with minimum pulsations.

From (2.237) and (2.238) we have

$$\left(\frac{B_m}{\sqrt{2}}\right)^2 = E_{eff}^2 = \frac{\sqrt{2}}{\pi\epsilon_0 \left(\frac{e}{m}\right)^{3/2}} \times \frac{I}{U_0^{1/2} r_{av}^2} = 69 \frac{I}{U_0^{1/2} r_{av}^2} \text{ [Gs, A, V, m]} \quad (2.258)$$

The calculations made by means of electronic computers show that formula (2.258) holds for the values of α and β not exceeding 0.2 or about quantity. In the case of higher values of α and β the minimum pulsations occur when $\alpha > \beta$ (Fig. 2.46). Figure 2.46 also gives the graphs of optimal ratio between the parameters α and β for $k \neq 0$, i.e. for the partially screened cathode of the gun.

In order to obtain minimum pulsations, besides the above-said optimal ratio of α to β , it is necessary to adequately satisfy the initial conditions of introducing the beam into the periodic magnetic system. It is of interest to note that in case of a failure to obtain an optimal ratio α/β , the character of the pulsation depends on the value of this ratio. With $\beta/\alpha < 1$ the beam pulsations are directed inwards, with $\beta/\alpha > 1$ they are directed outwards (Fig. 2.47).

Consequently, when it is necessary to run the beam through a channel of a specified cross section, it is best to have $\alpha > \beta$, since in this case the beam contour will not extend outside r_{av} (pulsations are directed inward the beam).

A periodically varying magnetic field can be also used for focusing a strip beam. The analysis similar to that given for the axisymmetric flow shows that a stable flow with a wavy boundary can be obtained for a strip beam with certain ratios between the beam parameters and the magnetic field.

2.7. Principle of Synthesis of Systems for Forming Intensive Beams

The solution of the problem of shaping (focusing) intensive electron beams, as well as the solution of the problem of geometrical electron optics without taking into account the effect of space charge of the beam itself can be found in two ways.

(1) The electron trajectories and the configurations of the beam are found analytically or by means of simulating on the basis of specified and calculated distributions of electrostatic and magnetic fields produced by systems of electrodes and magnetic circuits that are external with respect to the beam. In that when focusing intensive beams, one has to take into account the effect of space charge of the beam itself.

(2) The specified configuration of the beam, i.e. the boundary trajectories of the electrons can be used for determining such a distribution of electrostatic and magnetic fields which ensures an electron flow of a definite shape. The general theory of focusing electron beams developed by G. Grinberg (see Sec. 1.11) makes it possible to solve both the direct problems of geometrical electron optics (finding electrostatic and magnetic fields by the shape of electron trajectories) and the reverse problems (finding electron trajectories by the specified fields). However, the Grinberg theory is inapplicable

for focusing intensive beams, since it does not take into account the effect of space charge of the beam itself, i.e. it is assumed that the potential distribution along the path of the electron flow is described by the Laplace equation.

In the case of focusing intensive beams the potential distribution outside and inside the beam cannot be described by a single equation. Inside the beam the potential distribution is described by the Poisson equation and outside it, by the Laplace equation. Therefore, when analyzing the focusing of intensive beams, two problems have to be solved, i.e. the internal and external problems (see Sec. 2.3).

When considering the shaping of intensive beams, the solution of the direct problems is of far greater interest, since the calculations of the forming systems are usually based on the data on the beam boundary, i.e. given are the boundary trajectories of electrons determining the beam configuration. In this case the problem comes to finding such electrostatic and magnetic fields that ensure the existence of the given beam. Generally the finding of the electrostatic and magnetic fields forming a beam of a specified shape, as well as finding the configurations of the electrodes and magnetic circuits, the electrode potentials and magnetic induction value are called the *synthesis of systems forming intensive electron flows*.

An example of application of the method of synthesis is calculation or experimental finding of the shape of the electrodes used in the Pierce guns. Actually, using the Pierce method, a beam with linear boundaries is first cut out of an electron flow in a planar, cylindrical or spherical diode. This done, calculations or simulating technique are employed to find the shape of the anode and the focusing near-cathode electrodes providing for the boundary conditions similar to those existed before the removal of the electron flow surrounding the cut-out beam.

The general theory of synthesis of forming intensive electron beams has been lately developed by V. Ovcharov. In this theory the basic equations are written in the curvilinear orthogonal coordinate system, in which the position of any point of space is determined by three coordinates: q_1, q_2, q_3 . In this case one of the curved coordinate axes is so selected that it coincides with the specified electron trajectory or a coordinate system is constructed in such a manner that one of the surfaces $q_i = \text{const}$ coincides with the boundary surface of the beam. In the curvilinear orthogonal coordinate system the length of an elementary arc (in particular, a length of the electron trajectory) is expressed through the equation

$$ds^2 = h_1^2 dq_1^2 + h_2^2 dq_2^2 + h_3^2 dq_3^2 \quad (2.259)$$

where h_1, h_2, h_3 are the so-called metric Lamé coefficients dependent on the coordinates q .

The Lamé coefficients are associated with the Cartesian coordinates through the relations

$$h_i^2 = \left(\frac{\partial x}{\partial q_i} \right)^2 + \left(\frac{\partial y}{\partial q_i} \right)^2 + \left(\frac{\partial z}{\partial q_i} \right)^2 \quad (2.260)$$

where $i = 1, 2, 3$.

Now the equation of electron motion must be converted so as to obtain expressions which are independent of the selection of the coordinate system. Let us write the equation of motion of the electron in a vector form

$$\frac{d}{dt}(\mathbf{v}) = -\frac{e}{m} \mathbf{E} - \frac{e}{m} [\mathbf{v} \mathbf{B}] \quad (2.261)$$

Determine the derivative in the left-hand side of the equation by performing the differentiation in the Cartesian system of coordinates

$$\frac{d}{dt}(\mathbf{v}) = \frac{\partial \mathbf{v}}{\partial t} + v_x \frac{\partial \mathbf{v}}{\partial x} + v_y \frac{\partial \mathbf{v}}{\partial y} + v_z \frac{\partial \mathbf{v}}{\partial z} \quad (2.262)$$

where v_x, v_y, v_z are the velocity components along the coordinate axes OX, OY, OZ .

If one considers only the stationary process, then $\partial \mathbf{v} / \partial t = 0$. The sum of the three partial derivatives at the right-hand side of equation (2.262) is transformed by using the known vector analysis formula

$$v_x \frac{\partial \mathbf{v}}{\partial x} + v_y \frac{\partial \mathbf{v}}{\partial y} + v_z \frac{\partial \mathbf{v}}{\partial z} = \text{grad } \frac{v^2}{2} - [\mathbf{v} \text{ rot } \mathbf{v}] \quad (2.263)$$

If the initial velocity of the electrons is neglected, then, according to the law of conservation of energy we have

$$\frac{v^2}{2} = \frac{e}{m} U \quad (2.264)$$

$$\text{grad } \frac{v^2}{2} = \text{grad } \left(\frac{e}{m} U \right) = -\frac{e}{m} \mathbf{E} \quad (2.265)$$

and the left-hand side of equation (2.262) in the stationary case may be presented in the form

$$\frac{d}{dt}(\mathbf{v}) = -\frac{e}{m} \mathbf{E} - [\mathbf{v} \text{ rot } \mathbf{v}] \quad (2.266)$$

Equating the right-hand sides of equations (2.261) and (2.266), we obtain

$$\left[\mathbf{v} \left(\frac{e}{m} \mathbf{B} - \text{rot } \mathbf{v} \right) \right] = 0 \quad (2.267)$$

The relation (2.267) is the equation of electron motion in a vector form. It is written so that it does not depend on the selection of the coordinate system. It should be noted that equation (2.267) does not

contain the value of the electric field strength in an apparent form and this is convenient for further transformations.

The further transformations come to finding v as ds/dt and $\text{rot } \mathbf{v}$ by the formulas

$$\begin{aligned}(\text{rot } \mathbf{v})_{q_1} &= \frac{1}{h_2 h_3} \left[\frac{\partial (v_3 h_3)}{\partial q_2} - \frac{\partial (v_2 h_2)}{\partial q_3} \right] \\(\text{rot } \mathbf{v})_{q_2} &= \frac{1}{h_1 h_3} \left[\frac{\partial (v_1 h_1)}{\partial q_3} - \frac{\partial (v_3 h_3)}{\partial q_1} \right] \\(\text{rot } \mathbf{v})_{q_3} &= \frac{1}{h_1 h_2} \left[\frac{\partial (v_2 h_2)}{\partial q_1} - \frac{\partial (v_1 h_1)}{\partial q_2} \right]\end{aligned}\quad (2.268)$$

where v_1, v_2, v_3 are the velocity components in the direction of the tangents to the curved coordinate axes; h_1, h_2, h_3 are the Lamé coefficients calculated by formula (2.260).

The system of curved coordinates, as noted above, should be so selected that the equations for the given configuration of the beam have the simplest form. For example, in the case of axisymmetric beams the coordinate q_1 is preferably considered as a longitudinal coordinate of the beam, i.e. the length of the beam (in this case curvilinear) is measured along the axis OQ_1 ; the coordinate surface $q_2 = \text{const}$ should be matched with the boundary surface of the beam. For example, assume that on the beam surface $q_2 = 1$. In this case $q_2 < 1$ inside the beam, $q_2 = 0$ on the axis. The angle of turn ψ is preferably taken as the third coordinate q_3 . With such selection of coordinates all derivatives by q_3 go to zero because of the axial symmetry. Furthermore, since the electrons cannot pass beyond the beam boundary which is specified, the velocity component is $v_2 = \partial q_2 / \partial t = 0$.

Now assume that in the cylindrical coordinate system the contour (boundary surface) is specified by the equation

$$r = \Phi(z) \quad (2.269)$$

In this case the coordinate surfaces in the curvilinear orthogonal coordinate system are described by the equations

$$\left. \begin{aligned} r &= q_2 \Phi(z) \\ r^2 + 2 \int_{q_1}^z \frac{\Phi(z)}{\Phi'(z)} dz &= 0 \end{aligned} \right\} \quad (2.270)$$

In the general case the system of equations (2.270) cannot be solved, i.e. it is impossible to find $q_1 = q_1(z, r)$ and $q_2 = q_2(z, r)$ in their explicit form. Therefore, it is also impossible to calculate the Lamé coefficients necessary for conversion of equation (2.267). However, if we restrict ourselves to paraxial approximation, i.e. when we limit the class of the beams in question, the problem can be solved in many practical cases.

To carry out further transformations, it is necessary to use dimensionless (normalized) variables

$$x = \frac{q_1}{l}, \quad \varphi = \frac{\Phi(z)}{\Phi_0}, \quad u = \frac{U(z)}{U_0}, \quad \mu = \frac{\Phi_0}{l} \quad (2.271)$$

where l is the length of the beam; Φ_0 is the maximum radius of the beam; U_0 is the scale potential having the meaning of a potential along the axis of an equilibrium beam with the radius Φ_0 in a uniform magnetic field.

Taking into account the new variables, the condition of paraxiality takes the simple form

$$\mu \ll 1 \quad (2.272)$$

The condition of paraxiality in the curvilinear coordinates does not mean that during the conversion of the beam contour into a cylindrical system of coordinates the boundary trajectories will for sure form small angles with the axis. Sometimes a clearly nonparaxial beam (in a cylindrical system of coordinates) can be regarded as paraxial with good approximation by means of suitable selection of a curvilinear system.

In the paraxial approximation

$$\left. \begin{aligned} \frac{r}{l} &\approx \mu q_2 \varphi \left[1 - \mu^2 q_2^2 \frac{(\varphi')^2}{2} \right] \\ \frac{z}{l} &\approx x - \mu^2 q_2^2 \frac{\varphi \varphi'}{2} \end{aligned} \right\} \quad (2.273)$$

and the Lamé coefficients are

$$\left. \begin{aligned} h_1 &= 1 - \mu^2 q_2^2 \left(\frac{\varphi \varphi'}{2} \right) \\ h_2 &\approx l \mu \varphi [1 - \mu^2 q_2^2 (\varphi')^2] \\ h_3 &= r \end{aligned} \right\} \quad (2.274)$$

Let us transform equation (2.267) with due account for the Poisson equation in paraxial approximation

$$(\varphi^2 u')' + 4u\varphi\varphi'' + iF = \frac{i}{\sqrt{u}} \quad (2.275)$$

(the primes denote the differentiation with respect to x).

The equation obtained, which is similar to an ordinary equation of paraxial optics, describes the electron trajectories in intensive beams at any density of space charge, since the value $U = uU_0$ is the axial potential determined not only by the shape and potential of the external electrodes but also by the space charge of the beam itself.

In expression (2.275) $i = I/I_0$ is the normalized beam current (I is the beam current, I_0 is the scale current which means the current

in an equilibrium beam with the radius Φ_0 in a uniform magnetic field); F is the normalized magnetic field

$$F = \frac{\varphi^4 b^2 - \varphi_c^4 b_c^2}{\varphi^2 (1 - \varphi_c^4 b_c^2)} \quad (2.276)$$

where $b = B(z)/B_0$ is the dimensionless value of the magnetic induction on the beam axis in units B_0 ; B_0 has the sense of induction of a uniform magnetic field maintaining an equilibrium beam with radius Φ_0 , current I_0 and axis potential U_0 ; b_c is the value of b in the centre of the cathode (the point of intersection of the cathode surface with the beam axis).

In order to find the distribution of the magnetic field and the potential inside the beam by the given trajectories (the solution of the direct internal problem), or in order to find the electron trajectories in an explicit form by the given distributions of the electrostatic and magnetic fields (the solution of the inverse internal problem), it is necessary to solve equation (2.275) with the boundary conditions

$$u|_{x=x_c} = 0 \text{ and } u'|_{x=x_c} = 0 \quad (2.277)$$

where x_c is the normalized coordinate of the centre of the cathode surface.

However, under such boundary conditions the right-hand side of equation (2.275) turns into infinity and in this case integration is impossible. To overcome this obstacle, equation (2.275) is transformed into a system of two first order differential equations by using an auxiliary function

$$Y = (\varphi^2 u')^2 - 4i\varphi^2 \sqrt{u} \quad (2.278)$$

The transformation results in a set of equations

$$\left. \begin{aligned} u' &= \pm \frac{\sqrt{Y + 4i\varphi^2 \sqrt{u}}}{\varphi^2} \\ Y' &= 8i\varphi\varphi' \sqrt{u} - (8\varphi^3\varphi''u + 2i\varphi^2 F) u' \end{aligned} \right\} \quad (2.279)$$

with boundary conditions

$$u|_{x=x_c} = 0 \text{ and } Y|_{x=x_c} = 0 \quad (2.280)$$

The system of equations (2.279) can be solved by the method of numerical integration. Thus, in a paraxial approximation the internal problem (both direct and inverse) can be solved for the whole space occupied by the beam — from the cathode to the collector of electrons.

Under the above-said selection of a coordinate system and the absence of intersecting trajectories, i.e. assuming that the beam is laminar, any trajectory lies on the surface $q_2 = \text{const}$. In this case

the potential distribution $U(x, q_2)$ and the space charge density $\rho(x, q_2)$ inside the beam are described by the equations

$$U = u + \mu^2 q_2^2 \left(u \varphi \varphi'' + \frac{1}{4} i F \right)$$

$$\rho = - \frac{1}{\varphi^2 \sqrt{u}} \quad (2.281)$$

Here ρ is the normalized density of the space charge in units of ρ_0

$$\rho_0 = \frac{j_0}{w_0} \quad (2.282)$$

$$j_0 = \frac{I}{\pi \Phi_0^2} \quad (2.283)$$

$$w_0 = \sqrt{\frac{2e}{m} U_0} \quad (2.284)$$

The values φ , u and F in (2.281) are connected by equation (2.275).

Now it is necessary to solve the external problem, i.e. to find the potential distribution outside the beam. In the curvilinear orthogonal coordinate system the Laplacian of the potential $U = U(q_1, q_2)$ has the form

$$\nabla^2 U = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial q_1} \left(\frac{h_2 h_3}{h_1} \times \frac{\partial U}{\partial q_1} \right) + \frac{\partial}{\partial q_2} \left(\frac{h_1 h_3}{h_2} \times \frac{\partial U}{\partial q_2} \right) \right] \quad (2.285)$$

where h_1 , h_2 , h_3 are the Lamé coefficients calculated by formulas (2.274).

Substituting the Lamé coefficients into (2.285) and making the Laplacian equal to zero and $-i\varphi^2\rho$, we obtain the Laplace and Poisson equations describing the potential distribution outside and inside the beam (in the coordinates x, q_2)

$$\frac{\partial}{\partial x} \left(\varphi^2 \frac{\partial U}{\partial x} \right) + \frac{1}{\mu^2 q_2} \times \frac{\partial}{\partial q_2} \left(q_2 \frac{\partial U}{\partial q_2} \right) = 0 \quad (2.286)$$

$$\frac{\partial}{\partial x} \left(\varphi^2 \frac{\partial U}{\partial x} \right) + \frac{1}{\mu^2 q_2} \times \frac{\partial}{\partial q_2} \left(q_2 \frac{\partial U}{\partial q_2} \right) = -i\varphi^2 \rho \quad (2.287)$$

Substituting the expression of ρ from (2.282) into (2.287) and proceeding from I and U_0 to the dimensionless coordinates, we obtain the Poisson equation in the form

$$\frac{\partial}{\partial x} \left(\varphi^2 \frac{\partial U}{\partial x} \right) + \frac{1}{\mu^2 q_2} \times \frac{\partial}{\partial q_2} \left(q_2 \frac{\partial U}{\partial q_2} \right) = \frac{i}{\sqrt{u}} \quad (2.288)$$

At the beam boundary the potential and its first derivative along the normal to the boundary remain continuous, while the other derivative of the potential along the normal to the boundary is broken when crossing the beam boundary. The value of the jump

of the latter derivative, when passing the boundary, is obtained as the difference between equations (2.286) and (2.288)

$$\left(\frac{\partial^2 U}{\partial q_2^2}\right)^+ - \left(\frac{\partial^2 U}{\partial q_2^2}\right)^- = -\frac{\mu^2 i}{\sqrt{u}} \quad (2.289)$$

where the upper index ($\pm$) indicates the value of the second derivative at the external (outside the beam) and internal (inside the beam) sides of the boundary surface of the beam.

Since the shape of the electrodes of the focusing system in the vicinity to the beam boundary is very important when forming intensive beams (see Sec. 4.2), the potential distribution outside the beam can be sought as an expansion by the powers of low difference ($q_2 = \kappa$) restricting ourselves to the terms of the series including the second power

$$U = U_\kappa + (q_2 - \kappa) U'_\kappa + \frac{(q_2 - \kappa)^2}{2} U''_\kappa \quad (2.290)$$

where κ is the value of q_2 at the beam boundary; U_κ , U'_κ , U''_κ are respectively the values of the potential, its first and second derivatives at the external side of the boundary surface of the beam.

Using (2.281) and (2.289), find the values U_κ , U'_κ , and U''_κ .

$$\left. \begin{aligned} U_\kappa &= u + \mu^2 \kappa^2 \left(u \varphi \varphi'' + \frac{1}{4} i F \right) \\ U'_\kappa &= 2\mu^2 \kappa \left(u \varphi \varphi'' + \frac{1}{4} i F \right) \\ U''_\kappa &= 2\mu^2 \left(u \varphi \varphi'' + \frac{1}{4} i F \right) - \frac{\mu^2 i}{\sqrt{u}} \end{aligned} \right\} \quad (2.291)$$

Substitution of (2.291) into (2.290) results in the expression as follows

$$U = u + \mu^2 q_2^2 \left(u \varphi \varphi'' + \frac{1}{4} i F \right) - \frac{\mu^2 i}{\sqrt{u}} (q_2 - \kappa)^2 \quad (2.292)$$

In this expression the first two terms at the right-hand side describe the potential distribution inside the beam [compare with (2.281)], the third term takes account for the jump of the second derivative when passing the beam boundary. Thus, equation (2.292) describes the "analytical continuation" of the potential distribution inside the beam beyond its boundary with account for the jump of the second derivative when passing the beam boundary.

Equation (2.292) satisfies the boundary conditions. To enable this equation to describe the potential distribution outside the beam (even near its external boundary), the following conditions must be satisfied: the Laplacian of the potential calculated by (2.292) must be turned into zero, i.e. the Laplace equation must be satisfied

The Laplacian of expression (2.292) is written as

$$\begin{aligned} \nabla^2 U = & \frac{1}{\varphi^2 l^2} \left[\frac{\partial}{\partial x} \left(\varphi^2 \frac{\partial U}{\partial x} \right) + \frac{1}{\mu^2 q_2} \times \frac{\partial}{\partial q_2} \left(q_2 \frac{\partial U}{\partial q_2} \right) \right] = \\ = & \frac{1}{\varphi^2 l^2} \left\langle \left[(\varphi^2 u')' + 4u\varphi\varphi'' + iF - \frac{i}{\sqrt{u}} \right] + \right. \\ & + \mu^2 \left\{ q_2^2 \left[\varphi^2 \left(u\varphi\varphi'' + \frac{i}{4} F \right)' \right]' - \right. \\ & \left. \left. - \frac{i(q_2 - \kappa)^2}{q_2} \left[\varphi^2 \left(\frac{1}{\sqrt{u}} \right)' \right]' \right\} - \frac{q_2 - \kappa}{q_2} \times \frac{i}{\sqrt{u}} \right\rangle \quad (2.293) \end{aligned}$$

In this equation the first term in square brackets is equal to zero [see (2.275)]. The term in the braces has an order of μ^2 , i.e. in a paraxial approximation it is a small second-order value and may be neglected. In this case we have

$$\nabla^2 U \approx -\frac{1}{\varphi^2 l^2} \times \frac{q_2 - \kappa}{q_2} \times \frac{i}{\sqrt{u}} \neq 0 \quad (2.294)$$

i.e. equation (2.292) generally does not satisfy the Laplace equation. However, the nearer the considered region to the beam boundary (lower the difference $q_2 - \kappa$), the better approximation is provided by equation (2.290) for description of the potential outside the beam.

Further transformations are conveniently effected by using an auxiliary function $S(x, q_2)$ determining it so that the expression

$$U = u + \mu^2 q_2^2 \left(u\varphi\varphi'' + \frac{i}{4} iF \right) - \frac{\mu^2 i}{2 \sqrt{u}} (q_2 - \kappa)^2 + S \quad (2.295)$$

satisfies the boundary conditions and the Laplace equation with an accuracy to the second-order terms of smallness (μ^2 order). To satisfy the boundary conditions, i.e. the continuity of U and U' and the jump U'' in accordance with (2.289), it is necessary to meet the following requirements

$$S|_{q_2=\kappa} = 0, \quad \frac{\partial S}{\partial q_2} \Big|_{q_2=\kappa} = 0 \quad (2.296)$$

Furthermore, since the first three terms of the right-hand side of equation (2.295) satisfy the boundary conditions, it is necessary that the Laplacian expression include no U term which is not proportional to μ^2 . This requirement will be met if the function S satisfies the equation

$$\frac{1}{\mu^2 q_2} \times \frac{\partial}{\partial q_2} \left(q_2 \frac{\partial S}{\partial q_2} \right) = \frac{q_2 - \kappa}{q_2} \times \frac{i}{\sqrt{u}} \quad (2.297)$$

The solution of equation (2.297) with boundary conditions (2.296) has the form

$$S = \frac{\mu^2 i}{4 \sqrt{u}} \left[(q_2 - \kappa) (q_2 - 3\kappa) + \kappa^2 \ln \frac{q_2}{\kappa^2} \right] \quad (2.298)$$

The final potential distribution of the beam satisfying the Laplace equation (to the accuracy of μ^2 order smallness terms) and the boundary conditions is found by substituting (2.298) into (2.295)

$$U = u + \mu^2 q_2^2 \left(u \varphi \varphi'' + \frac{1}{4} i F \right) + \frac{\mu^2 i}{4 \sqrt{u}} \left(\kappa^2 - q_2^2 + \kappa^2 \ln \frac{q_2^2}{\kappa^2} \right) \quad (2.299)$$

for $0 < q_2 \leq \kappa$

$$U = u + \mu^2 q_2^2 \left(u \varphi \varphi'' + \frac{1}{4} i F \right) + \frac{\mu^2 i}{4 \sqrt{u}} (1 - q_2^2 + \ln q_2^2) \quad (2.300)$$

for $q_2 \geq 1$.

Similar methods can be employed for solving the external problem for a hollow axisymmetric (tubular) beam and a plane-symmetric (strip) beam.

Thus, we have equations for potential distribution both inside and outside the beam, i.e. have the solutions for the internal and external problems (in paraxial approximation) of the theory of forming intensive beams. Of course, the practical application of the general equation is associated with some difficulties but these are of calculative rather than principal character. They can be overcome by means of additional transformations, use of new variables, etc. Yet, the general theory of synthesis of systems for forming intensive beams is a great achievement in the development of the forming systems.

Devices for Collimating Intensive Beams

3.1. General Parameters and Classification of Electron Guns

An intensive electron beam of a given configuration (cylindrical, conical, tubular, strip-type, etc.) with sufficiently large value of space charge coefficient (perveance) is formed by electron-optical systems commonly referred to as electron guns.

Since an intensive beam cannot be converged (focused) into a point as stated in Section 2.1, the requirement for a minimum diameter beam, as applied to the intensive flows, is not predominant. The beam at the output of the gun must have a limited (not necessarily small) cross section. The intensive beam produced by the gun and travelling through a space free of electrostatic and magnetic fields expands due to the action of space charge (see Section 2.1). The beam can be confined by means of electrostatic or magnetic fields, i.e. the gun must be combined with a focusing system limiting the beam expansion. In many electron-beam devices the limiting field acts in the gun either, for example, in the case of a cathode that is not shielded from the magnetic field. On the contrary, in magnetron injection guns the magnetic field collimating the flow in the gun also serves as a limiting factor. It is clear that in these cases the division of a beam collimating system into a gun and a limiting device is purely arbitrary.

In addition to the basic requirements concerning the generation of a beam having a specified shape and a high perveance, the guns collimating intensive beams must meet some general and specific requirements governed by the particular characteristics of the electron device for which the gun has been designed. In general, the best current path must be provided, i.e. the number of electrons entrapped by the gun electrodes should be minimized. In other words, the beam current produced by the gun must approximate the cathode current. It is also desirable that the gun flow be as close to the ideal laminar flow of electrons as practicable. Since the intensive beams are collimated by using very high accelerating voltages (tens of kilovolts), the dielectric strength of the system must be taken into account in addition to the principle requirement for a desired focusing field when selecting the configuration and position of the gun electrodes. In many guns used in practice the electron flow is com-

pressed in order to reduce the cathode current density (the current load on the cathode) at a given beam current, because, first, the specific emissivity of a real cathode is limited and, second, the lower the current load the longer is the service life of a given cathode. Hence, one of the requirements for an electron gun is adequate compression of the electron flow. Finally, in some cases the gun must provide for possibility to control the beam current.

Modern electron guns collimating intensive electron flows can be classified in terms of the focusing method, i.e. be subdivided into systems with electrostatic, magnetic and combined focusing systems (with superposed electrostatic and magnetic fields). It should be remembered that it is impossible to develop a purely magnetic gun (without electrostatic field), since the magnetic field cannot control the electron energy, i.e. accelerate the electrons (see Section 1.2). However, if an electrostatic field is used only for properly accelerating the beam electrons and the beam is formed by a magnetic field, such guns may be referred to as guns with a magnetic focusing system.

The guns can be also classified in terms of the character of the beam produced, i.e. one may consider the guns as collimating axisymmetric beams (cylindrical, conical, tubular or strip beams) and parallel or converging. Other systems of classification may of course be proposed, for example, in terms of perveance, presence (or absence) of compression, possibility to control the beam current, application, etc. In this book the gun classification is by the beam shape, although such division of guns is somewhat arbitrary, since a compressed electron beam is converging (conical or wedge-shaped) within the gun and often becomes cylindrical or planar (strip) outside the gun.

It should be noted that a great success has been presently achieved with respect to one of the basic requirements—high perveance. If in the forties the microperveance in commercial microwaves electron devices did not exceed tenths of one $\mu\text{A}/\text{V}^{3/2}$, then in the recent years guns have been developed that produce electron beams having $p > 30\mu\text{A}/\text{V}^{3/2}$. The modern guns for collimating intensive beams are usually single-potential systems, i.e. have no electrodes with potentials other than zero (cathode potential) and other than the anode potential. The systems with a control (modulating) electrode whose potential is varied to modulate the field in the near-cathode region are employed only in those cases when the beam current must be controlled during the operation of the device.

Generally an electron gun collimating an intensive beam consists of a cathode (source of free electrons), an anode (accelerating electrode) and one or several focusing (collimating) electrodes at the cathode potential (zero) or the anode potential. An integral part of the guns with magnetic focusing is magnetic coils (solenoids) or permanent

magnets producing a focusing magnetic field. These components are provided with magnetic circuits.

Since the configuration of the focusing electrostatic fields is determined by the potentials, shape and location of the electrodes and that of magnetic fields by the magnetizing ability, shape and arrangement of the electromagnets and ferromagnetic screens, determination of the shape and mutual arrangement of the electrodes and magnetic elements is the fundamental problem of the gun design. In particular, the configuration of the cathode and that of the focusing electrodes are of importance in collimating an electron flow.

3.2. Guns for Collimating Axisymmetric Beams

Many electron devices with the use of intensive axisymmetric beams are based on the so-called Pierce guns (see Section 2.4). Widespread use of these guns is due to a possibility of producing beams with rather high perveance (a few $\mu\text{A}/\text{V}^{3/2}$) by means of the Pierce systems and due to their comparatively simple construction. More than that, the shape of the electrodes of the Pierce gun can be calculated accurately. Figure 3.1 shows Pierce guns for collimating a cylindrical (parallel) and a conical (converging) beams of the axisymmetric type.

The electron system of the gun consists of a cathode, a near-cathode focusing electrode at the cathode potential, and an anode positive with respect to the cathode.

As discussed in Section 2.4, a parallel beam is produced when the cathode is planar, while the near-cathode electrode is shaped as a truncated cone with a generatrix inclined at an angle of 67.5° to the perpendicular drawn to the cathode edge. The anode may be either a flat disc with an aperture or be convex towards the cathode in correspondence with the

shape of one of the equipotential surfaces remote from the cathode (Fig. 2.19). In order to form a converging axisymmetric beam, the cathode and anode must be parts of concentric spheres, while the near-cathode focusing electrode must be cup-shaped (Fig. 2.23).

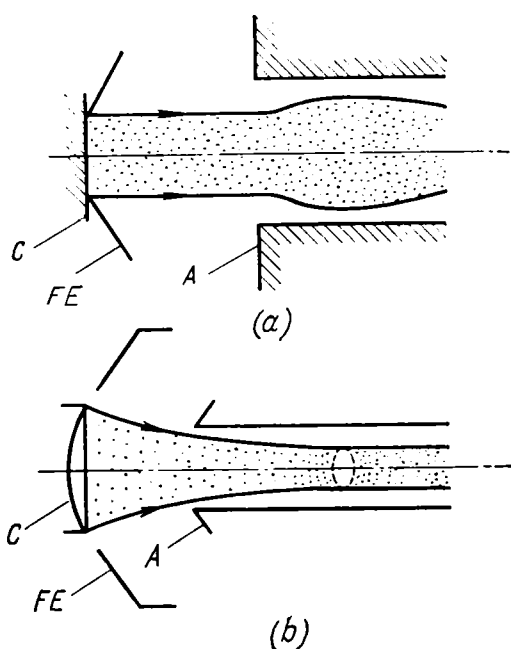


Fig. 3.1. Schematic of Pierce guns for collimating cylindrical (a) and conical (b) beams

As shown in the figures, the equipotential surfaces have rather a complex shape and accurately shaping the electrodes in compliance with the design is an engineering challenge. At the same time, the potential distribution in the immediate vicinity to the beam boundary is a decisive factor in its collimating. Investigations have shown that changes in the electrodes beyond the electron flow have a little effect on the potential distribution along its boundary. In practice it is sufficient to maintain a required (design) potential distribution at a distance of one to one and a half diameter of the beam from its boundary. Furthermore, it should be taken into account that the presence of an anode aperture results in a diverging lens in the region

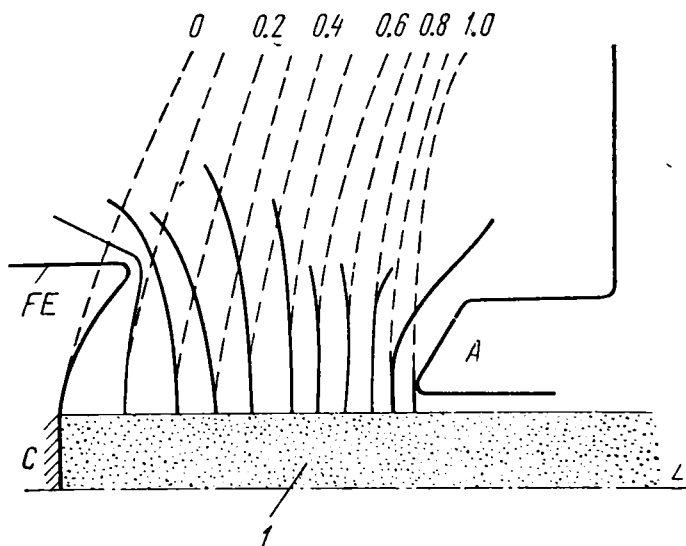


Fig. 3.2. Simplified shape of electrodes of Pierce gun
1—beam boundary

of the anode electrode (Sec. 2.4). To compensate for the diverging action of the anode lens, one has either to change the shape of the anode or (more often) to place the gun in a longitudinal magnetic field. In the presence of a confining magnetic field the shape of the anode electrode practically has no effect on the beam configuration.

The above considerations show that, when designing the guns, it is quite possible to select a simplified shape of electrodes providing the required potential distribution only near the beam boundary. Far outside the beam boundary the electrode shape is selected depending on the construction features: simple making, convenient mounting, etc. A simplified shape of electrodes can readily be selected by simulating in an electrolytic tank. Figure 3.2 shows an exemplary cross section of the electrode system providing the Pierce potential distribution.

As shown in the drawing, the shape of the electrodes is far from the theoretical (ideal) one.

From the analytical calculations it follows that the zero equipotential surface must approach the beam boundary near the cathode surface at an angle of 67.5° . In practice this condition can be met only when the cathode and near-cathode focusing electrode are made as an integral member—truncated cone whose smaller opening is closed by the cathode. This solution, however, is unacceptable due to the following reasons: the focusing electrode having a metal contact with the incandescant cathode will function as a radiator removing the heat from the cathode peripheral zone so that the input power of the heater must be much increased to keep the operating temperature of the cathode at a level providing for the required emission

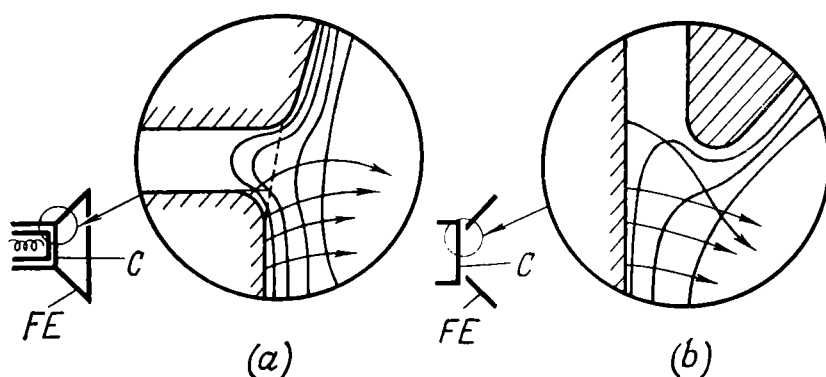


Fig. 3.3. Construction of near-cathode electrodes

current. What is more, the cathode operation is accompanied by migration of the active material (barium) from the oxide-coated cathode to the surface of the focusing electrode and this results in an electron-emission stray current from the surface of the heated electrode located near the cathode. The stray emission current can break the potential distribution in the near-cathode region thus changing the conditions of forming the beam. Consequently, in real guns a small angular gap is to be provided between the edges of the cathode and focusing electrode. Two constructions are possible. When the cathode is sufficiently large, the hole in the near-cathode electrode is made with a radius exceeding that of the cathode by the width of the gap (Fig. 3.3a). In the case of small cathodes, where the heater is mounted in a space with a diameter exceeding that of the emission part of the cathode, the focusing electrode is arranged in front of the cathode (Fig. 3.3b). In both cases the field near the gap is distorted and the equipotential surfaces "sag" into the gap. This "sagging" of the field results in distortion of the trajectories of the electrons emitted by the peripheral part of the cathode. Disturbance to the outer trajectories caused by the gap field is a very undesirable phenomenon, since it is the outer electrons that determine the configuration of the beam and cause settling of a portion of the electron

flow on the electrodes of the focusing system. The distortion of the field near the cathode depends not only on the width of the gap itself but also on the shape of the cathode edges and the focusing electrode. The technological rounding of the edges leads to an increase in the "sagging" of the field and the disturbance of a higher portion of the electrons. Calculations show that with a width of the gap of 0.1 mm and a bending radius of the cathode edge of the same order the portion of disturbed electrons can be as high as 10-15% of the total electron flow emitted by the cathode. Thus, when designing electron guns, it is necessary to reduce the width of the gap and to make the edges of the electrodes as sharp as possible. Some reduction of the portion of disturbed electrons can be obtained by applying to the focusing electrode a small correcting voltage which is negative with respect to the cathode. In this case a damping field is produced at the cathode edges, which prevents the emission of electrons from these edges. Of course, the total current of the beam is thus reduced, but the settling of the electrons on the positively charged electrodes of the shaping system can be reduced considerably by controlling the correcting voltage.

The Pierce gun with a cylindrical (parallel) flow can be used for collimating a beam with a radius approximately equal to the radius of the emission surface of the cathode. In this case the current density in the beam principally cannot exceed the specific emission of the cathode. Taking into account that the latter is limited, we arrive at a conclusion that such systems are expedient only for collimating comparatively low-current beams. The smaller the beam radius, the smaller a possible value of the current. Since up-to-date microwave electron devices employ beams with radii not exceeding a few millimetres and currents from fractions of an ampere to tens of amperes with moderate accelerating voltages ($p > 1 \mu\text{A}/\text{V}^{3/2}$), the current density in the beam is substantially higher than the maximum value of the specific emission of commercial cathodes. Therefore, widely used in radio electronics are guns with electron flow compression, i.e. those collimating converging electron beams. The value of the compression, i.e. the ratio of the emission area of the cathode to the cross-sectional area of the collimated beam may be equal to 100 and more.

Besides reducing the cathode current load and, therefore, producing a beam with a high current density at specified emissivity of the cathode which ensures a fairly long life of it, the compression guns have certain other advantages. As mentioned in Sections 2.5 and 2.6, the behaviour of the beam in the space behind the gun anode, where the beam is limited by a uniform or periodic field, is largely governed by the initial conditions of introducing the beam into the focusing system. The lower the initial radial accelerations, the lower the amplitude of the beam pulsations. To obtain a more or less smooth beam in the space behind the anode, the initial conditions must

be well satisfied. A diverging lens formed near the anode aperture results in significant radial accelerations of the outer electrons of the beam (outward from the beam axis, see Section 2.4). When using a gun forming a cylindrical beam, the magnetic induction of the focusing system must be increased to compensate for these accelerations, yet, the amplitudes of these pulsations will be still high. If the gun is used for collimating a converging beam, the action of the anode lens results in a decrease in the radial accelerations acquired in the gun field and directed towards the axis. Suitable selection of the anode electrode can help to obtain practically parallel beam

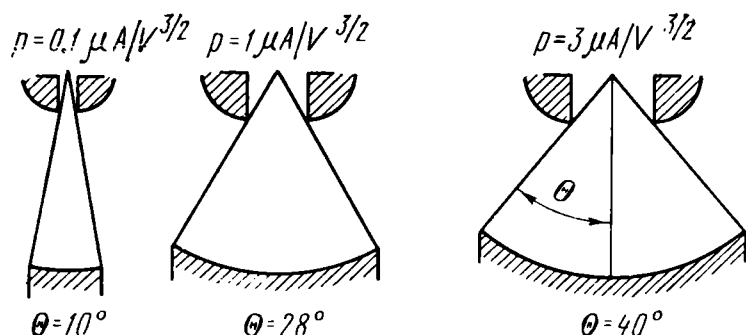


Fig. 3.4. Beam convergence angle versus microperveance

after leaving the anode aperture, i.e. to minimize the radial acceleration of the electrons and, consequently, the amplitude of the pulsations of the beam boundary in the space behind the anode. Finally, in the guns with a converging beam only a small central portion of the cathode surface is subjected to the bombardment by the positive ions formed near the anode aperture and this also reduces premature wear of the cathode.

The shape of the electrodes of the guns used for collimating conical (converging) beams is determined by the shape of the equipotential surfaces near the beam boundary (see Fig. 2.23). The maximum possible value of perveance of a converging beam, as follows from equation (2.94) is expressed by

$$P = 14.68 \times 10^{-6} \frac{1 - \cos \Theta}{(-\alpha)^2} [A/V^{3/2}] \quad (3.1)$$

where Θ is half the angle of convergence of the beam; $\alpha = f \left(\frac{r_c}{r_a} \right)$ (see Fig. 2.22).

From (3.1) it follows that, in order to increase the perveance, one must increase the angle of convergence of the beam and reduce the ratio r_c/r_a , i.e. reduce the distance between the cathode and anode. Figure 3.4 shows the influence of the angle of convergence of the beam on the perveance value.

As shown in the drawing, an increase in the perveance results in an increase in the radius of the anode aperture. With $p > 1 \mu\text{A}/\text{V}^{3/2}$ this radius becomes commensurable with the distance between the cathode and anode. In this case the calculation given in Section 2.4 may produce considerable errors. The investigation of high-perveance guns with a converging beam has shown that the value of perveance obtained in practice is much lower than the theoretically calculated value. Moreover, the beam emitted from the anode aperture becomes non-homocentric, i.e. the continuations of the trajectories of electrons emerging at different distances from the cathode centre do not intersect at one point. Finally, the current density is substantially different at different cathode points: the current load of the peripheral portion of the cathode is significantly higher than that of the central portion thereof.

The perveance drop is due to a decrease in the accelerating field at the middle of the cathode. In fact, due to the presence of the anode aperture, the distance between the cathode and anode is not the same for different cathode points: the distance from the cathode centre to the edge of the anode aperture exceeds that from the cathode edge to the anode. Thus, at high convergence angles, i.e. at high ratios of the anode aperture radius to the cathode-anode distance, the real fields and current densities are close to the design values only at the peripheral circular zone of the cathode, while in the central part of the cathode the current density can be 3 to 5 times less than the peripheral current density due to the field attenuation. A drop in the current density in the central part of the cathode results in a decrease of the total beam current and, consequently, in a decrease of the perveance. It should be also noted that, in order to reduce the fraction of electrons intercepted by the anode, the radius of the anode aperture is practically made greater than the theoretical one by a factor of 1.2 to 1.5, i.e. the angle of opening of the anode aperture $\Theta_a = (1.2-1.5)\Theta$. Such an increase of the anode aperture also results in a drop of the perveance. As an example, Fig. 3.5 illustrates the angle of convergence at different ratios Θ_a/Θ for a gun with $r_c/r_a = 2$.

From this drawing it is evident that the influence of the anode aperture on the value of perveance may be neglected only for $p \leq \leq 0.8 \times 10^{-6} \text{ A/V}$ and $\Theta \leq 15^\circ$.

The perveance drop due to an increase in the anode aperture can be to some extent compensated for by reducing the distance between the cathode and anode. The example of simulating in an electrolytic bath has shown that the satisfactory results are obtained when varying the radius of the anode aperture to the value

$$r'_a = \frac{r_a}{\cos \Theta} \quad (3.2)$$

where r'_a is the theoretically found value of the anode radius in an ideal (without an aperture) collimating system.

The breaking of the laminar flow is explained by considerable aberrations of the lens formed in the anode aperture. The field configuration in the anode lens is such that the peripheral trajectories are refracted to a lesser extent than the trajectories passing nearer to the central region of the cathode. As a result, intersecting electron trajectories appear in the beam to make it nonlaminar.

One of the methods of reducing the aberrations of the anode lens is a decrease in the radius of curvature of the cathode surface compared with the design value. Experiments have shown that when

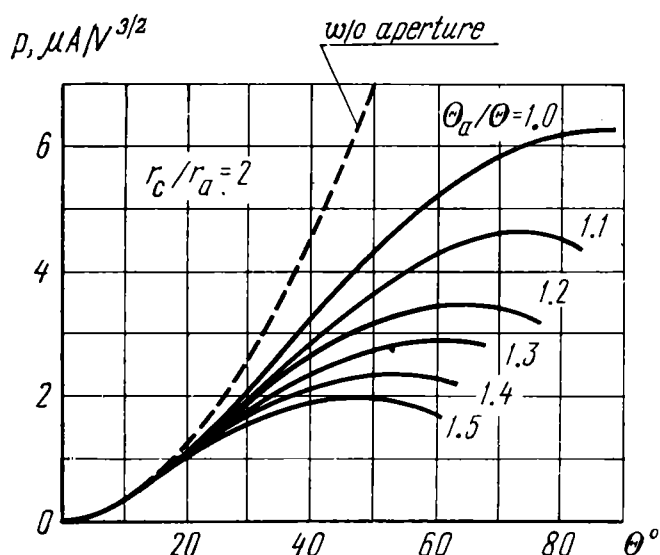


Fig. 3.5. Influence of anode aperture on microperveance at different beam convergence angles

the microperveance is $p > 1 \mu\text{A}/\text{V}^{3/2}$, the use of more concave cathodes allows one to reduce the nonuniformity of current density distribution through the beam section and to restore the flow laminarity. In this case, the higher the microperveance, the lesser should be the radius of curvature of the cathode surface as compared to the design value. For example, when $p = 1.2 \mu\text{A}/\text{V}^{3/2}$, the optimal radius of curvature of the cathode surface should be approximately by 16% below the design value and when $p = 2 \mu\text{A}/\text{V}^{3/2}$, by 25%. However, such correction of the aberrations of the anode lens results in undesirable redistribution of the current density on the cathode: the current load of the peripheral zone of the cathode rises up, while that in the central zone drops down.

The above considerations show that the analytic calculation of the guns producing intensive beams can be regarded only as a first approximation and corrections on the basis of the electrolytic-

tank simulating and experimental study are often necessary when designing the electrode system of such a gun.

As indicated above, the classical Pierce guns for collimating solid cylindrical flows with a radius equal to the radius of the emitting part of the cathode have limited applications, since the current density in the beam can never be higher than the specific cathode

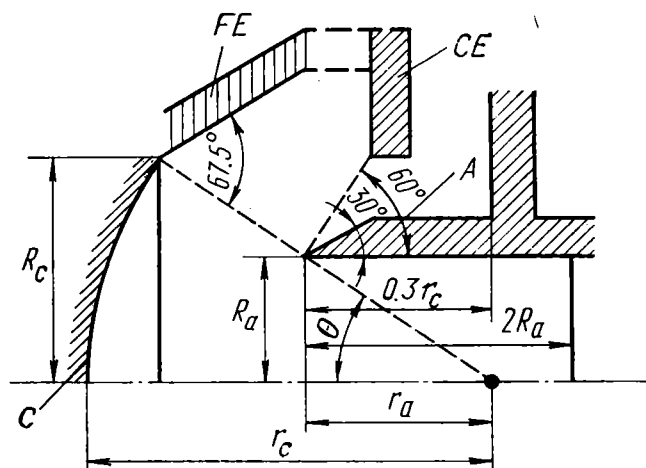


Fig. 3.6. Müller gun

emission. The guns with compression, i.e., those collimating conical (converging) beams in the cathode-anode space have found more extensive use.

The making of electrodes of an involved shape corresponding to the shape of the calculated equipotential surfaces is a very difficult technological process. Therefore, nowadays there are widely used practical constructions of the guns having simple electrodes in the form of cones, apertured discs and cylinders. The cathode of the guns with compression is an exception, since in such devices the cathode is a part of a sphere with a radius equal to the design value or smaller than that to compensate for the aberrations of the anode lens.

Widely used in the field of electron optics are also so-called Müller guns (Fig. 3.6) the characteristic feature of which is the presence of an anode "branch pipe" with a sharp edge and a special concentrating electrode at a potential close to the cathode potential. The concentrating electrode may be in the form of a bent edge of the near-cathode focusing electrode (Fig. 3.7).

Still more simple construction of the electrodes for the guns producing cylindrical and converging beams has been proposed by S. Treneva. The sectional view of the electrode system of one of such guns is shown in Fig. 3.8.

All the electrodes of the gun, except for the cathode having a concave shape, consist of cylinders and discs. The electrode shape was selected by simulating in an electrolytic tank.

The above examples of the constructions show that the shape and disposition of the electrodes of the guns collimating solid axisym-

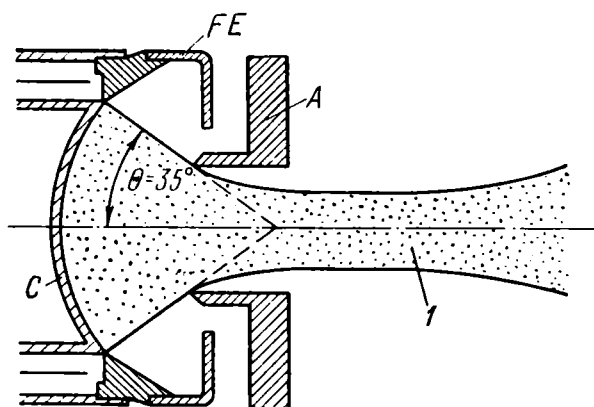


Fig. 3.7. Modification of Müller gun
1—beam boundary

metric beams can be varied within a wide range. However, in all cases the boundary conditions (specified potential distribution along the beam boundary and zero component of the electric field strength normal to the beam boundary) must be satisfied.

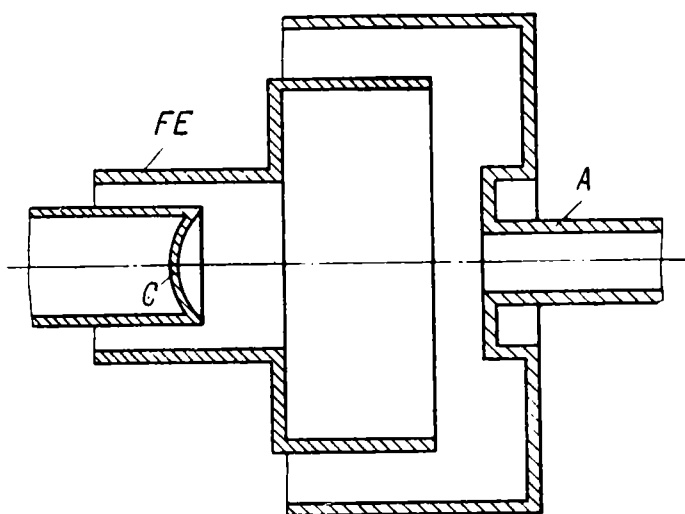


Fig. 3.8. Treneva's gun

The above-discussed guns are purely electrostatic systems, since the beam is formed by an electric field produced by a system of electrodes (cathode, near-cathode electrode and anode) whose shape and potentials are specified. It is quite clear that in the space behind the

anode the beam expands due to the repulsive action of space charge. In many practical cases expansion of the beam in the space behind the anode is confined by using a longitudinal magnetic field. The minimum value of the magnetic induction of the confining field is obtained in the case of the Brillouin flow (see Section 2.5). Generation of a Brillouin flow requires an ideal screening of the gun zone from the magnetic field and strict observance of the initial conditions of insertion of the beam into the magnetic field (the beam radius must be equal to the equilibrium Brillouin value, and there must be no radial components of the electron velocities). Due to the heat velocities of the emitted electrons, the aberrations of the lens near the anode aperture, and some other factors (the presence of positive ions, secondary electrons, "sagging" of the magnetic field in the gun zone, etc.) the initial conditions of introducing the beam into the magnetic field cannot be strictly observed in practice. Owing to this fact the equilibrium Brillouin flow has not yet been obtained experimentally. To confine the flow, i.e. to obtain a beam with comparatively small pulsations (ripple) of the boundary, the magnetic field should practically exceed the Brillouin value by a factor of 1.5 to 2. The calculations and experiments show that the guns shielded from the limiting action of the magnetic field provide for sufficiently stable (slightly pulsating) electron flows with microperveance of a few $\mu\text{A}/\text{V}^{3/2}$ in the presence of a magnetic field with induction higher than the Brillouin value in the space behind the anode.

As it has been mentioned above, it is difficult to completely shield the gun from the limiting magnetic field. More than that, the penetration of the magnetic field to the cathode is sometimes even desirable, since the focusing is better in the presence of a magnetic field in the gun area (see Section 2.3). Therefore, when the beam collimating system must meet stringent requirements for high perveance, stability, and low beam pulsations, there is an advantage in using the guns with a cathode partially shielded from the magnetic field. The systems with partially shielded guns require a higher value of magnetic induction as compared to the Brillouin field. However, since it is practically impossible to obtain an ideal Brillouin flow and the induction of the practically employed magnetic fields exceeds the Brillouin value, an increase in the field is not a significant disadvantage of the systems with partially shielded guns.

Consider a gun with compression of the electron flow and a partially shielded cathode (Fig. 3.9).

As shown in the drawing, the trajectories of the undisturbed electrons in the gun space between the cathode and anode coincide with the magnetic lines of force. It is clear that the magnetic field does not act on these electrons, since the Lorentz force is zero when $\mathbf{v} \parallel \mathbf{B}$. When disturbed electrons appear in the flow, the Lorentz force twists

the electron trajectories about the corresponding magnetic flux lines. Thus, the magnetic field in the gun zone substantially improves the conditions for collimating the beam by suppressing the electrons transverse velocity components expanding the flow. In other words, the magnetic field in the gun as though "accompanies" the electrons and does not allow them to fly away from the magnetic flux lines. Since in the compression guns the beam section decreases with the distance from the cathode, while the magnetic field inside the flow remains constant (this follows directly from the coincidence of the electron trajectories with the magnetic flux lines), the

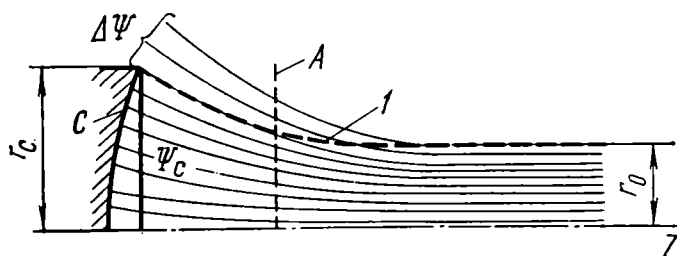


Fig. 3.9. Gun with partially shielded cathode
1—outer trajectory

magnetic induction must rise with the distance from the cathode.

The equation describing the contour of an axisymmetric beam emitted by an arbitrarily shielded cathode is written as follows [see (2.240)]

$$\frac{d^2R}{dZ^2} + \alpha R - \frac{\alpha k^2}{R^3} - \frac{\beta}{R} = 0 \quad (3.3)$$

where $R = \frac{r}{r_0}$, $Z = \frac{z}{l}$ are dimensionless (normalized) coordinates; α is the magnetic field parameter; β is the space charge parameter; k is the parameter of the cathode conditions [see (2.236), (2.237), (2.238), 2.239)].

The analysis of equation (3.3) shows that the equilibrium flow in a uniform longitudinal magnetic field in the space behind the anode can be obtained with definite relations among the magnetic field parameter α , the space charge β and the cathode conditions k . In fact, a stable non-pulsating beam with a constant radius r_0 ($R = 1$) is characterized by the zero first and second derivatives dR/dZ and d^2R/dZ^2 at any Z . This requirement, in turn, leads to the ratio

$$\alpha - \alpha k^2 - \beta = 0 \quad (3.4)$$

Expressing the parameter of the cathode conditions from (3.4), we obtain

$$k^2 = 1 - \frac{\beta}{\alpha} \quad (3.5)$$

anode the beam expands due to the repulsive action of space charge. In many practical cases expansion of the beam in the space behind the anode is confined by using a longitudinal magnetic field. The minimum value of the magnetic induction of the confining field is obtained in the case of the Brillouin flow (see Section 2.5). Generation of a Brillouin flow requires an ideal screening of the gun zone from the magnetic field and strict observance of the initial conditions of insertion of the beam into the magnetic field (the beam radius must be equal to the equilibrium Brillouin value, and there must be no radial components of the electron velocities). Due to the heat velocities of the emitted electrons, the aberrations of the lens near the anode aperture, and some other factors (the presence of positive ions, secondary electrons, "sagging" of the magnetic field in the gun zone, etc.) the initial conditions of introducing the beam into the magnetic field cannot be strictly observed in practice. Owing to this fact the equilibrium Brillouin flow has not yet been obtained experimentally. To confine the flow, i.e. to obtain a beam with comparatively small pulsations (ripple) of the boundary, the magnetic field should practically exceed the Brillouin value by a factor of 1.5 to 2. The calculations and experiments show that the guns shielded from the limiting action of the magnetic field provide for sufficiently stable (slightly pulsating) electron flows with microperveance of a few $\mu\text{A}/\text{V}^{3/2}$ in the presence of a magnetic field with induction higher than the Brillouin value in the space behind the anode.

As it has been mentioned above, it is difficult to completely shield the gun from the limiting magnetic field. More than that, the penetration of the magnetic field to the cathode is sometimes even desirable, since the focusing is better in the presence of a magnetic field in the gun area (see Section 2.3). Therefore, when the beam collimating system must meet stringent requirements for high perveance, stability, and low beam pulsations, there is an advantage in using the guns with a cathode partially shielded from the magnetic field. The systems with partially shielded guns require a higher value of magnetic induction as compared to the Brillouin field. However, since it is practically impossible to obtain an ideal Brillouin flow and the induction of the practically employed magnetic fields exceeds the Brillouin value, an increase in the field is not a significant disadvantage of the systems with partially shielded guns.

Consider a gun with compression of the electron flow and a partially shielded cathode (Fig. 3.9).

As shown in the drawing, the trajectories of the undisturbed electrons in the gun space between the cathode and anode coincide with the magnetic lines of force. It is clear that the magnetic field does not act on these electrons, since the Lorentz force is zero when $\mathbf{v} \parallel \mathbf{B}$. When disturbed electrons appear in the flow, the Lorentz force twists

the electron trajectories about the corresponding magnetic flux lines. Thus, the magnetic field in the gun zone substantially improves the conditions for collimating the beam by suppressing the electrons transverse velocity components expanding the flow. In other words, the magnetic field in the gun as though "accompanies" the electrons and does not allow them to fly away from the magnetic flux lines. Since in the compression guns the beam section decreases with the distance from the cathode, while the magnetic field inside the flow remains constant (this follows directly from the coincidence of the electron trajectories with the magnetic flux lines), the

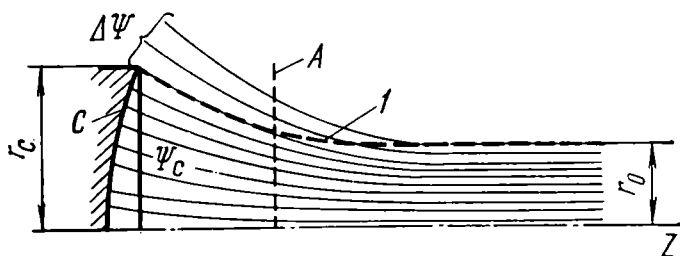


Fig. 3.9. Gun with partially shielded cathode
1—outer trajectory

magnetic induction must rise with the distance from the cathode.

The equation describing the contour of an axisymmetric beam emitted by an arbitrarily shielded cathode is written as follows [see (2.240)]

$$\frac{d^2R}{dZ^2} + \alpha R - \frac{\alpha k^2}{R^3} - \frac{\beta}{R} = 0 \quad (3.3)$$

where $R = \frac{r}{r_0}$, $Z = \frac{z}{l}$ are dimensionless (normalized) coordinates; α is the magnetic field parameter; β is the space charge parameter; k is the parameter of the cathode conditions [see (2.236), (2.237), (2.238), 2.239)].

The analysis of equation (3.3) shows that the equilibrium flow in a uniform longitudinal magnetic field in the space behind the anode can be obtained with definite relations among the magnetic field parameter α , the space charge β and the cathode conditions k . In fact, a stable non-pulsating beam with a constant radius r_0 ($R = 1$) is characterized by the zero first and second derivatives dR/dZ and d^2R/dZ^2 at any Z . This requirement, in turn, leads to the ratio

$$\alpha - \alpha k^2 - \beta = 0 \quad (3.4)$$

Expressing the parameter of the cathode conditions from (3.4), we obtain

$$k^2 = 1 - \frac{\beta}{\alpha} \quad (3.5)$$

It is clear that when the cathode is completely shielded ($k = 0$), we have the equality $\alpha = \beta$ or the case of the Brillouin flow [conf. with (2.145)]. In this case an equilibrium Brillouin flow is provided if the magnetic induction value [see (2.147)] is as follows

$$B_{0B}^2 = \frac{\sqrt{2}}{\pi \epsilon_0} \left(\frac{m}{e} \right)^{3/2} \frac{I}{\sqrt{U_0} r_{0B}^2} \quad (3.6)$$

where r_{0B} is the radius of the equilibrium Brillouin flow.

When the cathode is partially shielded ($k \neq 0$), an equilibrium flow is maintained by using a magnetic field with induction exceeding the Brillouin value ($B > B_{0B}$). Comparing the values α , β and B_{0B}^2 and making allowance for the Brillouin radius (2.145), we obtain

$$\frac{\beta}{\alpha} = \left(\frac{r_{0B}}{r_0} \right)^2 = \left(\frac{B_{0B}}{B} \right)^2 \quad (3.7)$$

Substituting (3.7) into (3.5) we shall find a relative (in comparison with the Brillouin value) increase in the magnetic induction required for maintaining the equilibrium flow produced by the gun with a cathode partially shielded from the magnetic field

$$\frac{B^2}{B_{0B}^2} = \frac{1}{1 - k^2} \quad (3.8)$$

As follows from (3.8), in the case of a gun with a partially shielded cathode the equilibrium flow will exist in a magnetic field with induction

$$B = B_{0B} \sqrt{\frac{1}{1 - k^2}} \quad (3.9)$$

exceeding the Brillouin value. It is clear that the larger the part of magnetic field that pierces the cathode, i.e. the higher $k^2 = \psi^2 c / \psi_0$, the higher must be the excess of the magnetic induction over the Brillouin value. The value of magnetic induction defined by formula (3.9) is optimum, since only this relationship of B and k^2 makes it possible to obtain an equilibrium flow with a partially shielded gun. The connection between the values B/B_{0B} and k^2 , which provides for an equilibrium flow according to formula (3.8), is graphically illustrated in Fig. 3.10.

As seen in the drawing, a small excess of the magnetic induction over the Brillouin value requires a substantial change in the parameter k^2 . But at $B/B_{0B} > 3$ the value of k^2 approaches unity and features comparatively small dependence on the ratio B/B_{0B} . Representing k^2 as a ratio ψ_c^2 / ψ_0^2 and determining k^2 from (3.8), we obtain the value of magnetic flux that must pierce the cathode to provide for an equilibrium flow with a partially shielded gun

$$\Psi_c = \Psi_0 \sqrt{1 - \left(\frac{B_{0B}}{B} \right)^2} \quad (3.10)$$

From (3.10) it follows that the higher the excess of the "operating" magnetic field B over the Brillouin value B_{0B} , the greater part of the magnetic flux pierces the cathode. Thus, for example, with $B/B_{0B} = 2$ the magnetic flux passing through the cathode is expected to be equal to 86 % of the magnetic flux in the space behind the anode.

Thus, the guns with a partially shielded cathode feature a certain relationship between the perveance (space charge parameter), the magnetic field and the degree of shielding the cathode which provides for an equilibrium nonpulsating beam. When this condition is

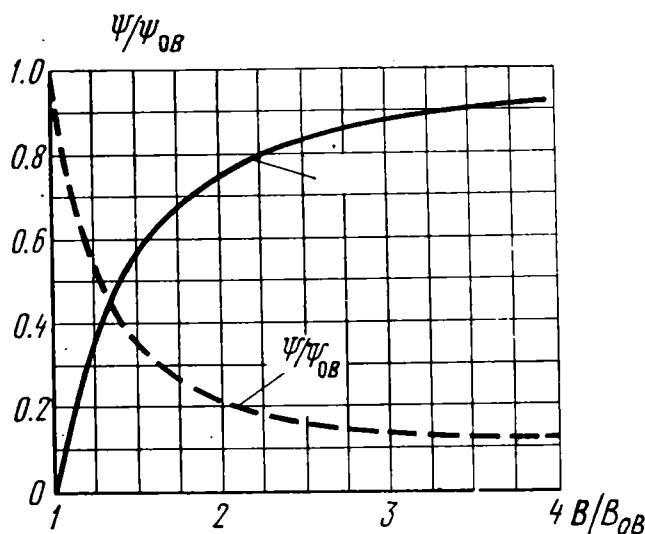


Fig. 3.10. Cathode conditions versus B/B_{0B} ratio

not satisfied, the beam boundary starts pulsating. It is interesting to note that with $k = 1$ (the entire magnetic flux pierces the cathode) it is impossible to obtain a nonpulsating beam; when $k \rightarrow 1$ the optimum value of magnetic induction is $B \rightarrow \infty$ [see (3.9)]. At the same time, an increase in k (at $k < 1$) and, therefore, an increase in the optimum value of the magnetic field contributes to a decrease in the influence of the radial components of the electron velocities including the initial heat velocities. Owing to this fact the beam collimated by the guns with partially shielded cathodes are more stable, and the focusing with $k \neq 0$ is more hard than in the case of a Brillouin flow. It should be also noted that the azimuthal velocity of rotation of the beam about the axis is reduced with an increase in the magnetic induction at ($k \neq 0$). Using the Bush theorem of the (2.129) type and expressing $\left(\frac{B_c}{B}\right)^2 \left(\frac{r_c}{r_0}\right)^4$ through k^2 find the azimuthal velocity of the outer electron of the beam

$$\dot{\psi} = \frac{e}{2m} B (1 - k) \quad (3.11)$$

Meanwhile, the azimuthal velocity of rotation of the Brillouin flow [see (2.133)] is

$$\dot{\psi}_{0B} = \frac{e}{m} B_{0B} \quad (3.12)$$

Setting a relation from (3.11) and (3.12) and using (3.9), we obtain

$$\frac{\dot{\psi}}{\dot{\psi}_{0B}} = \frac{B}{B_{0B}} \left[1 - \sqrt{1 - \left(\frac{B_{0B}}{B} \right)^2} \right] \quad (3.13)$$

From (3.13) it follows that the flow rotates but slightly in strong magnetic fields ($B \gg B_{0B}$). The dependence $\dot{\psi}/\dot{\psi}_{0B} = f(B/B_{0B})$ is illustrated graphically in Fig. 3.10. A low azimuthal velocity is also one of the advantages of the beams collimated by a gun with a partially shielded cathode.

Thus, the guns with partially shielded cathodes have a number of advantages including a sharper boundary of the beam due to smaller influence of the heat velocities, and this improves the flow of electric current in the device. The partially shielded guns are less sensitive to fluctuation of the anode voltage and the focusing solenoid current. As shown above, the ideal Brillouin flow practically cannot be obtained, and in real systems producing intensive beams we always have $B > B_{0B}$, for which reason the guns with a partially shielded cathode and beam compression have presently found wide application.

Hollow (tubular) cylindrical and conical beams can be produced by electron guns of three types: (1) guns based on the Pierce system for forming parallel or converging flows; (2) guns with centrifugal electrostatic focusing; (3) magnetron injection guns.

The guns of the first type are the simplest ones and with correctly selected geometry of the electrode system can collimate tubular beams with micropervance of up to a few $\mu\text{A}/\text{V}^{3/2}$. As an example, Fig. 3.11 shows a sketch of an electrode system of a gun collimating a parallel tubular beam.

As shown in the drawing, the focusing system of the gun is a Pierce system coiled into a ring for forming a rectilinear flow. The cathode emissive surface has the form of a flat ring. Adjoining the cathode at both sides are outer and inner electrodes of a conical shape; the anode has a circular slit. Blurring of the beam in the space behind the anode is prevented by a uniform magnetic field partially penetrating into the gun zone. This field partially compensates for the diverging action of the circular lens formed in the anode aperture. The gun can be roughly calculated by the formulas for a strip beam (see Section 2.4). The accuracy of the calculations is the higher, the lesser the thickness (difference between the outer and inner radii) of the beam as compared to its mean radius. The disadvantage of

such guns consists in the absence of compression, and this does not allow one to obtain a current density in the beam higher than the specific emission of the cathode.

The forming of a tubular cylindrical beam with compression of the electron flow can be effected by means of a ring-shaped Pierce system collimating a converging wedge-shaped flow (Fig. 3.12).

The cathode of this gun has the shape of a toroidal surface so that the above-described system is referred to as a *toroidal gun*. The shape of the gun electrodes and the magnetic circuits shielding the cathode against the uniform longitudinal field limiting the beam in the space

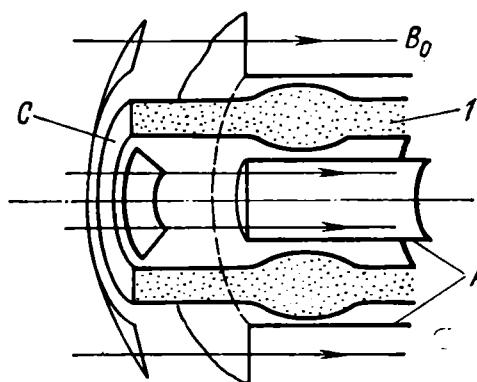


Fig. 3.11. Collimating parallel tubular beam
 $\vec{r}$ —beam

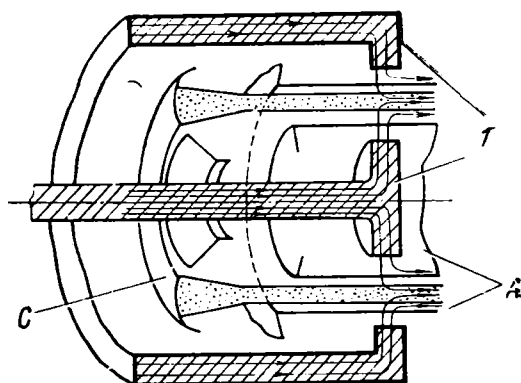


Fig. 3.12. Collimating tubular cylindrical beam with compression of electron flow
 1—magnetic circuits

behind the anode is clear from the drawing. The theoretical and experimental study of toroidal electron guns has shown that these systems can provide for sufficiently stable beams with micropervance up to $10 \mu\text{A}/\text{V}^{3/2}$ and higher with compression of up to 10.

A tubular conical (converging) beam can be collimated in a similar way. Figure 3.13 illustrates a cross section of the electrodes of the gun for collimating tubular conical beam proposed by S. Treneva.

When using such guns, it should be kept in mind that due to the dispersing anode lens disturbing the homocentricity of the flow the tubular conical beam formed by the gun is transformed into a solid beam in the space behind the anode. For this reason such guns have not gained wide recognition.

The guns with centrifugal electrostatic focusing proposed by Z. Chernov (see Section 2.4) are advantageous in a low weight and high economy of the focusing system due to the absence of magnetic coils and magnetic circuits, impossibility of penetration of positive ions to the cathode and, as a result, a long service life of the cathode. However, such guns are featured by high pervance values and the action of the space charge in these systems cannot be compensated

for by an increase in the potential difference between the plates of the cylindrical capacitor. The beam becomes unstable, the settling of electrons on the electrodes increases. Owing to this fact the guns with centrifugal electrostatic focusing find limited applications in devices where microperveance is not to be higher than a few tenths of $1 \mu\text{A}/\text{V}^{3/2}$.

At the present time in many electron-optical devices there are used magnetron guns providing stable tubular flows with very high perveances, up to $40 \mu\text{A}/\text{V}^{3/2}$ and higher. As mentioned in

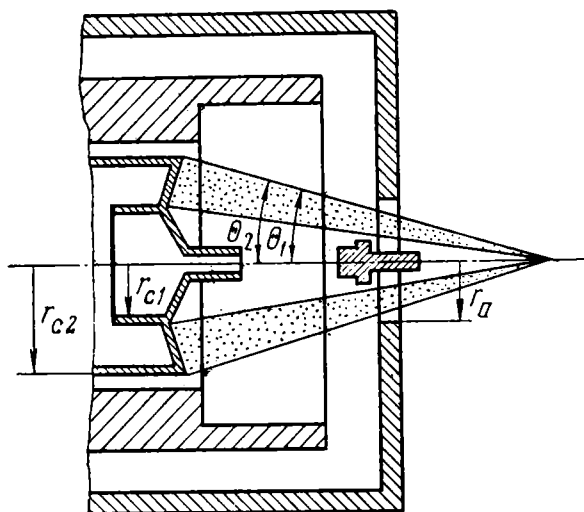


Fig. 3.13. Collimating tubular conical beam (Treneva's gun)

Section 2.5, the magnetron guns relate to the systems with crossed electrostatic and magnetic fields. The diagram of the electrode system of a magnetron gun is typical of the majority of real guns of the magnetron type. The cathodes of magnetron guns, as a rule, are made in the form of truncated cones with an angle of inclination of the generatrix to the axis within a range of 2 to 5° . The anode can be made in the form of a body of revolution shaped as a funnel with a curved generatrix or be composed of linked cones having somewhat different angles of inclination of their generating lines to the axis. Also known are magnetron guns with a cylindrical cathode and a conical anode, as well as guns with a conical cathode and a cylindrical anode and the so-called inverted magnetron guns with an internal anode and a cathode in the form of a truncated cone encircling the anode. Experiments have shown that it is possible to collimate a tubular beam by means of a magnetron gun whose anode and cathode are coaxial cylinders.

Design calculation of axisymmetric magnetron guns (with conical electrodes) is very complicated, therefore, the calculations are often effected by the formulas for a planar system with a cathode

inclined at an angle Θ with respect to the magnetic field. After that the shape of the electrodes is corrected in an electrolytic tank with an inclined bottom.

Since the electrostatic field in a planar magnetron gun (planar diode) has only one component ($E_y \neq 0$) (see Section 2.5, Fig. 2.36), the potential at any point of the interelectrode space is uniquely defined by the expression

$$U = - \int_0^y E_y dy = - \int_0^t E_y \dot{y} dt \quad (3.14)$$

The field intensity component E_y in expression (3.14) is determined by equation (2.202) while the transverse velocity component is found by differentiating the second equation of system (2.206) with respect to time. Substituting these values into (3.14) and performing the integration, we obtain

$$U = \frac{e}{m} \times \frac{j_0^2}{\epsilon_0^2 \omega^2} \left(\frac{1}{8} \omega_y^2 t^4 + \frac{1}{2} t^2 \right) \quad (3.15)$$

where $\omega = \frac{e}{m} B_0$, $\omega_y = \frac{e}{m} B_y = \frac{e}{m} B_0 \sin \Theta$.

Let us introduce normalized variables: coordinates X , Y , Z , potential Φ and time T determined by the expressions

$$\begin{aligned} X &= \frac{\epsilon_0 \omega^3}{\frac{e}{m} j_0} x, & Y &= \frac{\epsilon_0 \omega^3}{\frac{e}{m} j_0} y, & Z &= \frac{\epsilon_0 \omega^3}{\frac{e}{m} j_0} z \\ \Phi &= \frac{\epsilon_0^2 \omega^4}{\frac{e}{m} i_0^2} U, & T &= \omega t \end{aligned} \quad (3.16)$$

In a normalized system of variables equations (2.206) and (3.15) have the form

$$\left. \begin{aligned} X &= X_0 - \frac{T^2}{2} \cos \Theta \\ Y &= T + \frac{1}{6} T^3 \sin \Theta^2 \\ Z &= Z_0 + \frac{1}{6} T^3 \sin \Theta \cos \Theta \\ \Phi &= \frac{1}{2} T^2 + \frac{1}{8} T^4 \sin^2 \Theta \end{aligned} \right\} \quad (3.17)$$

The system of parametric equations obtained makes it possible to determine the electron trajectories and the potential distribution inside the beam. To find the form of the anode, one has to solve an external problem, i.e. to calculate the potential distribution outside the beam. In this case any equipotential surface beyond the boundary

of the beam may be replaced by a conducting surface with the same potential, i.e. the form of the anode is defined by the shape of the equipotential surface lying outside the beam. The shape of the near-cathode focusing electrodes is determined by the zero ($\Phi = 0$) equi-

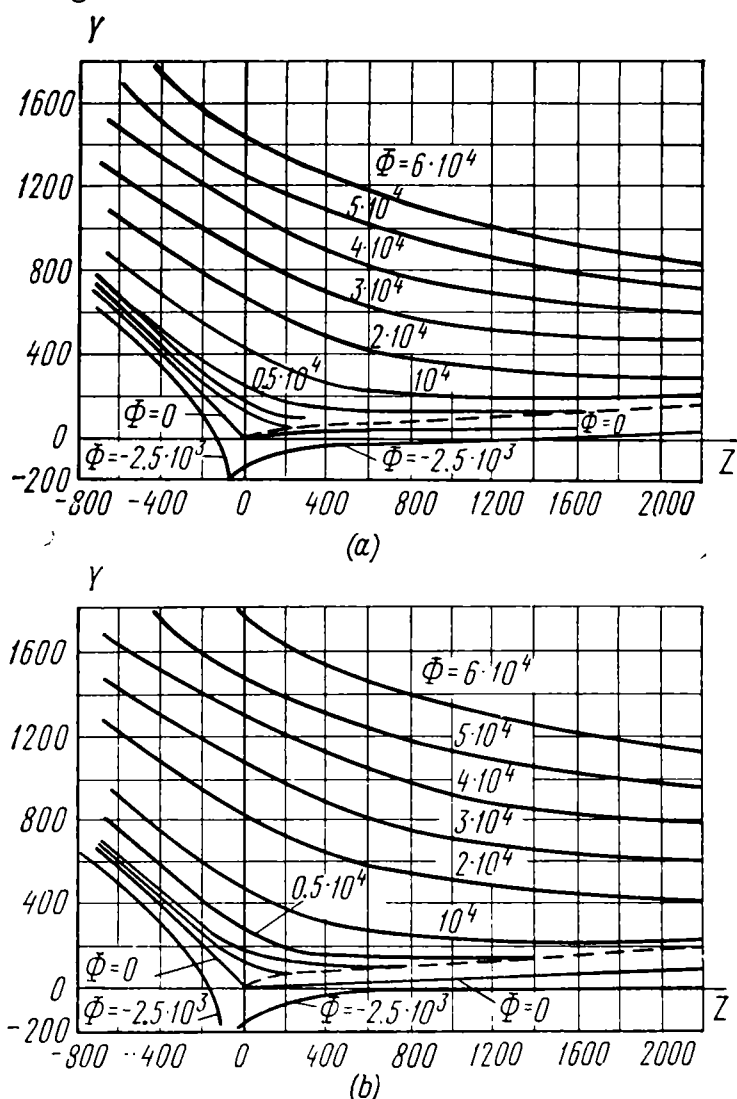


Fig. 3.14 Equipotential surfaces in flat magnetron injection guns with cathode inclination angles $\Theta = 2^\circ$ (a) and $\Theta = 4^\circ$ (b)

potential surface outside the beam, i.e. outside the edges of the emitting cathode surface. The solution of the external problem is reduced to finding the two-dimensional Laplace equation with boundary conditions obtained from (3.17) by calculating the potential distribution along the beam boundary.

The analytic solution of the external problem can be performed by the method of conformal reflection of the region outside the beam

on the complex variable plane. Since any function of the complex variable satisfies the Laplace equation, the solution of the above problem under such transformation is much simplified. The graphical solution of the external problem for two angles of inclination of the cathode surface to the direction of the magnetic field in projection on the plane YOZ is given in Fig. 3.14. The same drawing illustrates the electron trajectories calculated through solving the system of equations (3.17).

Using the above graphs, one can easily determine the shape of the gun electrodes. The shape of the near-cathode focusing electrodes is determined by the curve $\Phi = 0$ outside the beam by recalculating the values Y, Z into the preceding coordinate system (y, z) through

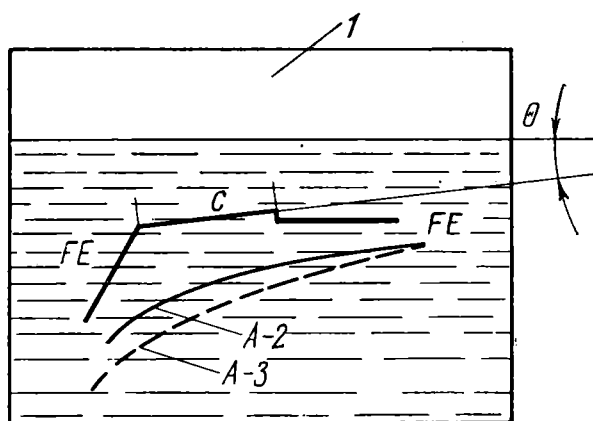


Fig. 3.15. Selecting shape of anode of axisymmetric gun
1—"shore line"; 2—flat anode; 3—axisymmetric anode

relations (3.16). The anode shape is determined by any curve $\Phi = \text{const}$ ($\Phi > 0$), when recalculated into the system (y, z) . The electrode system calculated by such a method is simulated in a two-dimensional electrolytic tank and the electric field strength near the cathode is measured. Then the tank bottom is inclined so that the "shore line" coincides with the axis of the axisymmetric gun being designed and the coincidence of the values of the field intensity near the cathode of the axisymmetric system and the field strength values measured in the planar system are achieved by changing the shape of the anode (Fig. 3.15).

It should be noted that the lower the thickness ($r_{out} - r_{in}$) of the beam as compared to its mean radius, the lesser the difference between the shape of the anode of the axisymmetric gun and that of the planar gun. When $(r_{out} - r_{in}) \leq 0.1 r_{av}$ the correction of the anode shape when proceeding to the axisymmetric system is not great.

Figure 3.16 illustrates a section view of one of modern magnetron injection guns. The experimental study of magnetron injection guns

has shown that the current density difference at various points of the cathode is not higher than 10%, i.e. the current load of the cathode is approximately the same at any point of its surface. The magnetron injection guns are presently widely used in electron optics and are recognized as very promising due to their advantages over other types of electron guns (high perveance, a distinct boundary of

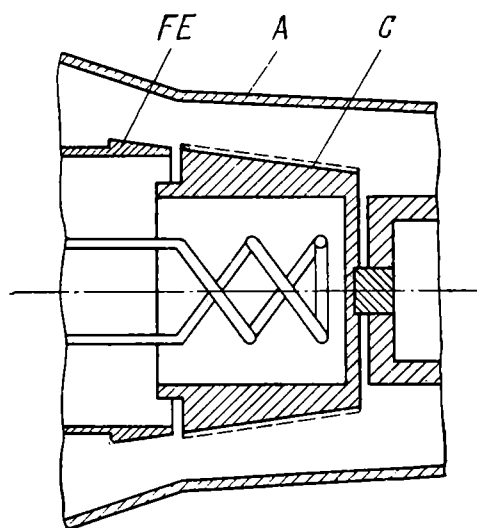


Fig. 3.16. Sectional view of magnetron gun

a formed beam, sufficiently high compression and uniform current distribution in the cathode.) The basic disadvantage of magnetron injection guns is a comparatively high value of magnetic induction which is 2 to 3 times the Brillouin value. Another disadvantage is that the magnetron injection guns feature rather high noise level.

3.3. Strip Beam Guns

Some microwaves devices such as high-power travelling wave tubes and backward-wave tubes employ intensive electron beams of a rectangular section (strip beams). The simplest method of col-

limating a strip beam is based on utilization of a Pierce system with a planar rectilinear cathode and focusing cathode electrodes shaped as wings adjoining the long sides of the cathode and forming angles of 67.5° with regard to the normal to the cathode surface (See Section 2.4.). The anode also has two wings following the shape of one of the equipotential surfaces ($U > 0$) Fig. 2.18, with a slot between them equal to the beam thickness. A sketch of the electrode system of a strip beam gun is shown in Fig. 3.17.

The beam emerging from the anode slit, in which a diverging lens is formed that curves the electron trajectories from the centre-plane of the beam, will certainly be expanded under the action of space charge. The expansion of the beam in the space behind the anode is usually limited by a uniform longitudinal magnetic field, like that used in the systems collimating axisymmetric beams.

The guns collimating a parallel strip beam have found limited applications, in spite of their simple construction, mainly because of the absence of compression.

The guns forming converging (wedge-shaped) beams in the space between the cathode and anode are more popular. In this case it is possible to obtain sufficient compression and, therefore, to produce a beam with current density much exceeding the specific emission

of the cathode. The wedge-shaped beam formed by the gun becomes parallel in the space behind the anode due to the action of space charge and the diverging anode lens. Then the beam starts expanding, if there are no limiting fields. The expansion of the beam is limited by a system in the form of a uniform longitudinal magnetic field or a periodic focusing system (electrostatic or magnetic). It is clear that when the initial conditions of introducing the strip beam into the magnetic field are satisfied, particularly, when the cathode is completely shielded from the magnetic field, in principle, it is possible to obtain a Brillouin strip flow. However, owing to the fact

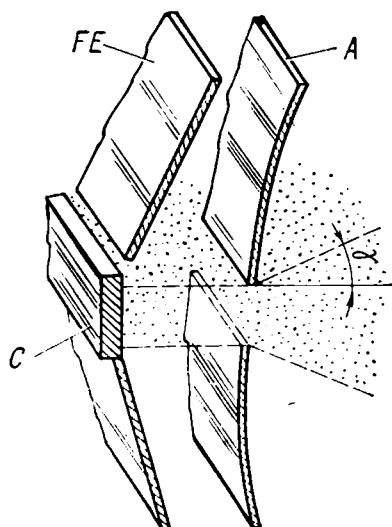


Fig. 3.17. Electrode system of gun for collimating strip beam

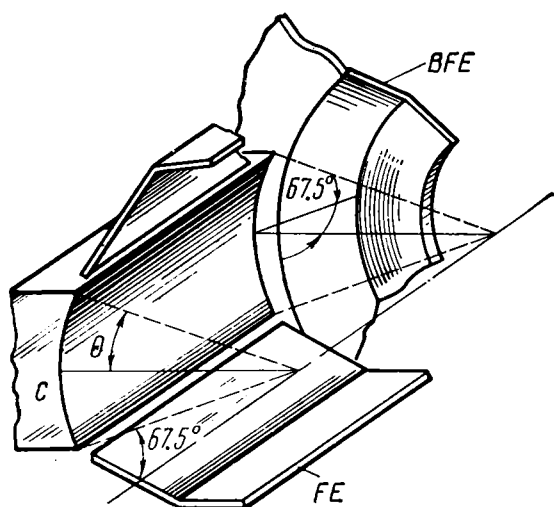


Fig. 3.18. Electrode system of gun for collimating wedge-shaped beam

that in practice the beam is limited by magnetic fields with induction exceeding the Brillouin value (see Section 2.5) and the ideal shielding of the cathode from the magnetic field is a difficult engineering problem, strip beam guns with partial shielding of the cathode are often employed.

The cathode of the gun collimating a wedge-shaped beam is made as a cylindrical surface. The anode has a shape defined by one of the equipotential surfaces ($U > 0$), Fig. 2.21. At both sides the cathode is adjoined by focusing electrodes whose shape is determined by the equipotential $U = 0$, Fig. 2.21. However, the curved electrodes are inconvenient in manufacture. Taking into account the fact that during the development of the gun design the shape of the electrodes is corrected by simulating in an electrolytic tank, the focusing electrodes are often made in the form of two planes with generatrices inclined at 67.5° to the normals drawn from the cathode edges.

Discussing the formation of wedge-shaped flows (see Section 2.4), an assumption is made that beam width is unlimited. In real systems the beam width, which in the case of a strip beam is much greater than its thickness, must be limited. The simplest way of doing so is to use two additional focusing electrodes made in the form of a conical surface with generating lines inclined at 67.5° to the normals drawn to the cathode edges.

The correction of the shape of the main focusing electrodes on a model in the electrolytic tank is effected by using a two-dimensional tank, i.e. what is simulated is a field infinitely expanding along the beam width. The shape of the additional side focusing electrodes is determined using an electrolytic tank with an inclined bottom.

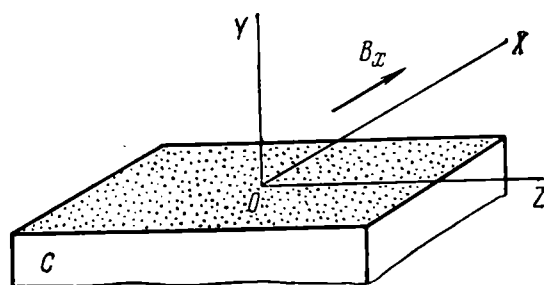


Fig. 3.19. On calculation of crossed-field system

In this case the model is installed so that the "shore line" coincides with the axis of the cylindrical surface of the cathode. A sketch of the electrode system of the gun for collimating a wedge-shaped beam is shown in Fig. 3.18.

Since the main and side focusing electrodes are at the same (zero) potential, in real devices both pairs of focusing electrodes can be made in the form of a single member with a rectangular aperture matching the cathode shape.

The disadvantages of the guns forming converging strip beams are complex calculation and simulating. Furthermore, such guns have comparatively large overall dimensions due to the presence of side focusing electrodes.

A strip beam can be also collimated by a focusing system with crossed electrostatic and magnetic fields. In contrast to the magnetron injection gun collimating an axisymmetric tubular flow, in such a system the electrons are forced from the space between the cathode and anode by the magnetic Lorentz force and not by an electrostatic field.

Let us consider the motion of electrons in a crossed-field gun. Let a planar cathode be disposed in a plane XOZ and the origin of coordinates be aligned with point O on the cathode surface (Fig. 3.19).

The uniform magnetic field is directed along the axis OX ($B = B_x$). Assume that the potential, the current density and the space charge

density depend only on coordinate y . In this case the path of the electron leaving the cathode at the point 0 is a flat curve lying in the plane YOZ . This assumption stems directly from the relationship $E_x = E_z = 0$ [$U = U(y)$] and from the expression of the Lorentz force

$$F_m = -e[\mathbf{vB}] = -ev_y B_x = F_z \quad (3.18)$$

In the same manner, the electrons flying from the other points of the cathode are acted on by the electrostatic force F_y and the magnetic force F_z . The absence of the resultant force F_x indicates that the trajectories of all the electrons are in this case flat curves lying in planes parallel to the plane YOZ (it is supposed that the electrons have no x - and z -components of the initial velocity: $v_{0x} = v_{0z} = 0$). These assumptions are of course somewhat artificial but the obtained solution of the problem and the experimental verification of this solution indicate to their validity.

Let us set up equations of motion of the electrons under the above assumptions

$$\left. \begin{aligned} \ddot{x} &= 0 \\ \ddot{y} &= -\frac{e}{m} E_y - \omega \dot{z} \\ \ddot{z} &= \omega \dot{y} \end{aligned} \right\} \quad (3.19)$$

where $\omega = \frac{e}{m} B$.

This system must be solved together with the Poisson equation

$$\frac{d^2 U}{dy^2} = -\frac{dE_y}{dy} = -\frac{\rho}{\epsilon_0} \quad (3.20)$$

and the continuity equation

$$j = j_y = -\rho \dot{y} = \text{const} \quad (3.21)$$

where ρ is space charge density.

Let us integrate the third equation of system (3.19) with an initial condition $z = 0$ at $t = 0$ ($v_{0z} = 0$)

$$\dot{z} = \omega y \quad (3.22)$$

Now let us express ρ through j_y on the basis of (3.21), substitute into (3.20) and perform the integration

$$E_y = -\frac{j_y}{\epsilon_0} t + C = \frac{j_y}{\epsilon_0} t \quad (3.23)$$

The integration constant C is equal to zero, since when the current is limited by the space charge, the minimum of the potential coincides with the cathode surface, i.e. $(E_y)_{t=0} = 0$.

Substituting (3.22) and (3.23) into the second equation of system (3.19), we obtain an equation determining the "transverse" motion of the electron

$$\ddot{y} + \omega^2 y = \frac{e}{m\epsilon_0} y_y t \quad (3.24)$$

Double integration of this equation with initial conditions $\dot{y}|_{t=0} = \dot{y}_0$ and $y|_{t=0} = 0$ yields an expression for y

$$y = \frac{e}{m\epsilon_0} \times \frac{\dot{y}_y}{\omega^3} (\omega t - \sin \omega t) + \frac{\dot{y}_0}{\omega} \sin \omega t \quad (3.25)$$

Now, substituting expression (3.25) into (3.22) and performing integration with an initial condition $z|_{t=0} = z_0$, we obtain an expression for z

$$z = z_0 + \frac{e}{m\epsilon_0} \times \frac{\dot{y}_y}{\omega^3} \left[\frac{(\omega t)^2}{2} + \cos \omega t - 1 \right] + \frac{\dot{y}_0}{\omega} (1 - \cos \omega t) \quad (3.26)$$

determining the "longitudinal" motion of the electron.

Equations (3.25) and (3.26) in the parametric form determine the trajectory of the electron leaving the cathode at a point $y = 0$, $z = z_0$, at an initial velocity y_0 . The motion of the electrons in a planar magnetron injection gun may physically be interpreted in the following way: the electrostatic field accelerates the electrons in the transverse direction (along the axis OY), while the magnetic field shifts the electron flow in the longitudinal direction (along the axis OZ). The presence of terms with $\sin \omega t$ and $\cos \omega t$ in the solution shows that generally the trajectories of the electrons, including the outer ones, determining the beam configuration are wavy.

The potential distribution inside the beam can be obtained from the relation

$$U = - \int_0^y F_y dy = - \int_0^t E_y \dot{y} dt \quad (3.27)$$

Substituting E_y from (3.23) and $\dot{y}$ from (3.25) into (3.27) (this equation is preliminarily differentiated with respect to t) and performing the integration, we obtain

$$U = \frac{e j_y}{m\epsilon_0 \omega^4} \left[1 - \cos \omega t + \frac{(\omega t)^2}{2} - \omega t \sin \omega t \right] + \frac{\dot{y}_0 j_y}{\epsilon_0 \omega^2} [\omega t \sin \omega t + \cos \omega t - 1] \quad (3.28)$$

The obtained equation determines the potential distribution inside the beam.

Let us introduce normalized variables: the coordinates Y , Z , the time T and the potential Φ determined by relations (3.16). In the

normalized system of variables equations (3.25), (3.26) and (3.28) assume the form

$$\left. \begin{aligned} Y &= T - \left[1 - \frac{\varepsilon_0 \omega^2 \dot{y}_0}{\frac{e}{m} j_y} \right] \sin T \\ Z &= Z_0 + \frac{T^2}{2} - \left[1 - \frac{\varepsilon_0 \omega^2 \dot{y}_0}{\frac{e}{m} j_y} \right] (1 - \cos T) \\ \Phi &= \frac{T^2}{2} - \left[1 - \frac{\varepsilon_0 \omega^2 \dot{y}_0}{\frac{e}{m} j_y} \right] (T \sin T - \cos T + 1) \end{aligned} \right\} \quad (3.29)$$

The analysis of the first two equations of system (3.29) shows that the character of motion of the electrons generally depends on the expression in the square brackets. Here two particular cases are possible. In the first case

$$\frac{\varepsilon_0 \omega^2 \dot{y}_0}{\frac{e}{m} j_y} \ll 1 \quad \text{or} \quad \dot{y}_0 \ll \frac{\frac{e}{m} j_y}{\varepsilon_0 \omega^2} \quad (3.30)$$

and the expression in the square brackets is close to unity. In this case system (3.29) assumes the form

$$\left. \begin{aligned} Y &= T - \sin T \\ Z &= Z_0 + \frac{T^2}{2} - 1 + \cos T \\ \Phi &= \frac{T^2}{2} - T \sin T - \cos T + 1 \end{aligned} \right\} \quad (3.31)$$

In the second case

$$\frac{\varepsilon_0 \omega^2 \dot{y}_0}{\frac{e}{m} j_y} \approx 1 \quad \text{or} \quad \dot{y}_0 \approx \frac{\frac{e}{m} j_y}{\varepsilon_0 \omega^2} \quad (3.32)$$

and the expression in the square brackets is close to zero. Thus system (3.29) is reduced to the form

$$\left. \begin{aligned} Y &= T \\ Z &= Z_0 + \frac{T^2}{2} \\ \Phi &= \frac{T^2}{2} \end{aligned} \right\} \quad (3.33)$$

The value y in equations (3.29) has a sense of the initial velocity of the electron leaving the cathode. According to the law of con-

servation of energy the value of the initial velocity may be characterized by the equivalent potential difference u_0

$$u_0 = \frac{\dot{y}_0^2}{2 \frac{e}{m}} \quad (3.34)$$

The value $\frac{e}{m} j_y / \varepsilon_0 \omega^2$ having a dimension of velocity and compared in (3.30) and (3.32) with the initial velocity can be expressed through

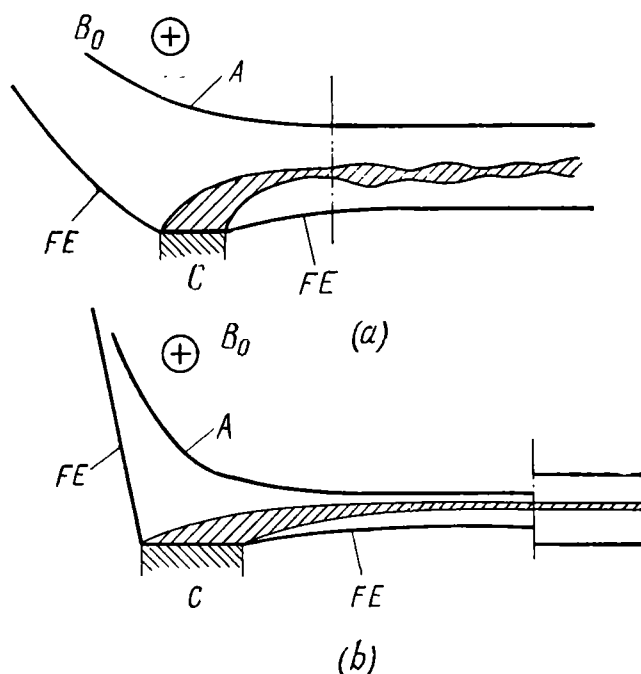


Fig. 3.20. Configuration of electron beam in short-focus (a) and long-focus (b) guns

the equivalent potential difference u_m

$$u_m = \frac{1}{2 \frac{e}{m}} \left(\frac{\frac{e}{m} j_y}{\varepsilon_0 \omega^2} \right)^2 = 1.15 \times 10^8 \frac{j_y^2}{B^4} [V, A/m^2Gs] \quad (3.35)$$

With this estimation we obtain the following relationships: for the first particular case

$$u_0 \ll u_m \quad (3.36)$$

for the second particular case

$$u_0 \approx u_m \quad (3.37)$$

The calculation of the trajectories of the electrons emerging from the cathode edges by means of formulas (3.31) and (3.33) shows that in the first case the beam has pulsating boundaries [the presence of

trigonometric terms in equations (3.31)]. In the second case the beam boundaries are parabolic, i.e. the beam becomes approximately parallel to the axis OZ only at sufficiently large values of Z . Approximate configurations of the beam for both these cases are shown in Fig. 3.20.

As shown in the drawing, in the first case the beam becomes approximately parallel to the axis OZ near the cathode edge, in the other

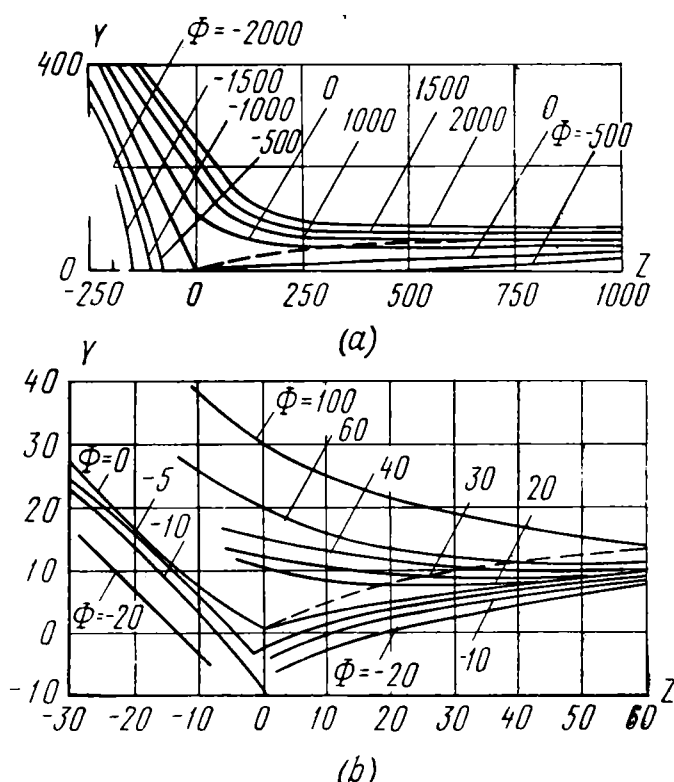


Fig. 3.21. Equipotential surfaces in long-focus (a) and short-focus (b) guns

case the approximately parallel beam is formed at a comparatively large distance from the cathode. In this connection the crossed-field guns forming strip beams are conventionally called short-focus, when condition (3.36) is satisfied, and long-focus, when condition (3.37) is satisfied. In both cases the beam current density can be substantially higher than the specific emission of the cathode, i.e. compression is possible.

When determining the shape of the near-cathode focusing electrodes and the anode, it is necessary to solve the Laplace equation for the region outside the beam. This problem is solved by the method of conformal representation of the external area on the plane of the complex variable.

The graphical solutions in a projection on the plane YOZ are given in Fig. 3.21. The equipotential $\Phi = 0$ during the transition

to the coordinates y, z according to (3.16) determines the shape of the near-cathode focusing electrodes, while any equipotential $\Phi > 0$ (in the coordinates y, z) can be replaced by an electrically conducting surface, i.e. the anode.

The crossed-field guns collimating strip beams are employed in high-power travelling-wave tubes of the M-type. Compared to the Pierce guns forming wedge-shaped beams, the crossed-field guns have somewhat higher noise level.

3.4. Guns with Control Electrodes

Some microwaves devices operate under pulse conditions or under beam current modulation conditions. The pulse operating conditions are provided, when the anode voltage is applied only during a pulse, at the intervals between the pulses the gun is inoperative. Such anode modulation dictates the use of a high-power modulator switching the high anode voltage and, therefore, the total beam power. Much better economy is attained by using the grid modulation at which the modulating voltage ranging from less than 1 % to 10-15 % of the anode voltage is applied to a special control electrode of the gun. In this case the current in the circuit of the control electrode is low and the beam current can be controlled by low-power switching devices.

In principle, the grid modulation is similar to the control of the anode current in electron valves by varying the potential of the control grid resulting in variation of the field in the near-cathode region. The beam current modulation can be easily performed by changing the potential (with respect to the cathode) of the near-cathode focusing electrode, which is not electrically connected to the cathode. When the potential of the near-cathode electrode is negative with respect to the cathode, the accelerating field in the near-cathode region, particularly in the peripheral part of the cathode, adjoining the focusing electrode is substantially attenuated and the cathode current (therefore, the beam current) is reduced. However, the use of a focusing control electrode, particularly in the guns with high perveance and beam compression, is not effective, since cutoff of the beam requires a negative voltage of the focusing electrode commensurable in magnitude with the accelerating anode voltage. The weak control action of the near-cathode electrode stems from high permeability of such a "control grid". The modulation efficiency can be increased by placing a diaphragm between the cathode and anode. The diaphragm has an aperture the radius of which is somewhat smaller than that of the cathode, i.e. in this case we employ the cathode—modulator—focusing electrode (anode) optical system of the electron gun of cathode-ray devices (see Section 4.3). In this case, however, the modulator cutoff voltage is equal to 50-60 % of

the anode voltage. The cause of poor control action of the near-cathode electrode is rather high permeability of the modulator which, in turn, is due to the fact that the radius of the aperture in the modulator diaphragm cannot be made much smaller than the cathode radius. A decrease in the radius of the modulator aperture inevitably results in a drop of the beam current and reduction of the perveance. Therefore, the control of the beam current by means of a near-

cathode focusing electrode or a special diaphragm (modulator) in the guns forming intensive beams has not gained wide recognition.

The guns having control electrodes in the form of a rod or a grid (Fig. 3.22) used for modulating the beam current have found practical application.

In the first case an axisymmetric concave cathode has a central

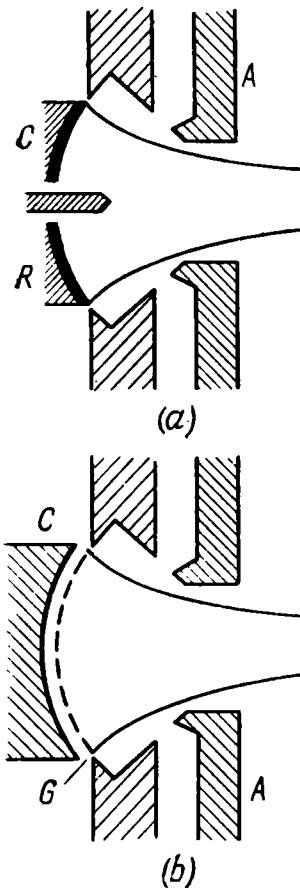


Fig. 3.22. Guns with rod (a) and grid (b) control electrode

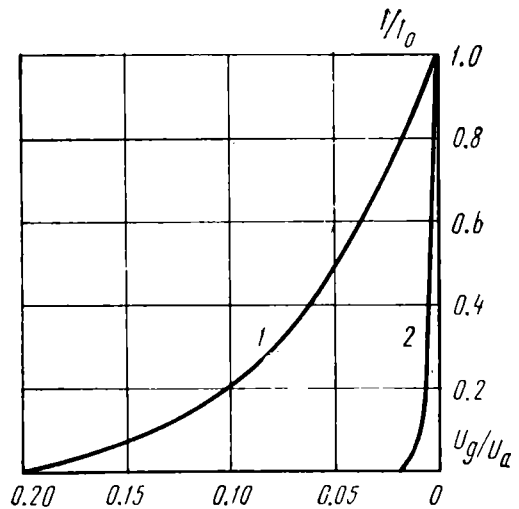


Fig. 3.23. Modulation characteristics of guns with rod (1) and grid (2)

aperture into which is entered a cylindrical rod insulated from the cathode and terminated in a cone. The conical end of the rod extends beyond the boundary of the emitting surface of the cathode. The rod is electrically connected to the near-cathode focusing electrode. In the other case a fine-mesh grid is mounted near the cathode surface, the edges of the grid being secured on the focusing electrode. In the guns with a rod the modulating voltage (which is negative with respect to the cathode) is equal in magnitude to 15-20 % of the anode voltage. In the guns with a control grid the modulating voltage does not exceed 1-2 % of the anode voltage. As an example, Fig. 3.23

illustrates in dimensionless coordinates the modulation characteristics of the guns with a rod and with a grid.

As shown in the drawing, the control action of the grid is much more effective than the action of the rod shaped control electrode. At the same time, both these systems are more effective than systems in which the current is controlled by a focusing electrode and a diaphragm modulator.

In spite of the high efficiency of controlling the beam current, the systems with control grids cannot be used in high-power high-perveance guns, since the control grid collects a considerable portion of the electrons emitted by the cathode. The heavy current flowing through the grid circuit results in its heating to a high temperature, since in practical constructions it is difficult to withdraw the heat from the grid. Such a high-power gun cannot operate without grid currents ($U_g < 0$), like an ordinary valve, because the accelerating field near the cathode surface, even in the case of a high anode voltage, is inadequate for taking off a required current from the cathode due to comparatively small permeability of the grid. The guns collimating intensive beams are rendered fully conducting only with the grids at a positive potential with resultant heavy grid currents.

In the systems with a central rod the potentials of the rod and focusing electrode are zero with respect to the cathode, when the gun is fully conducting. Therefore, such a system operates practically with no current take-off.

The calculation of the guns with control electrodes is much more complicated as compared with the calculation of the guns without beam current modulation. The initial data are usually based on a gun with no control electrode and having specified values of perveance, compression and anode voltage. In the case of designing a gun with a control rod the area of the cathode emission surface is calculated with due account for the central aperture. After that the rod is introduced into the gun designed by usual technique. In order to provide for minimum disturbance of the conditions of forming an electron beam in the near-cathode region by the presence of the rod, the angle of inclination of the generatrix of the conical part of the rod to the boundary of the electron flow is selected within the range of $65-70^\circ$. When the rod and focusing electrode are at zero potential (with respect to the cathode), the conditions of collimating the beam differ but slightly from those in the system with no control electrode.

The control action of the rod and the associated focusing electrode can be estimated by the static amplification factor μ similar to the same parameter in electron valves. Calculation of the amplification factor of the system with a rod is difficult. It is known that this factor to a great extent is determined by the geometrical sizes of the rod: the higher the diameter of the cylindrical part and the greater the portion of the conical part extending beyond the cathode surface,

the greater the amplification factor. The final correction of the geometry of the electrodes is usually conducted by the method of simulating in an electrolytic tank taking into account the space charge and by the method of selection with dummy electrodes.

The design of the gun with a control electrode in the form of a grid is also based on an ordinary diode system. Then the so-called intrinsic potential of the grid is determined, i.e. the potential of an equipotential surface located in place of the grid. It is clear that if a grid having an intrinsic potential is placed into the gun, the electric field in the gun will not change and, therefore, the conditions of taking off the cathode current will be invariant. However, since the intrinsic potential of the grid is positive, a portion of the electron flow will be collected by the grid. The fraction of the current intercepted by the grid depends on the filling factor α of the grid, i.e. on the ratio of the area occupied by the grid wires to the total surface of the grid.

Thus, in the presence of the grid, i.e. in a triode gun, the beam current is smaller than the cathode current. Therefore, when designing an initial diode gun, it is necessary to have some current reserve taking into consideration that in the presence of a grid a portion of the cathode current is branched to the grid.

The control action of the grid is determined by the static amplification factor which in the case of compression guns can be calculated by the formulas for a spherical diode. The amplification factor depends on the filling factor of the grid and on the cathode—grid and grid—anode distances.

At a low distance between the grid and cathode the amplification factor can be calculated approximately by a simpler formula for a planar diode

$$\mu \approx \frac{L'_g (d_{ca} - d_{cg}) - \Delta}{T} \quad (3.38)$$

where L'_g is the length of the grid wire per unit of its surface; d_{ca} is the cathode—anode distance; d_{cg} is the cathode—grid distance; Δ and T are functions of the grid filling factor given graphically in Fig. 3.24.

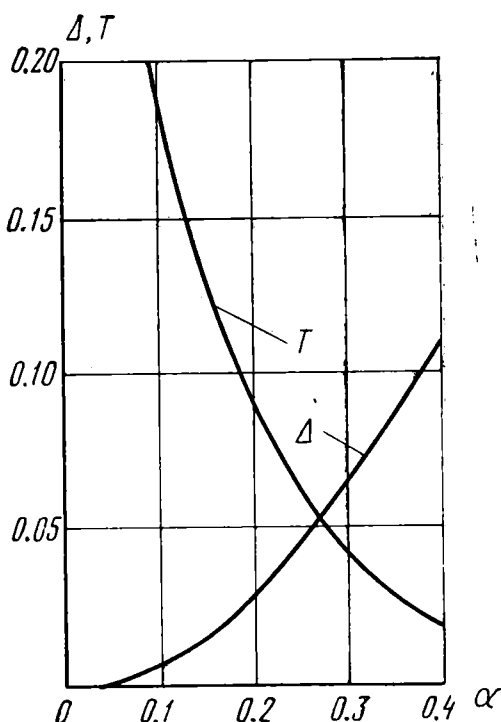


Fig. 3.24. Functions for calculation of amplification factor

The grid filling factor is chosen within the range of 0.1 to 0.25. With greater values of α the portion of the current intercepted by the grid increases materially with resultant drop of the beam perveance. More than that, the heat removal from the grid becomes difficult.

Thus, having selected a filling factor by the specified amplification factor, one can determine the distance between the grid and the cathode using formula (3.38). It should be noted that too low distance between the grid and cathode is inexpedient, since a grid close to the incandescent cathode is heated by the cathode and may be distorted with resultant distortion of the field in the near-cathode region, and disturbance to the conditions under which the beam is collimated. What is more, with the distance d_{cg} commensurable with the spacing between the grid wires (grid pitch), there appears island effect well-known in the theory of electron tubes, which results in a drop of the control action of the grid.

In conclusion, it should be noted that when designing guns with control grids, as well as guns with a rod control electrode, the final adjustment of the system requires correction of the geometrical relations by simulating in an electrolytic tank or by the dummy electrode method.

Part Two

Electron-Beam Devices

Electron Gun

4.1. Applications, General Requirements and Optical System

Many electron-beam devices (oscillograph tubes, camera TV tubes, kinescopes, radar tubes, etc.) employ a narrow, focused electron stream, usually of a circular cross section, the so-called electron beam. The electron beam is produced by one of the essentials of an electron-beam device—an electron gun (the gun for collimating high-perveance beams was described in Chapter 3).

The design and parameters of electron guns depend on the field of application. Their parameters and construction features may greatly differ. For example, the accelerating voltage may vary from a few hundred volts in small oscillograph tubes to tens of kilovolts in projection kinescopes, while the beam current may vary from fractions of one microampere in some camera tubes to several milliamperes in kinescopes with a large screen.

Regardless of the electrical parameters and construction features, the guns must meet some general requirements.

The electron-optical system collimating the beam (focusing electrode system) must provide a minimal cross-sectional area of the beam on the electron receiver (screen or target). In many cases the beam diameter must not exceed fractions of one millimetre, while in some devices it must be not less than 0.1 mm. The gun should be adapted for continuous control of the beam current from zero (the gun is cut off) to a maximum value determined by the type and field of application of the device. Furthermore, the gun construction must not be too complicated, the gun must be compatible with the apparatus, economical, and have a long service life. The last requirement is particularly important for the guns of sophisticated and expensive electron-beam devices, since a failure of the gun makes the whole device worthless. Finally, the materials used for making the gun elements must have good vacuum properties: they must not evolve vapours or gases and allow heating to relatively high temperatures in the process of welding and evacuating the device. These elements are preferably made of non-magnetic materials, since

the presence of ferromagnetic parts in the region of formation of an electron beam can result in amplification of the stray magnetic fields impeding the focusing or curving of the beam path.

The electron gun collimating a focused electron beam consists of a source of electrons (cathode) and a system of electrodes operating at potentials differing from that of the cathode. The electrode system produces an electric field accelerating and focusing the electrons emitted by the cathode. Furthermore, one of these electrodes functions as a control grid of an electron valve, i.e. it controls the beam current. Since the beam is of a circular cross section, in the majority of electron-beam devices, the electrons are focused by electrostatic fields having rotation symmetry.

As it was noted above, the provision of the minimum cross section of the beam in the screen plane is one of the basic requirements placed on the gun. It is possible to focus the electrons emitted by the cathode by means of a single electron lens (Fig. 4.1a). In this case an image of the cathode is produced in the screen plane.

Let us use the Langrange—Helmholtz theorem (see Section 1.7)

$$r_1 n_1 \tan \gamma_1 = r_2 n_2 \tan \gamma_2$$

Replacing n_1 and n_2 through $\sqrt{U_1}$ and $\sqrt{U_2}$ (U_1 is the potential in the near-cathode region, U_2 is the potential in the image region), and $\tan \gamma_1$ and $\tan \gamma_2$ through γ_1 and γ_2 (restricting ourselves to small angles), we may evaluate the magnitude of the cathode image

$$r_2 \approx \frac{r_1 \sqrt{U_1} \gamma_1}{\sqrt{U_2} \gamma_2} \quad (4.1)$$

Expression (4.1) shows that the cross section of the beam in the screen plane (reducing r_2), can be obtained by reducing the cathode size (r_1), the potential in the near cathode region (U_1) and the aperture angle γ_1 at the cathode side. However, the reduction of the numerator of expression (4.1) can hardly be accomplished in practice: the cathode emissivity cannot be as much as desired and, in order to obtain the beam current necessary for normal operation of the device, one has to use a cathode with a comparatively large emissive surface. Reduction of the potential in the near-cathode region

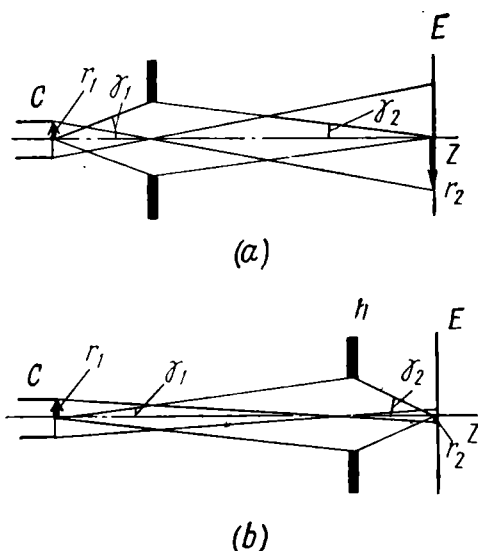


Fig. 4.1. Optical diagram of single-lens gun

is inexpedient for two reasons: (1) the lower the potential, the greater the influence of the initial velocities of the electrons, the higher the chromatic aberration of the lens; (2) the slower the electrons, the greater the effect of space charge leading to expansion of the beam due to the Coulomb repulsive forces.

A decrease in the aperture angle at the cathode side results in a drop of the current beam, since, the lower the angle γ_1 , the smaller the fraction of the electrons emitted by the cathode surface passes through the diaphragm aperture.

The value of the cathode image can be reduced by increasing the denominator of expression (4.1). An increase in the potential in the region of the image is possible only to a certain value limited by

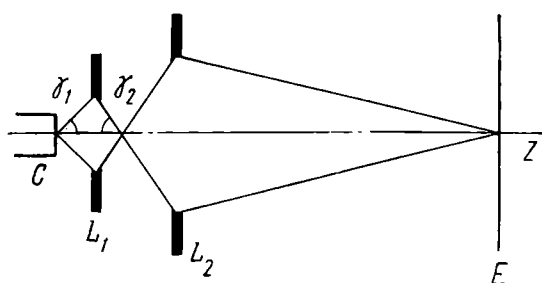


Fig. 4.2. Optical diagram of two-lens gun

the dielectric strength of the device, as well as by the necessity of obtaining sufficient sensitivity when deflecting the electron beam (see Section 5.1). An increase in the angle γ_2 would require a decrease in the distance from the lens to the screen (Fig. 4.1b) and this, again, is practically inexpedient, since, first, it is necessary to have adequate space between the focusing system and the screen for mounting the deflecting elements and, second, when the deflecting elements are shifted closer to the screen, the efficiency of their deflecting action is sharply reduced.

The considerations given above show that the gun with a single electron lens cannot provide for a beam having a small cross-sectional area in the screen plane when the requirements for obtaining sufficient beam current and sensitivity of deflection systems are satisfied. Consequently, the electron guns based on a single-lens optical scheme have not found wide application in electron-beam devices.

Most of modern electron-beam devices employ a two-lens optical scheme (Fig. 4.2). When a short-focus lens is used as a first one, it is possible to obtain a sufficiently great angle γ_2 and, therefore, a reduced cathode image. The second, relatively weak lens should be adjusted so as to obtain on the screen an image of the least cross section of the beam formed between the first lens and the cathode image created by this lens. The use of the two-lens technique provides

a simple means for obtaining a beam with a radius of the order of 0.1 mm in the screen plane with a radius of the emissive cathode surface of about 1 mm.

An approximate view of the paths of the electrons in the electron gun built around a two-lens optical system is given in Fig. 4.3.

The first lens of the gun, besides the focusing, must accelerate the electrons, i.e. the field of the lens must be extended to the cathode surface and collect the electrons emitted thereby. Thus, the cathode is "immersed" in the field of the first lens which is an immersion

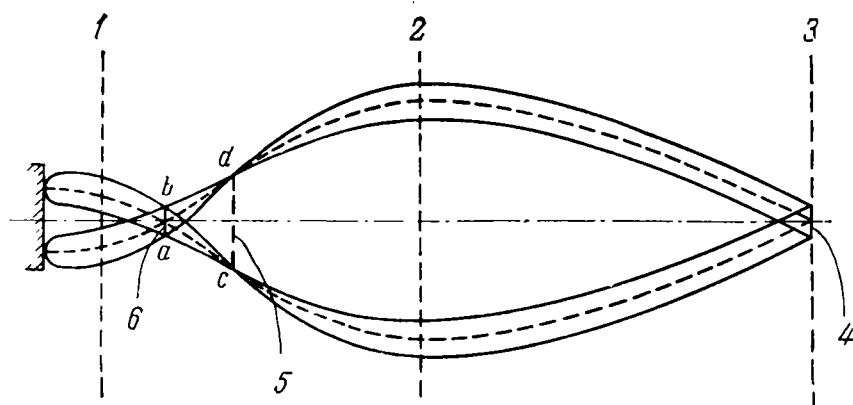


Fig. 4.3. The path of rays in gun

1—first lens; 2—second lens; 3—screen; 4—crossover image spot; 5—cathode image; 6—beam crossover

(bipotential) lens from the optical point of view. Since the first lens must produce a field accelerating the electrons in the near-cathode region, it is clear that this lens must be electrostatic. The second lens of the gun may be electrostatic or magnetic. In this connection, the guns with electrostatic focusing have the following optical scheme: immersion objective + immersion lens or immersion objective + single lens, while the guns with magnetic focusing have the following scheme: electrostatic immersion objective + magnetic lens.

In some types of electron-beam devices, for example in kinescopes with large screens, the gun is built around a three-lens system: immersion objective + immersion lens + single lens. The use of an intermediate immersion lens between the immersion objective and the main focusing lens makes it possible to reduce the beam divergence angle and its cross-sectional area at the input of the main lens with resultant decrease in the aberrations of the main lens, therefore, resultant decrease of the cross section of the beam in the screen plane.

Thus, the use of a two- or three-lens optical system allows one to build electron guns well satisfying the basic requirements given at the beginning of this Section. A distinctive feature of such guns is in that the cross section of the beam in the screen plane is deter-

mined not by the area of the emissive surface of the cathode but by the crossover radius which, as it will be shown below, can be much lower than the cathode radius.

4.2. First Lens of Gun

As indicated above, the first lens of the gun is an immersion objective from the optical point of view. The task of the immersion objective is to form a crossover which is an object for the second lens. Furthermore, the first lens usually has a means for controlling the beam current.

The complete calculation of an immersion lens is associated with considerable difficulties. However, taking some simplifying assumptions, one can approximately find the shape of the paths of the

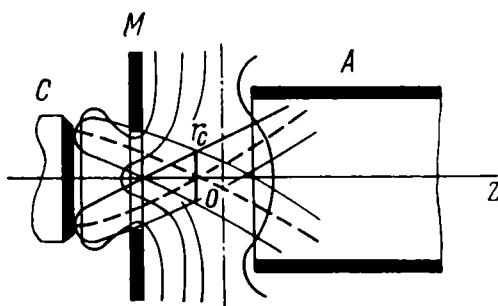


Fig. 4.4. Field in near-cathode region of electron gun

electrons emerging from the cathode and determine the crossover radius. Assume that the electrons are emitted by the cathode at a certain initial velocity v_0 or with energy $\frac{mv_0^2}{2} = 2kT$ (k is the Boltzmann constant, T is the cathode temperature). The initial energy of the electrons can conveniently be expressed through an equivalent potential difference on the basis of the law of conservation of energy as $\frac{mv_0^2}{2} = eu_0$. For the cathode operating at a temperature of 1000°K the value of u_0 is equal to about 0.17 volt. Assume that the lens is ideal, i.e. has no aberrations. Finally, ignore the Coulomb repulsive force acting on the electrons in the crossover region.

Now let us consider the picture of the electrostatic field in the near-cathode region of the gun (Fig. 4.4).

As shown in the drawing, this field is formed by three electrodes: cathode C whose potential is taken equal to zero, diaphragm (modulator) M at a small negative potential, and positively charged electrode A —the first anode or an accelerating electrode. Of course, in order to obtain a beam of a circular cross section, we need a field featuring axial symmetry; in other words, all the electrodes of the gun must be coaxial bodies of revolution.

If the electrons leaved the cathode without initial velocities, they would emerge from the cathode along paths close to the field lines of force near the cathode. Such paths are shown in Fig. 4.4 by dashed lines.

It is shown that they intersect the axis at the same point O , i.e. the crossover radius is zero. The presence of initial velocities results in that the real paths intersect the axis nearer or further than the point O , depending on the direction of the initial velocity. In this

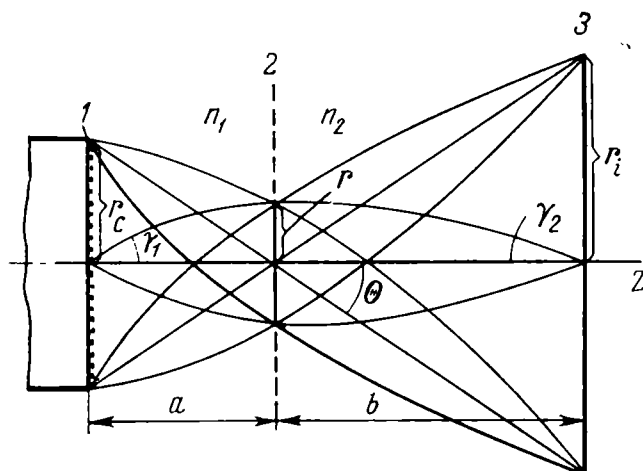


Fig. 4.5. Formation of electron trajectories crossover
1—cathode; 2—crossover plane; 3—cathode image

case within the plane passing through the point O the electron paths lie inside the circle with a radius r_c . It is clear that the value of r is the crossover radius.

To determine the crossover radius, use the Lagrange—Helmholtz theorem [see (1.154)]. Since in the case under consideration one of the aperture angles (at the cathode side) can vary from 0 to 90° , the Lagrange—Helmholtz equation is preferably presented in the form

$$r_c n_1 \sin \gamma_1 = r_i n_2 \sin \gamma_2 \quad (4.2)$$

where r_c and r_i are the radii of the cathode and image, respectively; n_1 and n_2 are refraction indices at the cathode and image sides; γ_1 and γ_2 are the aperture angles.

The formation of the crossover of the electron paths is shown in Fig. 4.5, from which we find that the crossover radius is

$$r = b \tan \gamma_2 \quad (4.3)$$

On the other hand, the image radius

$$r_i = b \tan \Theta \quad (4.4)$$

Restricting ourselves to small angles, we may approximately consider that $\sin \gamma_2 \approx \tan \gamma_2$. In this case

$$r = b \sin \gamma_2 \quad (4.5)$$

Using the Lagrange—Helmholtz theorem (4.2), we obtain

$$r = \frac{r_c n_1 \sin \gamma_1}{n_2 \tan \Theta} \quad (4.6)$$

In expression (4.6) the refraction index n_1 in the cathode plane is equal to $\sqrt{u_0}$.

The potential in the crossover plane is usually close to that in the image plane, therefore, we may consider that $U_i \approx U_c$ and $n_2 = \sqrt{U_c}$. Thus, the radius of the least cross section of the beam (crossover radius) is

$$r = \frac{r_c}{\tan \Theta} \sqrt{\frac{u_0}{U_c}} \sin \gamma_1 \quad (4.7)$$

Since the direction of the initial velocities is equiprobable in the whole range of γ_1 from 0 to 90° , the maximum value of $\sin \gamma_1$ is equal to unity and the maximum crossover radius for an electron beam with initial energy eu_0

$$r = a \sqrt{\frac{u_0}{U}} \quad (4.8)$$

where $a = r/\tan \Theta$ is the distance from the cathode surface to the crossover plane.

Expression (4.8) shows that in a first approximation the crossover radius does not depend on the area of the emissive surface of the cathode and is determined solely by the ratio of the initial energy of the electrons to the energy of the electrons in the crossover region. Equation (4.8) was derived under an assumption that all the electrons emerging from the cathode have the same initial velocity. In this case the crossover is represented by a circle with a radius having a sharp boundary.

The constancy of initial velocities is rather a rough assumption, since the electrons emitted by the thermocathode have a Maxwell velocity distribution. Consequently, (4.8) should be considered as an equation for determining the radius of a crossover formed by a group of electrons having initial energy eu_0 and emerging from the cathode parallel to its surface. It is obvious that all electrons having energies lower than eu_0 or a direction of the initial velocity different from the perpendicular to the axis, will intersect the crossover plane inside a circle with a radius r . The electrons having higher initial velocities will intersect the crossover plane outside the circle having the above radius. In other words, each group of electrons with similar initial velocities corresponds to a specific

circle in the crossover plane and the higher the initial velocity, the greater the radius of this circle.

These considerations show that the concept "crossover radius" is arbitrary, since among all the electrons emitted by the cathode there always are those having an initial velocity considerably exceeding the value $\sqrt{\frac{2e}{m}}u_0$. However, the fraction of high-speed electrons in the total electron flow decreases with an increase in the initial velocity according to the Maxwell velocity distribution. This results in that the density of the electron flow in the crossover plane is nonuniform: the highest density is at the axis of the system and this density decreases exponentially with the distance from the axis. Therefore, the crossover value can conventionally be estimated by a radius of a circle along which the current density constitutes a small part (not larger than 0.1) of the current density on the axis.

In order to estimate the crossover value, one has to know the current distribution in the crossover, depending on the distance from the axis. According to the Maxwell law the number N of electrons emitted from 1 cm^2 of the cathode surface per unit of time and normal to this surface, said electrons being related to a unit of a solid angle and having an initial energy from eu to $e(u + du)$, is equal to

$$N(u) du = N_0 \frac{eu}{kT} e^{-\frac{eu}{kT}} d\left(\frac{eu}{kT}\right) \quad (4.9)$$

where N_0 is the number of electrons having all possible energies and emitted by the cathode per unit of time from 1 cm^2 of its surface and normal thereto in a unit of the solid angle; k is the Boltzmann constant; T is the absolute temperature of the cathode.

The value N_0 can be determined by the density of the emission current of the cathode

$$j_c = e\pi N_0 \quad (4.10)$$

Assuming that the electron emission of the cathode obeys the Lambert law (it has been proved experimentally), i.e. assuming that the electron current in any direction is proportional to the cosine of the angle between the normal to the cathode surface and the given direction, the current in the solid angle from γ to $\gamma + d\gamma$ is equal to

$$dI_\gamma = 2\pi Q N e \sin \gamma \cos \gamma d\gamma \quad (4.11)$$

where Q is the area of the emissive surface.

Consider the circular zone in the crossover plane between the circles with radii r_c and $r_c + dr_c$ (Fig. 4.6).

This circular zone corresponds to the angles γ_1 and $\gamma_1 + d\gamma_1$. Then, according to (4.11), the current flow in the circular zone is

equal to

$$dI_{\gamma_1} = 2\pi QN_1 e \sin \gamma_1 \cos \gamma_1 d\gamma_1$$

where N_1 is the number of electrons emitted by the cathode with energies from eu_0 to $e(u_0 + du_0)$.

The current density in the circular zone

$$j_{r_1} = \frac{dI_{\gamma_1}}{d(\pi r^2)} = \frac{QN_1 e \sin \gamma_1 \cos \gamma_1 d\gamma_1}{2rdr} \quad (4.12)$$

Determine rdr from (4.7) and substitute it into (4.12). In this case we have

$$j_{r_1} = \frac{QN_1 e}{a^2} \times \frac{U_c}{u_0} \quad (4.13)$$

Equation (4.13) shows that in the circular zone the density of the current produced by electrons having energies from eu_0 to $e(u_0 + du_0)$ does not depend on the electron departure angle γ .

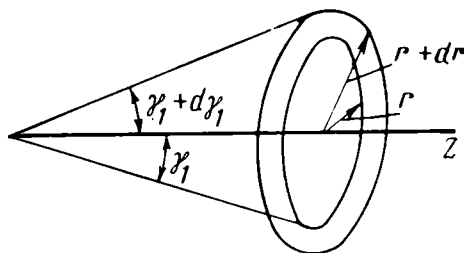


Fig. 4.6. On calculation of current density in crossover plane

In order to determine the full current density in the crossover, it is necessary to sum up the densities of the currents of all crossovers formed by the electrons with energies higher than eu_0 (in this case it is assumed that all crossovers are formed in a single plane). Taking into account the Maxwell velocity distribution, we obtain

$$j_r = \frac{Qe}{a^2} \int_{u_0}^{\infty} \frac{U_c}{u_0} N(u_0) du_0 \quad (4.14)$$

Express u_0 from (4.8), substitute it into (4.14) and make the integration

$$j_r = QN_0 e \frac{1}{a^2} \left(1 + \frac{eU_c}{kT} \right) e^{-\frac{eU_c}{kT}} \times \frac{r^2}{a^2} \quad (4.15)$$

As shown in equation (4.8), the crossover radius is the lower, the higher the potential in the crossover region. In practice the crossover is formed at a potential not lower than several hundred volts. Assuming that the cathode operates at a temperature of 1000°K , we may

estimate the value eU_c/kT

$$\frac{10^2 \text{ eV} \times 1.6 \times 10^{-19} \text{ Joule/eV}}{1.38 \times 10^{-23} \text{ Joule/degree} \times 10^3 \text{ degree}} \approx 1.2 \times 10^3 \gg 1$$

Using this estimation, we may simplify expression (4.15) as follows

$$j = j_0 e^{-\frac{r^2}{\kappa}}$$

where $j_0 = QN_0 \frac{e^2}{a^2} \times \frac{U_c}{kT}$ is the current density in the crossover centre ($r = 0$); $\kappa = \frac{a^2 kT}{eU_c}$ is the constant for the given conditions of operation of the gun.

The current density in the crossover centre (j_0) is associated with the density of cathode emission current (j_c) through the equation (Langmuir formula)

$$j_0 = j_c \left(\frac{eU_c}{kT} + 1 \right) \sin^2 \Theta \quad (4.17)$$

which can be obtained from (4.15) when replacing a^2 by $r_c \sin^2 \Theta$ (taking into account that for small angles $\sin \Theta \approx \tan \Theta$) and substituting N_0 from (4.10).

From equation (4.17) it follows that an increased current density in the crossover centre can be obtained at the expense of increasing the specific emission of the cathode while simultaneously decreasing its operating temperature. It is clear that in the case of thermal cathodes these requirements are contradictory, since a decrease in the cathode temperature inevitably causes a decrease in the emission current density.

Cathode-ray tubes have been recently developed which have cold (autoelectronic) cathodes providing for adequately high current from a very small surface of the emitter, i.e. high density of the emission current. Such cathodes surpass the thermal cathodes by many parameters (low temperature, high density of the emission current, high economy) and are promising for electron guns of many electron-beam devices. However, their low stability and short service life do not allow them to be used in mass-production cathode-ray tubes.

The crossover radius can theoretically be evaluated on the basis of the obtained equations assuming that the crossover boundary is a circle along which the current density constitutes a certain fraction of the current density at the crossover centre. For example, taking $j_r \leq 0.1 j_0$, an operating temperature of the cathode of 1000°K and practically acceptable potential in the crossover area, one can easily obtain a theoretical crossover radius of $10 \mu\text{m}$.

The experimental study of the crossover radius and the distribution of the current density through the crossover section is associated with many difficulties. Consequently, direct experimental checking of formulas (4.8), (4.15) and (4.17) can hardly be realized.

However, since the second lens of the gun can be adjusted so that an enlarged image of the crossover is obtained on the screen, the crossover parameters can be estimated by studying the trace of the electron beam (luminous spot) on the screen. Of course, trustworthy results can be obtained only in the case when the optical parameters of the second lens are accurately determined and the second lens itself has sufficiently low aberrations. Furthermore, during these measurements it is necessary to provide a passage of the whole electron beam emerging from the crossover plane to the receiver (screen), i.e. there should be no diaphragms limiting the beam section in the region of the second lens. The crossover parameters estimation in terms of the parameters of the spot on the screen is usually made

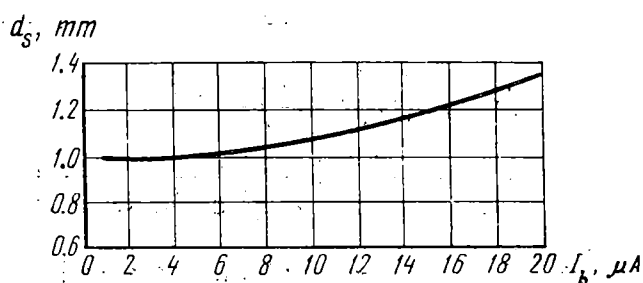


Fig. 4.7. Spot diameter versus beam current

using the guns with magnetic focusing, which employ a low-aberration magnetic lens as the second lens. The guns with magnetic focusing may have no limiting diaphragms while preserving good focusing.

The estimation of the crossover parameters by the experimentally found parameters of the spot on the screen shows that the crossover radius is practically several times the design value (30-100 μm), while the current density at the crossover centre is substantially lower than the theoretical value. Furthermore, the experiment shows that the radius of the spot on the screen and, therefore, the crossover radius do not depend on the beam current (as it follows from the given theoretical considerations) only at very low values of the current (Fig. 4.7). The spot radius (and crossover radius) markedly increase with an increase in the current.

These discrepancies are mainly due to two factors. The approximate theory of forming a crossover is based on an assumption that the electrostatic field in the near-cathode region (and in the crossover region) is uniquely determined by the potentials of the gun electrodes, as well as by their configuration and mutual arrangement, i.e. possible distortion of the field by the intrinsic space charge of the electrons is not taken into account. The effect of the space charge can be ignored only when the value of the space charge coefficient (perveance) is not greater than $10^{-8} A/V^{3/2}$ (see Section 2.1). Actu-

ally, in the near-cathode region (where the potential is close to zero) and in the crossover region having a high current density, the permeance is much greater than the above-said value and the presence of space charge has a substantial effect on the formation of the electron beam.

The theoretical calculation of the effect of the space charge on the formation of the crossover leads to complex integral equations which can be solved only for some particular cases.

Qualitatively, the presence of a perceptible effect of space charge in the crossover region results in appearance of a radial component of the electric field producing the force acting on the electrons in the direction from the axis (the Coulomb repulsive force, see Section 2.1). The repulsion of the electrons results in expansion of the beam, which is also observed as an increase in the crossover radius. In the same manner, the repulsive action of the space charge results in displacement of a portion of the electrons from the near-axis region of the beam thus reducing the current in the crossover centre.

The second factor which is not taken into account in the approximate theory of crossover formation is the aberrations of the immersion objective, first of all spherical aberration, observed even at low beam currents when the electrons are taken off only from the central part of the cathode surface. As the current increases, the electrons are taken off from a larger area of the cathode surface, the conditions of paraxiality near the cathode are disturbed and other kinds of geometric aberrations such as astigmatism, coma, distortion become noticeable. Therefore, a rise in the beam current is accompanied by a noticeable increase in the size of the crossover. Of course, the first factor, i.e. the repulsive action of the space charge, has a greater influence at higher beam currents.

As noted above, theoretically it is difficult to take into account the influence of the action of the space charge. Just the same the calculation of the aberrations of an immersion objective cannot be made analytically. Therefore, the guns are often calculated on the basis of approximate expressions, which are then corrected with due account for the space charge and the geometric aberrations, using the experimental data obtained in the guns of a similar design.

The beam divergence angle beyond the crossover plane is one of the parameters of the first lens of the gun. The value of this angle has influence on many parameters of an electron-beam device, for example, the size of the spot on the screen, the aberrations of the deflecting systems, and, as a result, the resolving power of the device.

Outside the crossover plane the electrons fly along paths slightly curved to the axis (Fig. 4.8).

This curving is caused by the fact that in real systems the crossover is formed at a potential which is somewhat lower than the anode

potential. Consequently, a small accelerating and focusing field is always present between the crossover plane and the anode.

We may approximately consider that the apex of the cone of the electron beam emerging from the crossover plane lies on the gun axis in the plane of the modulator diaphragm. On the basis of the Lagrange—Helmholtz theorem (4.2) we may arrive at a conclusion that, in order to obtain the least luminous spot on the screen, the aperture angle is preferably reduced at the crossover plane, i.e. the beam divergence angle outside the crossover plane.

Furthermore, the cross section of the beam in the region of the second lens and in the region of the deflecting systems depends on

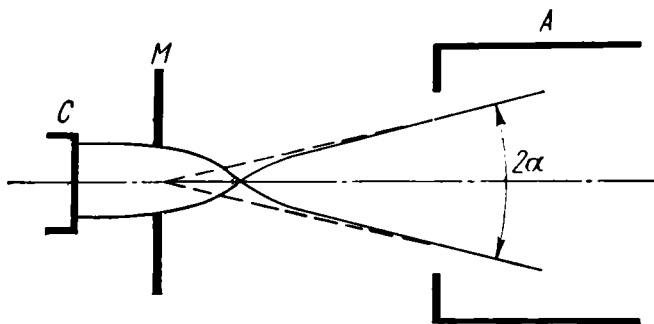


Fig. 4.8. Divergence of beam beyond crossover plane

the value of the beam divergence angle beyond the crossover plane. The larger this angle, the wider the beam and the greater the influence of the aberrations of the second lens and the deflecting systems (see Section 5.4). Thus, a decrease in the beam divergence angle outside the crossover plane is very expedient.

It has been found experimentally that the beam divergence angle greatly depends on the operating voltage in the plane of the modulator aperture $U_{em} = U_m - U_{m0}$ (U_{m0} is the modulator cutoff voltage, see Section 4.3). Furthermore, the divergence angle depends on the radius of the modulator aperture and the distances cathode—modulator and modulator—anode (accelerating electrode). The value of the beam divergence angle beyond the crossover plane can be approximated by means of a simple semiempirical formula proposed by Moss

$$\tan \alpha \approx \alpha \approx 0.6 \frac{R_m}{d_{ma}^{3/4}} \times \frac{U_m - U_{m0}}{U_{m0}} \quad (4.18)$$

where R_m is the radius of the modulator aperture; d_{ma} is the modulator—anode distance.

According to (4.18) there is linear dependence between the beam divergence angle and the modulator voltage, which is satisfied approximately at a negative modulator voltage having a sufficiently

high absolute value. As the value U_m approaches zero, formula (4.18) becomes more inaccurate.

The calculations of the electron paths in an immersion objective made with the aid of an electronic computer and the experimental

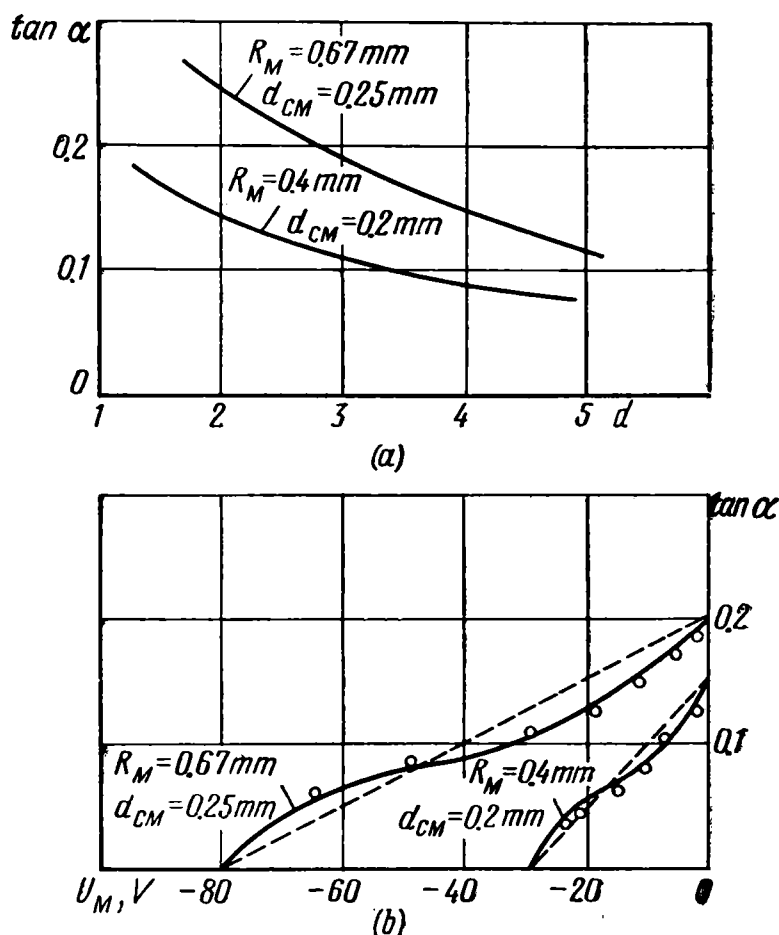


Fig. 4.9. Beam divergence angle *versus* modulator-anode distance d in mm (a) and modulator voltage (b)

study conducted by V. Martynova and V. Tsyganenko have shown that the beam divergence angle beyond the crossover plane can be defined much more accurately using the expression

$$\tan \alpha = \tan \alpha_0 \frac{r_c}{R_m} e^{0.6 \left(\frac{U_m - U_{m0}}{U_{m0}} \right)} \quad (4.19)$$

where

$$\tan \alpha_0 = \frac{0.6 R_m^{1/4}}{(d_{cm} + \delta_m)^{1/4} \gamma_0^{3/4}}$$

$$\gamma_0 = \left\{ 1 - \frac{2}{\pi} \left[\arctan \left(\frac{d_{cm} + \delta_m}{R_m} \right) + \frac{(d_{cm} + \delta_m) R_m}{R_m^2 + (d_{cm} + \delta_m)^2} \right] \right\} \times \frac{U_a}{U_{c0}}$$

where r_c is the radius of the effective cathode surface [see (4.54)]; d_{cm} is the cathode—modulator distance; δ_m is the thickness of the diaphragm (modulator).

Figure 4.9 illustrates the dependence of the beam divergence angle beyond the crossover plane ($\tan \alpha$) on the modulator—anode distance and the modulator voltage calculated by formulas (4.18) (a straight dashed line) and (4.19); the circles indicate the values of $\tan \alpha$ obtained experimentally.

As shown in the drawing, formula (4.19) rather accurately describes the dependence of the beam divergence angle on the modulator voltage observed in practice.

It should be noted that the above data on the beam divergence beyond the crossover plane have sense only when there is no diaphragms limiting the beam section in the space between the crossover and the second lens. In the presence of such diaphragms the angle under which the outer electrons of the beam enter the second lens is determined by the radius of the limiting diaphragm and its distance from the modulator plane. However, the beam divergence angle is expediently reduced in the presence of limiting diaphragms as well, since in this case a comparatively lower part of the electron flow is cut off by the diaphragm, i.e. the utilization of the electron beam is improved.

The beam divergence angle is reduced considerably when between the immersion objective and the main focusing lens there is an additional weak immersion lens which to some degree focuses the beam diverging beyond the crossover plane. As mentioned above, this is one of the advantages of the guns employing a three-lens optical system.

4.3. Cutoff Voltage and Modulation Characteristic

As a rule, the beam current in electron-beam devices can be controlled in a wide range. As the case is with electron valves, it is expedient to control the current by varying the electrostatic field near the cathode, i.e. in the region where electron velocities are low and a small variation of the field can considerably change the character of motion of the electrons and their paths. More than that, it is good to control the beam current without power consumption.

Since the cathode of the gun is “immersed” into the field of the immersion objective, it is obvious that the current of the cathode (therefore, the beam current) can be easily controlled by varying the field of the immersion objective. This can be done very simply by changing the potential of the electrode which is nearest to the cathode.

In the majority of guns of electron-beam devices the field in the near-cathode region is defined by the configuration, mutual arrangement and potentials of the three electrodes: the cathode itself at potential U_c , diaphragm having a circular aperture, called a modulator, at a potential U_m slightly negative relative to the cathode, and an anode or accelerating electrode in the form a cylinder or a disc with a circular aperture at a high positive potential U_a .

Any change in the modulator potential substantially affects the field in the near-cathode region and, as a result, the cathode current (and the beam current). Since the modulator potential is negative with respect to the cathode, the current in the modulator circuit is practically equal to zero, i.e. the control is effected without any power loss. Thus, the modulator is an analog of the control grid of an electron valve. But, in contrast to electron valves, the modulator, besides controlling the beam current (or rather the cathode current), takes part in the formation of the focusing field of the immersion objective, i.e. the variation of the modulator potential results in a change in the optical properties of the first lens. A change in the modulator potential results in a change in the focal length of the first lens, thus varying the position of the crossover plane. An increase in the modulator potential results in an increase in the distance between the cathode and the crossover plane. Therefore, when changing the modulator potential for controlling the cathode current, it is necessary to "trim" the second lens of the gun to obtain the minimum spot on the screen (see Section 4.4).

Since the effect of the modulator potential on the field in the near-cathode region is similar to the action of the voltage of the control grid of a triode, we may assume that the dependence of the cathode current of the gun on the modulator potential is described by the known equation of the three halves power law like the case is with electron valves. However, the experiment shows that the change in the cathode current of the gun due to a change in the modulator potential differs from the three halves power law even in a very rough approximation. This can be explained as follows.

On deriving the three halves power law for electron valves, it is assumed that the electrons can emerge from the whole surface of the cathode heated to the operating temperature, i.e. the area of the working surface of the cathode is equal to the area of its geometric surface. The variation of the cathode current during a change in the voltage of the control grid is explained by the variation of the field near the whole surface of the cathode. When a retarding field is created near the whole operating surface of the cathode, the electron valve is cut off. In other words, on deriving the three halves power law, it is assumed that only the remote, averaged grid field acts near the cathode surface. A closer field is provided only in valves with loose grids close to the cathode, which often cause

an island effect. In the presence of the island effect the equation of the three halves power law loses its meaning.

When discussing the operation of an electron gun with a modulator playing the role of a control grid, the field near the cathode cannot be regarded as remote even at very rough approximation. When the modulator is negative with respect to the cathode, the field is retarding at a considerable portion of the cathode and the electron can emerge only from a small central area of the cathode where there is still a positive potential gradient. In this case the area of the

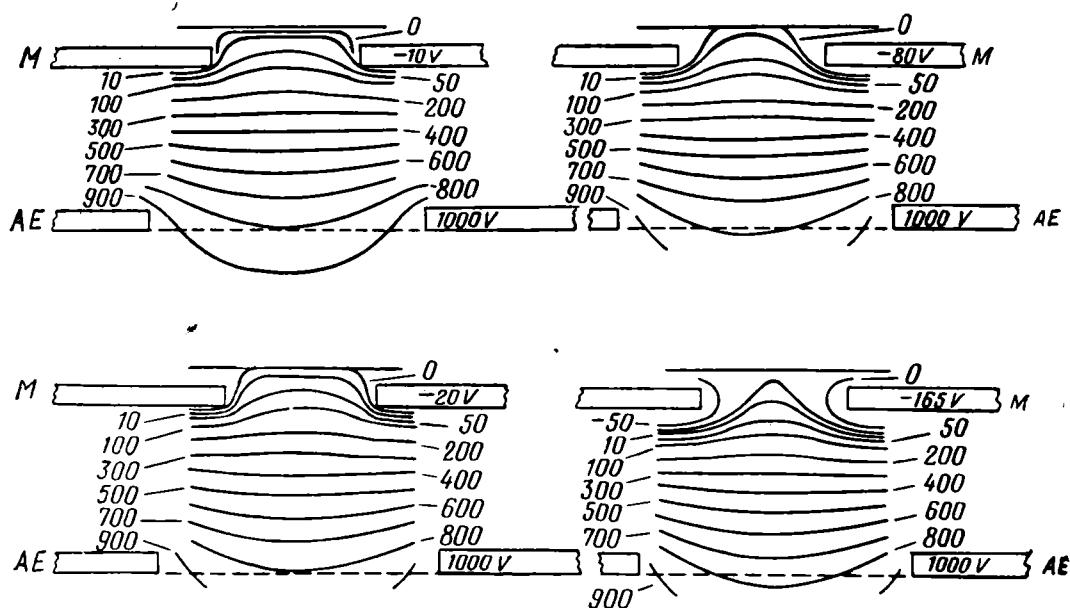


Fig. 4.10. Change in field near cathode surface when changing modulator potential

working surface of the cathode is much less than its geometric surface. With an approach to the cutoff voltage the cathode working area is reduced to a point and during the cutoff state the retarding field is produced at the whole surface of the cathode. The variation of the field near the cathode surface during the change in the modulator potential is shown diagrammatically in Fig. 4.10.

As shown in the drawing, the field near the cathode surface becomes more or less uniform only at a modulator potential close to zero or higher than zero. The condition $U_m > U_c$ is in principle possible but practically useless due to the fact that the greatest part of the electron flow is branched to the modulator unnecessarily overloading the cathode. What is more, when a positive modulator is placed near the cathode, it easily saturates and the cathode current control becomes ineffective.

A change in the working surface of the cathode with a change in the modulator voltage results in a higher rise of the cathode current with a simultaneous decrease in the magnitude of the modulator voltage. Owing to this fact the dependence of the cathode current of the gun on the modulation voltage (modulation characteristic) is described by an exponential function with an exponent higher than $3/2$.

Analytic calculations of the current characteristics of the gun with due account for all the factors affecting the emissive capacity of the cathode are associated with considerable difficulties. Therefore, several approximate relationships describing the dependence of the cathode current on the anode and modulator voltages have been proposed. Thus, for example, assuming that the radius of the operating surface of the cathode, r_c , is connected with the radius of the modulator diaphragm, R_m , through the relationship

$$r_c = \frac{(U_m - U_{m0})^{1/2}}{|U_{m0}|^{1/2}} R_m \quad (4.20)$$

where U_m is the modulator voltage (with respect to the cathode); U_{m0} is the cutoff voltage, i.e. the modulator voltage at which the cathode current is zero; then according to the three halves power law with due account for the variable surface of the cathode, its current is

$$I_c = k \frac{(U_m - U_{m0})^{5/2}}{|U_{m0}|} \quad (4.21)$$

In this formula proposed by E. Gundert the coefficient k is equal to 2.8 (I_c [μ A], U_m , U_{m0} [V]). Expression 4.21 satisfactorily describes the experimental dependence $I_c = f(U_m)$ for the guns with a ratio of the cathode-modulator distance d_{cm} to the radius of the modulator diaphragm R_m lying in the range of $0.7 \leq \frac{d_{cm}}{R_m} \ll 1.1$.

According to the data obtained by P. Tarasov, more accurate agreement is obtained with a value of k in formula (4.21) equal to 2.3.

For the guns without diaphragms limiting the beam H. Moss proposed the formula

$$I_c = k \frac{(U_m - U_{m0})^{7/2}}{U_{m0}^2} \quad (4.21a)$$

where $k \approx 3$ (I_c , [μ A], U_m and U_{m0} , [V]).

Generally the dependence of the cathode current on the modulator voltage of the gun can be given by the expression

$$I_c = k \left(\frac{U_m - U_{m0}}{U_{m0}} \right)^\gamma |U_{m0}|^{3/2} \quad (4.22)$$

where $k = 2.3-3$, $\gamma = 2.5-3.5$. The results of the investigations show that the exponent γ for the given gun is not a constant value.

As the cathode current increases, the exponent rises from $\gamma = 2.5$ (with a modulator voltage close to the cutoff voltage) to $\gamma = 3.5$, when the value U_m is close to zero.

The value of the cutoff voltage of the system depends on the geometric relationship of the system of electrodes of the gun: the distances d_{cm} (cathode—modulator) and d_{ma} (modulator—anode), the radius R_m of the modulator diaphragm aperture, the thickness of the modulator diaphragm δ_m , and the anode voltage. The value U_{m0} can be calculated by using semiempirical relationships, in particular the well-known formula proposed by M. Heine

$$U_{m0} = 3.4 \times 10^{-2} \frac{(2R_m - \delta_m)^2}{d_{cm}d_{ma}} U_a \text{ [V]} \quad (4.23)$$

The calculation by this formula gives satisfactory accuracy only at $d_{cm}/R_m \geq 1$.

The theoretical and experimental studies by Yu. Koyfman, V. Tsyganenko, V. Martynova, and P. Tarasov have lately resulted in an approximate theory of calculation of current characteristics of electron guns. These authors have obtained relationships that allow the guns of modern electron-beam devices to be calculated with an adequate practical accuracy, the geometric and electric parameters varying within a wide range.

Analytic calculation of an electrostatic field in the near-cathode region of the immersion objective, according to the works of the above authors, is reduced to the following. The electrostatic potential at any point of a closed region determined by the electrode system can be presented as

$$U(z, r) = \sum_{i=1}^n \varphi_i(z, r) U_i \quad (4.24)$$

where U_i is the potential of the i th electrode.

The functions $\varphi_i(z, r)$ —so-called functions of influence—determine the potential at the point (z, r) with the i th electrode at the potential $U_i = 1$ and all the other electrodes at the potential of $U_j = 0$ ($j \neq i$)

$$U(z, r)|_{U_i=1} = \varphi_i(z, r); U_j = 0 \quad (4.25)$$

The influence functions are interconnected through the relationship

$$\sum_{i=1}^n \varphi_i(z, r) = 1 \quad (4.26)$$

Differentiating expression (4.26) through the coordinates z, r , we obtain

$$\sum_{i=1}^n \frac{\partial \varphi_i(z, r)}{\partial z} = \sum_{i=1}^n \frac{\partial \varphi_i(z, r)}{\partial r} = 0 \quad (4.27)$$

Apply the given relationships to the cathode-modulator-anode system (Fig. 4.11), taking the origin of the potential on the modulator surface, i.e. assuming that $U_m = 0$.

In this case the potential at any point of the interelectrode space is equal to

$$U(z, r) = U_c \varphi_c(z, r) + U_a \varphi_a(z, r) \quad (4.28)$$

where U_c and U_a are the potentials of the cathode and anode respectively;

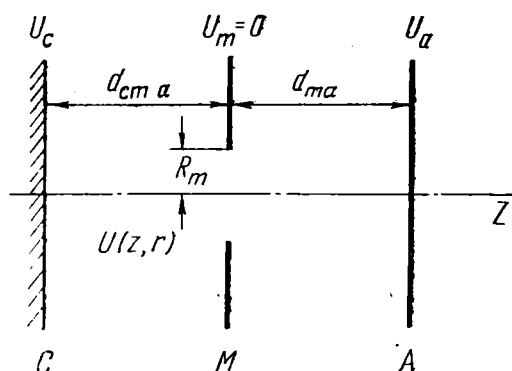


Fig. 4.11. Cathode-modulator-anode system

tively; φ_c and φ_a are the influence functions of the cathode and anode.

The potential gradient on the surface of the cathode ($z = 0$) can be presented in the form

$$E_z(r) = (U_c - U_{cr}) \varphi'_c(0, r) \quad (4.29)$$

where

$$U_{cr} = - \frac{\varphi'_a(0, r)}{\varphi'_c(0, r)} U_a \quad (4.30)$$

$$\varphi'_c(0, r) = \left. \frac{\partial \varphi(z, r)}{\partial z} \right|_{z=0} \quad (4.31)$$

Ignoring the initial velocities of the electrons emitted by the cathode, one makes certain that the current is taken only from that part of the cathode surface, which has a potential gradient of $E_z(r) \geq 0$. In this case the radius of the operating surface of the cathode can be found by solving the transcendental equation

$$U_c = - \frac{\varphi'_a(0, r)}{\varphi'_c(0, r)} U_a \quad (4.32)$$

From the comparison of expressions (4.30) and (4.32) it follows that the value U_{cr} determines the potential of the cathode at which the radius of its operating surface is equal to r . The case $E_z(r) = 0$ corresponds to the gun cutoff. The cutoff voltage

$$U_{c0} = - \frac{\varphi'_a(0, 0)}{\varphi'_c(0, 0)} U_a = - D_c U_a \quad (4.33)$$

at cathode modulation and

$$U_{m0} = -\frac{\varphi'_a(0, 0)}{\varphi'_m(0, 0)} U_a = -D_m U_a \quad (4.34)$$

at ordinary modulation.

The quantities D_c and D_m in expressions (4.33) and (4.34) stand for the permeability when controlling the cathode current by changing the cathode voltage (cathode modulation) and the modulator voltage (ordinary modulation).

The cathode current (under the conditions of limiting the current by the space charge) can be calculated by using the Poisson equation [see (2.48)]

$$\frac{d^2}{dz^2} U(z, r) = \frac{j(r)}{\varepsilon_0 \sqrt{\frac{2e}{m}} U^{1/2}(z, r)} \quad (4.35)$$

where $j(r)$ is the current density at an elementary cathode section with coordinates $(0, r)$.

Assuming that the potential at the near-cathode region features linear dependence on the coordinate z

$$U(z, r) = E(0, r) z \quad (4.36)$$

and introducing a concept of acting (effective) potential U_∂

$$U_\partial = \frac{U_m \varphi'_m(0, r) + U_a \varphi'_a(0, r)}{\varphi'_c(0, r)} \quad (4.37)$$

we shall find the field intensity near the cathode surface

$$E(0, r) = U_\partial \varphi'_c(0, r) \quad (4.38)$$

The solution of equation (4.35) with boundary conditions

$$\begin{aligned} U(z, r) \Big|_{z=0} &= 0, \quad \frac{dU}{dz} \Big|_{z=0} = 0 \\ U(z, r) \Big|_{z=\frac{1}{\varphi'_c(0, r)}} &= U_\partial \end{aligned}$$

has the form

$$U(z, r) = E(0, r) [\varphi'_c(0, r)]^{1/3} z^{4/3} \quad (4.39)$$

and the current density of the cathode elementary section is given by the expression

$$j(r) = \frac{4}{9} \varepsilon_0 \sqrt{\frac{2e}{m}} E^{3/2}(0, r) [\varphi'_c(0, r)]^{1/2} \quad (4.40)$$

The full current is obtained by integrating equation (4.40) within the range from 0 to r [r is the radius of the operating surface of

the cathode found from (4.32)]

$$I = 2\pi \int_0^r j(r) r dr \quad (4.41)$$

Thus, the analytic calculation of the potential distribution and the current characteristics of the gun can be made by finding the influence functions φ_i and their derivatives φ'_i , as well as on the basis of using the given relations for $U(z, r)$, cutoff voltage and cathode current.

The calculation of a triode gun, i.e. a system consisting of a cathode, modulator and anode, can be made under an assumption that the field produced by the electrode system may to sufficient accuracy be regarded as a field of a single diaphragm modulator with an aperture radius R_m placed at distances d_{cm} and d_{ma} from the cathode and anode. The potential distribution in the diaphragm field is found using the known analytic solution [see (1.3)]

$$U(z, r) = \sum_{i=1}^n A_i \left\{ |z + z_i| \left[\arctan \mu(z + z_i, r, R_i) + \frac{1}{\mu(z + z_i, r, R_i)} \right] - \right. \\ \left. - |z - z_i| \left[\arctan \mu(z - z_i, r, R_i) + \frac{1}{\mu(z - z_i, r, R_i)} \right] \right\} \quad (4.42)$$

where

$$\mu(z \pm z_i, r, R_i) = \frac{1}{\sqrt{2}R_i} \times \\ \times \sqrt{(z \pm z_i)^2 + r^2 - R_i^2 + V[(z \pm z_i)^2 + r^2 - R_i^2]^2 + 4(z \pm z_i)^2 R_i^2} \quad (4.43)$$

R_i are the radii of the apertures of the i th electrodes.

The constants A_i are found by solving a system of linear equations.

$$U(z, r) \Big|_{\substack{z=z_i \\ r=R_i}} = U_j, \quad (i \neq j) \quad (4.44)$$

The points z_i, R_i are located at the boundaries of j th electrodes.

System (4.44) is sufficiently stable and during the calculations we may restrict ourselves to a small amount (7-12) of the points z_i, R_i .

System (4.44) is solved comparatively simply by means of electronic computers with a low consumption of machine time. After that the influence functions and their derivatives are found by the known potential distribution and the current characteristics of the gun are calculated by the above-given formulas.

A tetrode gun having an additional accelerating (or shield) electrode between the modulator and anode is calculated in a similar way.

The use of computers in engineering design calculations is not always convenient. Therefore, the authors of the above works pro-

posed a number of more simple formulas approximating the basic relationships with sufficient accuracy. The investigations have shown that the maximum radius of the operating cathode surface ($r_{c \max}$) is approximately equal to the radius of the aperture of the modulator diaphragm, R_m , when the potentials of the cathode and modulator are equal ($U_c = U_m = 0$). Then

$$\varphi'_a(0, r_{c \max}) \approx \varphi'_a(0, R_m) = 0 \quad (4.45)$$

and

$$-\varphi'_c(0, R_m) \approx 0.75 \frac{1 + 0.2d_{cm}/R_m}{d_{cm}} = -\varphi'_m(0, R_m) \quad (4.46)$$

The functions $\varphi'_c(0, r)$ and $\varphi'_a(0, r)$ are described by the following expressions

$$\varphi'_c(0, r) = \varphi'_c(0, 0) \left[1 + a \left(\frac{r}{R_m} \right)^2 \right] \quad (4.47)$$

$$\varphi'_a(0, r) = \varphi'_a(0, 0) \left[1 - \left(\frac{r}{R_m} \right)^2 \right] \quad (4.48)$$

where

$$a = \frac{0.75(1 + 0.2d_{cm}/R_m)}{d_{cm}\varphi'_c(0, 0)} - 1 \quad (4.49)$$

and

$$\begin{aligned} \varphi'_c(0, 0) + -\frac{1}{2\pi d_{cm}} \left[\arctan \frac{d_{cm}}{R_m} + \frac{d_{cm}/R_m}{1 + (d_{cm}/R_m)^2} \right] = \\ = -\frac{1}{d_{cm}} f \left(\frac{d_{cm}}{R_m} \right) \end{aligned} \quad (4.50)$$

The function

$$f \left(\frac{d_{cm}}{R_m} \right) = \frac{2}{\pi} \left[\arctan \frac{d_{cm}}{R_m} + \frac{d_{cm}/R_m}{1 + \left(\frac{d_{cm}}{R_m} \right)^2} \right] \quad (4.51)$$

is determined by the geometric relationships of the gun and included into many design formulas. The graph of this function is given in Fig. 4.12.

The above approximate expressions allow the gun characteristics to be calculated rather simply. More compact formulas are obtained when selecting the origin of the potential on the modulator ($U_m = 0$, cathode modulation). Since in actual guns the thickness of the modulator diaphragm, δ_m , is often commensurable with the cathode-modulator distance, d_{cm} , necessary corrections should be made for the diaphragm thickness.

Taking into account the diaphragm thickness, the cathode cutoff voltage is expressed by

$$U_{c0} = \frac{\pi}{4} \times \frac{R_m}{d_{ma}} \left[1 - f \left(\frac{d_{cm} + \delta_m}{R_m} \right) \right] U_a \quad (4.52)$$

where $f\left(\frac{d_{cm} + \delta_m}{R_m}\right)$ is the same function (4.51) of the argument $\left(\frac{d_{cm} + \delta_m}{R_m}\right)$.

The calculation made by formula (4.52) gives values of cutoff voltage U_{c0} differing from those calculated by more accurate formula (4.33) by less than 10-15%, while the value of cutoff voltage

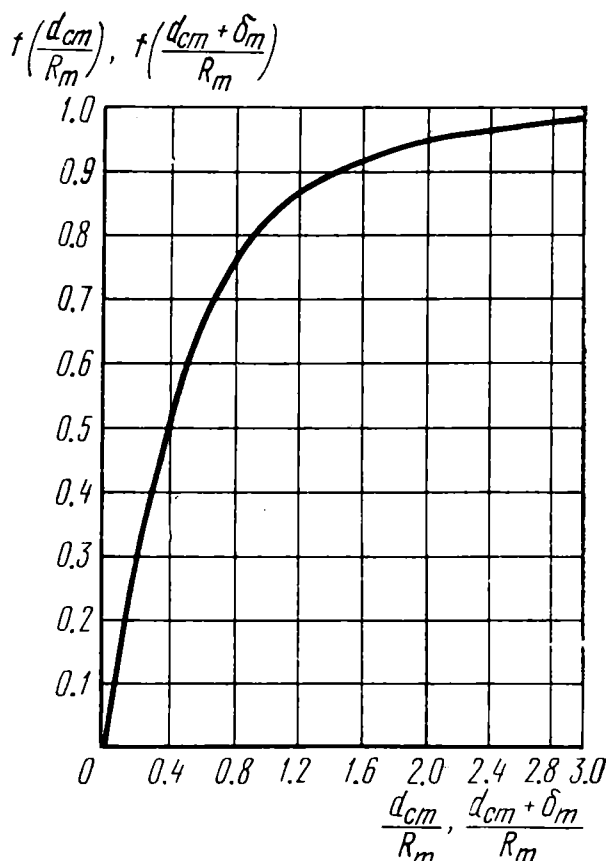


Fig. 4.12. Function $f\left(\frac{d_{cm}}{R_m}\right)$

U_{m0} found by formula (4.23) may be twice the value U_{c0} calculated by formula (4.33).

The electrical condition of the gun is conveniently characterized by a relative (dimensionless) voltage t

$$t = \frac{U_{c0} - U_c}{U_{c0}} = \frac{\Delta U_c}{U_{c0}} \quad (4.53)$$

The radius of the operating cathode surface is determined by the approximate expression

$$r_c \approx R_m \left[\frac{t}{1 + a(1-t)} \right]^{1/2} \quad (4.54)$$

where a is the coefficient (4.49) which, when replacing $\varphi'_c(0, 0)$ by $-\frac{1}{d_{cm}} f\left(\frac{d_{cm}}{R_m}\right)$ according to (4.50) is equal to

$$a = \frac{0.75 (1 + 0.2 d_{cm}/R_m)}{f(d_{cm}/R_m)} - 1 \quad (4.55)$$

The distribution of the current density over the cathode surface stems from the expression

$$j(r) = j(0) \left[1 - \left(\frac{r}{r_c} \right)^2 \right]^{3/2} \left[1 + a \left(\frac{r}{R_m} \right)^2 \right]^{1/2} \quad (4.56)$$

where $j(0)$ is the maximum (peak) current density at the cathode centre ($r = 0$)

$$j(0) = 2.33 \left[\frac{1}{d_{cm}} f\left(\frac{d_{cm}}{R_m}\right) \right]^2 U_{c0}^{3/2} t^{3/2} [\mu\text{A}/\text{mm}^2] \quad (4.57)$$

The modulation characteristic equation (the dependence of the cathode current on the control voltage) is expressed by

$$I_c = I_{c \max} t^{3/2} \left(\frac{r_c}{R_m} \right)^2 \frac{\gamma(\beta)}{\gamma(b)} \quad (4.58)$$

where

$$\gamma(\beta) = 1 + \frac{3}{8\beta^2} \left\{ \frac{\arcsin \beta^{1/2}}{[\beta(1-\beta)]^{1/2}} - 1 - \frac{2}{3} \beta \right\}, \quad \beta = bt, \quad b = \frac{a}{a+1} \quad (4.59)$$

The function γ characterizes the nonuniformity of the distribution of the current density over the cathode surface. The graph of the function $\gamma(\beta)$ is given in Fig. 4.13.

The maximum cathode current (at $U_c = U_m = 0$) used in equation (4.58) is determined by the formula

$$I_{\max} = 2.44 \left[\frac{R_m}{d_{cm}} f\left(\frac{d_{cm}}{R_m}\right) \right]^2 U_{c0}^{3/2} \gamma(b) \quad (4.60)$$

Thus, all the basic characteristics of the gun can be calculated rather easily and with sufficient accuracy. It is interesting to note that some of the above-mentioned semiempirical formulas can be obtained from general expressions at definite geometric relations or electrical conditions. Thus, for example, with $\beta \leq 0.2$ (low current) the function $\gamma(\beta)$ may approximately be considered a constant value equal to 1.2 (Fig. 4.13). In this case equation (4.58) is simplified as follows

$$I_c = I_{c \max} \frac{t^{5/2}}{1 + a(1-t)} \quad (4.61)$$

Furthermore, if we restrict ourselves to consideration of the system with a ratio d_{cm}/R_m lying within the interval 0.7 to 1.1 and assume that $d_{cm}/R_m \approx 0.9$, the modulation characteristic calculated by formula (4.61) matches the characteristic determined by formula (4.21) with $k = 2.3$.

The given relationships are also suitable for calculations of the basic characteristics of the guns at ordinary ($U_c = 0$) modulation. In this case U_c in the formulas must be replaced by U_m and U_{c0} must be replaced by its expression through the cutoff modulator voltage U_{m0}

$$U_{c0} = - \frac{U_{m0}}{1 - \frac{U_{m0}}{U_a}} \quad (4.62)$$

It should be noted that the expressions for the current characteristics of the guns are obtained without taking into account the initial velocities of the electrons emitted by the cathode. Noticeable

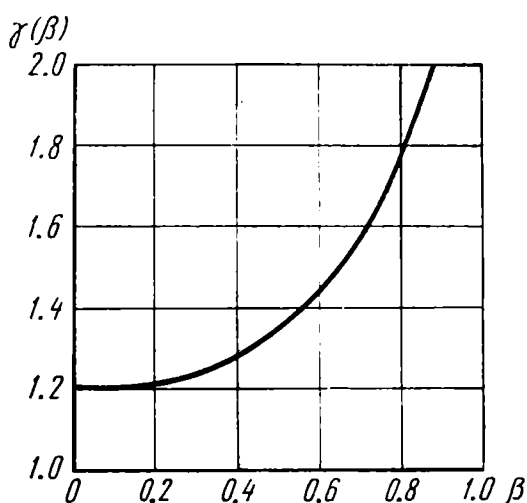


Fig. 4.13. Function $\gamma(\beta)$

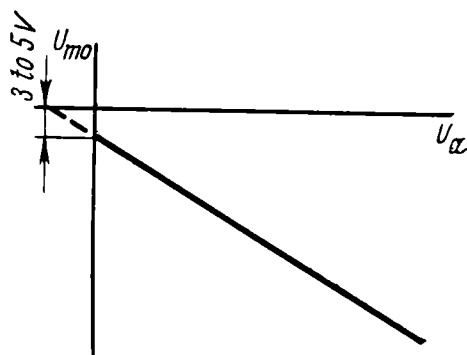


Fig. 4.14. Triode gun modulator cutoff voltage versus anode voltage

influence of the initial electron velocities is observed only at modulator voltages close to the cutoff value. The concept of "cutoff voltage" becomes indefinite due to the Maxwell velocity distribution. The zero field strength at the cathode centre does not mean that there is no cathode current. The electrons having a perceptible initial velocity can overcome the retarding field and leave the cathode. This phenomenon is well illustrated by the graph presenting the experimentally obtained dependence of U_{m0} on U_a (Fig. 4.14).

As is shown in this drawing, the experimental straight line does not pass through the origin of coordinates. A certain negative voltage of the modulator (usually about $-3V$) is necessary for cutting off the gun at $U_a = 0$.

Another difficulty encountered in the experimental determination of the value of U_{m0} is associated with the necessity to take measurements of very low currents. It is evident that the accuracy of determining $I_c = 0$ depends on the sensitivity of the instrument and the

experimentator's skill. Furthermore, the interelectrode leakage currents having the same order of magnitude as the cathode current I_c near the cutoff point can introduce substantial errors. Therefore, the cutoff voltage is conventionally accepted to be that voltage of the modulator or cathode (at the rated anode voltage) at which the cathode current (or the beam current) becomes lower than a predetermined value, for example, 1.0 or 0.5 μA .

Expression (4.52) determines the value of the cutoff voltage of triode systems with sufficient accuracy, i.e., the guns, in which the field in the near-cathode region is uniquely determined by the arrangement and the potentials of the three electrodes: the cathode, modulator and anode. Such triode systems are often used in cathode-ray tubes with magnetic focusing. In this case the gun employs the following optical scheme: an immersion objective (formed by the cathode, modulator and anode) plus a thin magnetic lens formed by a coil put on the tube neck.

In the guns with electrostatic focusing the second (electrostatic) lens is formed by one (second anode) or two (first and second anodes) electrodes at a positive potential, the electrodes being mounted behind the anode (or the accelerating electrode) nearest to the modulator. As the case is with electron valves such systems can be referred to as tetrode and pentode, respectively. It is clear that in a tetrode or pentode system the field in the near-cathode region and, therefore, the cutoff voltage are determined not only by the anode voltage but also by the voltages of the other electrodes of the gun. Using the analogy with electron valves, we may say that in the case of a tetrode system U_a in equation (4.52) should be replaced by the value of the operating voltage of the electrode nearest to the modulator. For example, if an accelerating electrode at a potential U_{ae} is located near the modulator and an anode at a potential U_a is located behind the accelerating electrode, the effective value of the voltage is

$$U_{\partial} \approx U_{ae} + D_{ae} U_a \quad (4.63)$$

where D_{ae} is the permeability of the accelerating electrode.

Approximate dependence of the cutoff voltage on the voltage of the accelerating electrode at a fixed value $U_a > 0$ for a tetrode system is shown in Fig. 4.15.

As is seen from the drawing, the straight line illustrating the dependence $U_{m0} = f(U_{ae})$ runs noticeably lower and to the left from the origin of coordinates. This is accounted for by the penetration of the positive field of the anode into the near-cathode region even at $U_{ae} < 0$.

In many modern electron guns with electrostatic focusing employing the scheme an immersion objection plus a single lens, the accelerating electrode located between the modulator and anode is

made in the form of a comparatively long cylinder accommodating one or two diaphragms. In this case the permeability of the accelerating electrode is very low, the field of the first anode practically does not affect the near-cathode region and the cutoff voltage is independent of the voltage of the first anode, i.e., as the case is with the triodes system [see Section (4.34)],

$$U_{m0} = -D_m U_{ae} \quad (4.64)$$

where U_{ae} is the accelerating electrode voltage.

It should be noted that the presence of limiting diaphragms has no practical effect on the value of the cutoff voltage determined by a decrease in the beam current, since the beam divergence angle

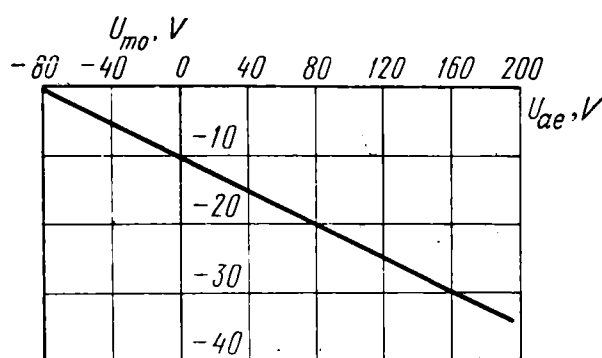


Fig. 4.15. Tetrode gun modulator cutoff voltage *versus* accelerating electrode voltage

reduces with a decrease in the beam current (with an approach to the cutoff potential) [see (4.19)] and near the cutoff point the whole electron beam passes through the gun without being affected by the diaphragms.

Experimental verification of formula (4.58) has shown that the experimental modulation characteristics are in satisfactory agreement with the theoretically calculated values for many types of electron guns. As an example, Fig. 4.16 illustrates an experimental modulation characteristic of a triode gun with indication of the points of the curve calculated by formula (4.58).

For tetrode and pentode guns expression (4.58) holds either if the cutoff voltage is calculated with due account for expression (4.63). Given in Fig. 4.17 is the gun modulation characteristic of a type 13L037I cathode-ray tube employing the scheme: an immersion objective plus a single lens.

In this case too, the experimental points are seen to be in satisfactory agreement with the theoretical curve.

The gun properties are characterized most fully by the modulation characteristic. However, in practical use of cathode-ray tubes it is

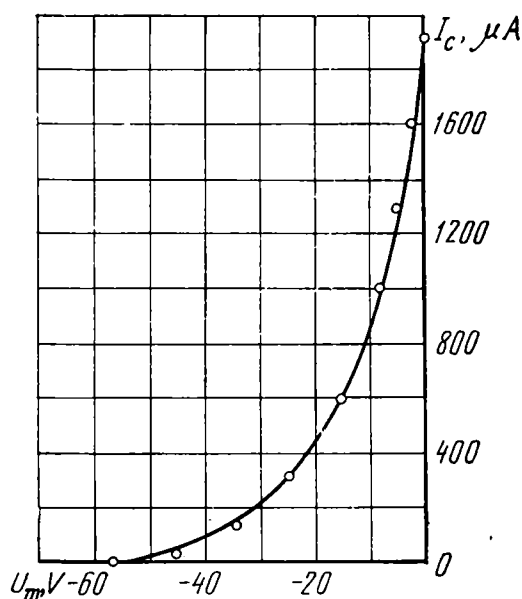


Fig. 4.16. Modulation characteristic of triode gun

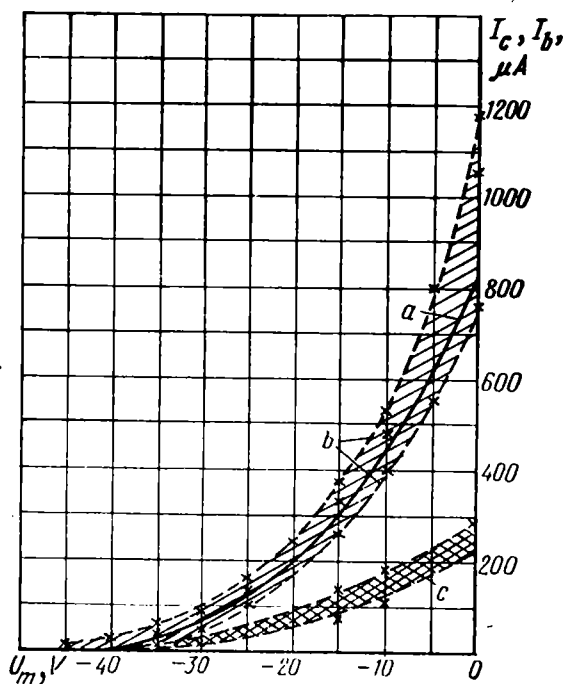


Fig. 4.17. Modulation characteristic of tetrode gun

a—calculated curve; *b*—region of experimental values of cathode current of several tubes; *c*—region of beam current values

often sufficient to know only two points of the modulation characteristic. These are the cutoff voltage (at a given value of U_a) and the modulator voltage at which a normal cathode current (or beam current) is ensured for the given device at its rating or maximum. In this case the decisive role is often played not by the absolute values of these voltages but their difference, the so-called modulation

$$V_m = U_{m0} - U_{mv} \quad (4.65)$$

where U_{mv} is the modulator voltage providing for the rated cathode or beam current.

The modulation is associated with the steepness of the modulation characteristic: the steeper the modulation characteristic, the lower the modulation depth. The steepness of the modulation characteristic is, as a rule, determined by the derivative of the cathode current with respect to the modulator voltage. Differentiating (4.58), we obtain an expression for the steepness of the characteristic

$$S = 3 \frac{I_{c \max}}{U_{c0}} t^{1/2} \left(\frac{r_c}{R_m} \right)^2 \times \frac{1 + \frac{\beta}{2(1-\beta)} \gamma(\beta)}{\gamma(b)} \quad (4.66)$$

As is clear from expression (4.66), a drop of the cutoff voltage raises the steepness of the characteristic.

Sometimes the modulation characteristic is referred not to the above-considered dependence of the cathode current on

the cathode or modulator voltage but to the dependence of the beam current and associated brightness of the screen due to the modulator voltage. If the gun has no diaphragms limiting the cross section of the electron beam, the entire electron flow emerging from the cathode takes part in the formation of the beam. In this case the beam current is equal to the cathode current and the modulation characteristic of the beam coincides with the current characteristic of the cathode.

If the gun employs limiting diaphragms, the beam current is equal to the cathode current only near the cutoff region when the influence of the aberrations and the Coulomb repulsive force is low due to the small aperture angles and low current density, and a well focused beam is practically not cut off by the diaphragms. As the diaphragms are filled with the electron beam, a portion of the cathode current is branched to the electrodes accommodating diaphragms. The beam current becomes lower than the cathode current. In this case an increase in the cathode current due to the increase in its operating surface has very little effect on the beam current, since the diaphragms cut off the outer paths of electrons emitted by those points of the cathode which are most remote from the axis. Therefore, after the diaphragms have been filled, the change in the beam current due to the change in the modulator voltage is similar to the variation of the plate current of an electron valve caused by changing the control grid voltage. The experience has shown that the modulation characteristic of the beam current of electron guns with diaphragms limiting the beam is well described by the equation of the three halves power law. Shown in Fig. 4.17 as an example, is a region of variation of the beam current for a gun with electrostatic focusing having limiting diaphragms in the accelerating electrode.

Of course, the origins of the characteristics (the cutoff point) of the cathode and beam currents coincide. The steepness of the characteristic of the beam current of the guns with limiting diaphragms is lower than the steepness of the modulation characteristic of the cathode current. The limiting diaphragms are usually installed in the guns having an electrostatic lens as a second lens. The guns with magnetic focusing having a magnetic lens as a second one are often not equipped with limiting diaphragms. Hence, the guns with magnetic focusing have more steep modulation characteristic of the beam (or smaller modulation depth) than the electrostatic guns with the same beam current.

The modulation characteristic (and the modulation depth) may to some extent depend on the emission properties of the cathode. In particular, when the cathode has an insufficient emissive capacity, the phenomenon of current saturation can appear at the central, most loaded region of the cathode. In this case a change in the

modulator voltage will cause a change in the current of the peripheral areas of the cathode, while the saturation current of the central area will practically be the same. Since the peripheral regions of the cathode provide only a part (usually smaller one due to not uniform field at the cathode surface) of the cathode current, the saturation in the central region causes a drop in the total current, considerable decrease in the steepness of the modulation characteristic and a greater modulation depth. At a sufficient emission of the cathode and at any value of modulator voltage (from the

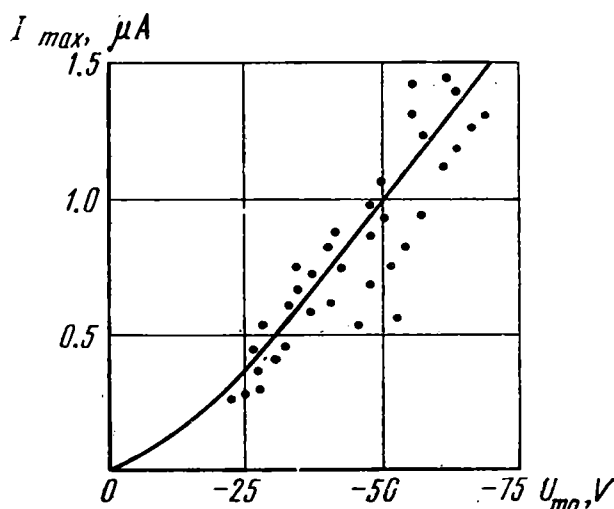


Fig. 4.18. Maximum cathode current *versus* cutoff voltage

cutoff value to zero) there will exist a space charge limiting the current over the entire surface of the cathode, the central region included. In this case the maximum cathode current will be proportional to the cutoff voltage in the power $3/2$ [see (4.60)].

The proportionality coefficient in equation (4.60) having the meaning of perveance is

$$p = \frac{I_{c \max}}{|U_{c0}|^{3/2}} \quad (4.67)$$

and can be used as a measure of the cathode emissivity. Sometimes, it is referred to as the conductivity of an immersion objective.

In practice cathodes of identical guns can be compared by plotting a graph of the dependence $I_{c \max} = p/U_{m0}^{3/2}$. Plotting the experimentally obtained values of $I_{c \max}$ at different values of U_{m0} , one can estimate the deviations of these points from the theoretically calculated curve. As an example, Fig. 4.18 illustrates such plotting for a series of guns employed by the 8L029I type cathode-ray tubes.

As can be seen from the drawing, most of the guns have almost the same conductivity. The experimental points approximate the

theoretical curve. Certain scatter of the experimental points can be explained by admissible variation of the geometric size of the guns of the same type. The experiment shows, however, that the influence of the cathode emissivity on the conductivity of an immersion objective materially exceeds the influence of the variations in the dimensions which are tolerable for the guns of the same type. Thus, the comparison of similar type guns by the above-described method is sufficiently reliable.

4.4. Main Projection Lens

As has been mentioned above, the guns of the most of modern electron-beam devices employ a two- or three-lens optical system. The second lens in a two-lens gun or the third lens in a three-lens gun is commonly referred to as a *main projection or focusing lens*. As it follows from the above-discussed optical systems (see Section 4.1), the main projection lens is used for reflecting the crossover produced by the first lens (immersion objective) on the plane of an electron receiver (screen or target). If the first lens of the gun (immersion objective) must be electrostatic due to the necessity to accelerate the electrons emitted by the cathode, the main projection lens can be either electrostatic or magnetic and, in this connection, the guns are subdivided into systems with electrostatic or magnetic focusing.

The general requirements placed upon this lens can be formulated regardless of the type of the main projection lens. In many cases a decisive parameter of an electron-beam device is its resolving power, which in general case is determined by the amount of information that can be "written" on the screen or target of this device. It is evident that the smaller the cross section of the electron beam in the receiver plane, the greater is the volume of information which can be written per unit area of the surface of screen or target.

The resolving power is often estimated by the number of separate lines laid over a certain height of the raster on the screen. For example, a good television picture is obtained at the resolving power of at least 600 lines in the centre of the screen.

The cross section area of an electron beam in the plane of electron receiver is usually estimated by the radius of the spot on the screen or target. Thus the requirement of high resolution underlies the basic requirement for the main focusing lens, i.e., its ability to produce an adequately small spot on the screen (target).

The size of the spot, i.e. the image of the crossover produced by the immersion objective, can roughly be determined from the Lagrange—Helmholtz theorem, which for the given case is written in the form

$$r_{cr} \sqrt{U_{cr}} \tan \gamma_1 = r_s \sqrt{U_s} \tan \gamma_2 \quad (4.68)$$

where r_{cr} is the radius of crossover; U_{cr} is the potential in the crossover plane; γ_1 is the aperture angle from the crossover (object) side; r_s is the radius of the spot (image); U_s is the potential in the spot (image) plane; and γ_2 is the aperture angle from the spot (image) side.

Restricting ourselves to small aperture angles (as is usually the case), from (4.68) we obtain an expression determining the radius of the spot

$$r_s \approx \frac{r_{cr}\gamma_1}{\gamma_2} \sqrt{\frac{U_{cr}}{U_s}} \quad (4.69)$$

Substituting into (4.69) the crossover radius determined by equation (4.8), we obtain

$$r_s = a \frac{\gamma_1}{\gamma_2} \sqrt{\frac{u_0}{U_s}} \quad (4.70)$$

From expression (4.70) it follows that at a first approximation the spot radius is independent of the potential in the crossover plane. At the same time, a decrease in the initial velocities of electrons (eu_0), i.e. the use of low-temperature cathodes, contributes to the reduction of spot radius.

The spot radius can also be reduced by increasing the potential in the image space. This rise of the screen potential is also advantageous in view of the following reasons: reduction of the effect of space charge (Coulomb repulsive forces), a higher luminance of the screen (see Section 6.2), a decrease in the influence of external electrostatic and magnetic fields, etc. Therefore, a decrease in the spot radius (an increase in the resolving power) due to formation of a spot at a higher potential is successfully realized in many types of electron-beam devices.

From expression (4.70) it also follows that the radius of the spot can be reduced, i.e. the resolving power can be increased, by reducing the aperture angle from the crossover side (γ_1) and increasing the aperture angle from the spot side (γ_2). The γ_1 can be somewhat reduced by decreasing the divergence of the beam after the crossover (see Section 4.2) or by using diaphragms with small apertures cutting out the central portion of the beam. In this case, however, a considerable amount of electrons are lost, i.e. the beam current is considerably reduced.

The divergence angle γ_2 can be increased either by increasing the beam section in the main lens region or by reducing the distance between the main lens and the screen. In the former case the aberrations of the main lens increase markedly and this results in an increase in the radius of the spot, i.e., this method does not improve the resolution. In the latter case the distance between the deflection centre of the beam and the screen becomes too small and this reduces

the deflection sensitivity (see Section 5.1). Hence, in actual devices selections are made of sufficiently small angles γ_2 .

It should be noted that in practically realizable electron-optical systems the quantities in equation (4.70) cannot vary independently. For example, an increase in U_s results in a noticeable decrease in γ_2 so that with an increase in U_s the resolving power rises less, than it follows from (4.70).

As has been mentioned in Section 4.2, the arbitrary crossover radius cannot be made infinitely small. Therefore, the spot, being an image of the crossover, cannot be point-small. From expression (4.69) the magnifying power of the main lens can be determined as the ratio of the spot radius to the crossover radius

$$M = \frac{r_s}{r_{cr}} = \frac{\gamma_1}{\gamma_2} \sqrt{\frac{U_{cr}}{U_s}} \quad (4.71)$$

With practically acceptable ratios of the aperture angles and the potentials, taking into account that in many types of electron guns the crossover is formed at potentials close to the potential U_s in the spot region, it is difficult to obtain $M < 1$. Hence, the spot is usually a slightly enlarged image of the crossover.

The aberrations of the main lens also result in an increase in the size of the spot. Since the crossover (scene) presented by the main lens lies on the axis of the system and has small dimensions, spherical aberration of the main lens is predominant among all the types of geometric aberrations. Therefore, the main lens must be characterized by the least spherical aberration. Practically observed asymmetry of the spot on the screen, appearing during adjustment of the main lens in the form of an elliptical spot rotating about the centre indicates an astigmatism phenomenon. Such near-axial astigmatism in many cases is due to disturbances to the axial symmetry of the field in the lens region which may result from poor workmanship of the manufacture and assembly of the electrodes producing the field of the electrostatic lens. In the guns with a magnetic focusing system pronounced astigmatism is observed, when the magnetic lens coil on the tube neck is misaligned.

The astigmatism of the main lens decreases the resolving power, which rises or lowers in two square with each other, directions depending on the adjustment of this lens. For example, when scanning the beam into a raster on a screen, one can obtain higher resolution along the line (horizontally) if the lens is adjusted so that the larger axis of the astigmatic ellipse is arranged vertically. On the contrary, with horizontal disposition of the axis of the astigmatic ellipse the vertical resolving power is greater. In this connection, particularly stringent requirements are placed on the workmanship used in the manufacture and assembly of the main projection lens.

Other kinds of geometric aberrations such as curving of the image surface, coma and distortion are of comparatively smaller practical importance, since the crossover in a correctly designed gun is adequately small, the aperture angles are small two, i.e. the conditions of paraxiality are well satisfied.

Since the spot is a crossover image, it is clear that the size of the spot depends on the crossover radius. But, as shown in Section 4.2, the concept of "crossover radius" is arbitrary due to nonuniform distribution of the current through the crossover section and the absence of a sharp crossover boundary. Consequently, the spot also has no sharp boundary and a certain conventional definition of the spot radius should be selected. The distribution of the current

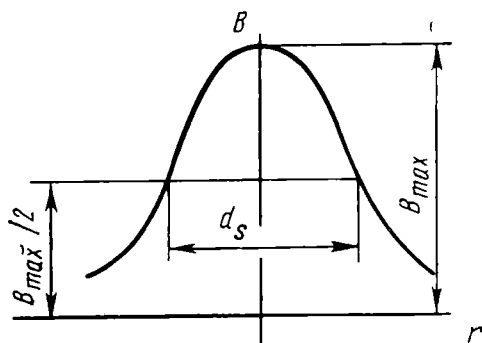


Fig. 4.19. On determining spot radius

through the spot area approximately corresponds to the current distribution in the crossover section. The brightness of the screen luminance is, in turn, approximately proportional to the beam current density (see Section 6.2). Thus, we may consider conventionally that the spot radius is the half-width of the brightness curve (Fig. 4.19), i.e. the radius of a circle at the boundary of which the brightness is equal to 50% the maximum brightness (in the centre of the spot).

At such definition two separate spots whose centres are located at a distance of their conventional radii cannot be distinguished by a human eye, i.e. they merge into a single elongated spot.

Strictly speaking, the brightness curve coincides with the current distribution curve in the crossover section only when the whole electron beam emerging from the crossover plane reaches the screen. The presence of limiting diaphragms results in some distortion to the brightness distribution curve. Nevertheless, experimental study of the brightness distribution through the cross section of the spot has shown that the brightness distribution curve is described by the same exponential law which governs the current density distri-

bution in the crossover [see (4.16)]

$$B = B_0 e^{-\frac{r^2}{b^2}} \quad (4.72)$$

where B_0 is the brightness in the centre of the spot; b is a constant.

Experimental verification of expression (4.72) on a number of guns showed sufficiently good coincidence of experimental curves with the theoretical curve. As an example, Fig. 4.20 illustrates an experimental curve of distribution of the brightness through the radius of a spot formed by a gun with magnetic focusing. The dash line shows the curve plotted by equation (4.72).

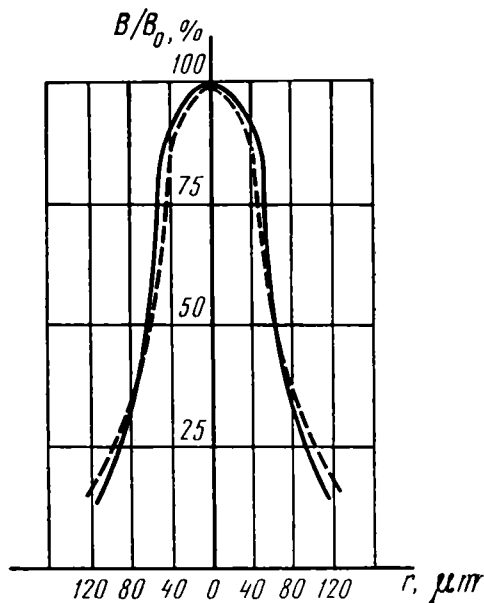


Fig. 4.20. Brightness distribution along spot radius

This dependence is markedly disturbed only when the limiting diaphragm is mounted in close vicinity of the crossover plane. Mounting of the diaphragm near the crossover plane with a radius of the aperture equal to the theoretically calculated radius of the crossover should result in formation of a spot having a sharper boundary due to the fact that a considerable portion of periphery electrons is cut off. However, the influence of the Coulomb repulsive forces effective near the screen and the presence of aberrations of the second lens cause an increase in the radius of the spot and blurring of its boundary. The limiting diaphragms with very small apertures mounted in the crossover plane are employed only in special guns having a uniform current density in the spot, which are also referred to as guns with artificial crossover.

It should be noted that the best results, i.e. the least spot or the highest resolving power, are obtained with such adjustment of the main lens at which the crossover image is formed not on the screen surface but in front of the screen (Fig. 4.21). This is explained by the influence of the Coulomb repulsive forces acting on the electrons near the screen.

It is clear that the smallest spot and, therefore, the highest resolving power are obtained when the receiver plane coincides not with the crossover image but with the plane of the second crossover formed by the main projection lens [see Section 2.1, equation (2.26)].

The above-discussed idealized theory of forming an image on

a screen leads to a conclusion of independence of the resolving power of the absolute value of the beam current. The experiments show, however, that the theoretically expected independence of the spot radius of the beam current is observed only at very low currents, which do not exceed a few microamperes, i.e. when the influence of space charge in the region of forming the spot may be neglected.

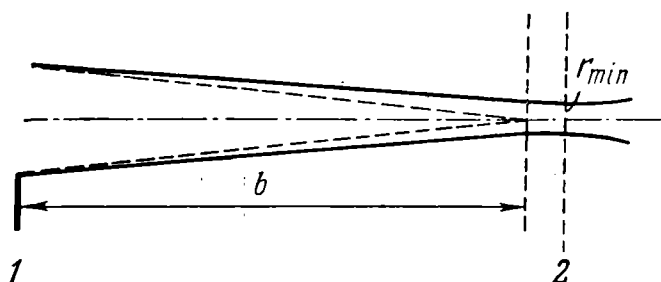


Fig. 4.21. Adjustment of main projection lens
1—limiting diaphragm; 2—screen plane

The influence of the space charge rises with a rise of the beam current; furthermore, the rise of the beam current results in greater aperture angles increasing the aberrations of the main lens and, as a consequence, reducing the resolving power. A drop of the resolving power

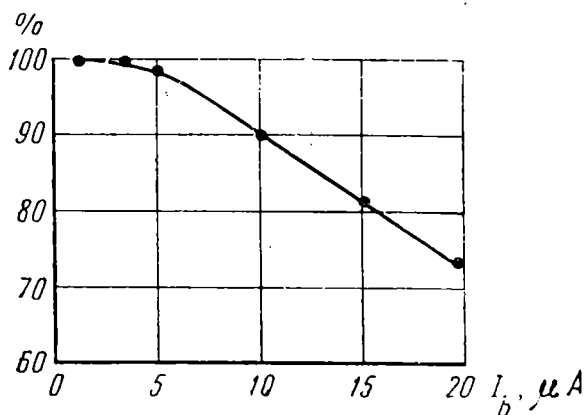


Fig. 4.22. Resolution versus beam current

with an increase in the beam current is particularly perceptible in the guns with electrostatic focusing systems, since they operate with lower aperture angles and even insignificant rise of the apertures results in considerable aberrations of the main projection lens. In the guns with magnetic focusing without limiting diaphragms and with a comparatively large diameter of the focusing coils the drop of the resolving power with a rise of the beam current is comparatively small. The analytical calculations of the dependence of the resolving power on the beam current present many difficulties

and the analysis of this dependence is usually based on experimental data. Figure 4.22 illustrates an experimental curve of the dependence of the resolving power on the beam current obtained on an experimental gun with magnetic focusing. The resolution at a beam current of $1\mu\text{A}$ is taken for 100 %.

At the same time, a rise of the potential in the image space (near the screen) helps to increase the resolution. Therefore, where it is necessary to maintain high resolving power and high brightness of the screen, it is expedient to increase the accelerating voltage [thus increasing the brightness (see Section 6.2)] at a comparatively low beam current.

Finally, it should be noted that the Coulomb repulsive forces and the aberrations of the second lens result in much lower values of the current density in the spot plane as compared with the design values. Though the distribution of the current in the spot is rather close to the distribution of the current density in the crossover section, the maximum value of the current density measured in the centre of the beam in many cases does not exceed 50 % the theoretically calculated value.

4.5. Electron Gun with Electrostatic Focusing

The gun with electrostatic focusing, as follows from its name, employs an electrostatic immersion lens or a single lens as the second lens of the system. Thus, such a gun employs an optical scheme "an immersion objective + an immersion lens" or "an immersion objective + a single lens". The three-lens guns have an additional weak (usually immersion) lens between the immersion objective and the second lens. In this case the gun employs an optical system "an immersion objective + an immersion lens + a single (or immersion) lens".

Schematic diagrams of the guns with electrostatic focusing are given in Fig. 4.23.

The gun employing the scheme shown in Fig. 4.23a is the most simple one and, at the same time, makes it possible to obtain satisfactory results. Such a gun was frequently used in the earlier types of electron-beam tubes.

One of the disadvantages of this gun, which resulted in gradual replacement of such a system with more advanced types, is interconnection between the first and the second lenses. Indeed, the first anode (A_1) forms the field of an immersion objective by its edge facing the cathode, while its opposite edge is one of the electrodes forming the second (immersion) lens. Owing to this fact a change in the potential of the first anode simultaneously changes the optical properties of both of the lenses. In particular, adjustments of the second lens simultaneously cause changes in the operating voltage

in the near-cathode region and, therefore, the value of the cathode and beam current. Focusing the beam (by the second lens) and the control of the beam current (brightness) become interrelated and this presents inconveniences in the operation of the devices equipped with guns of this type.

This interrelationship particularly manifests itself when feeding all the electrodes of the gun from a common voltage divider as the case is with oscilloscope cathode-ray tubes. Varying the cathode current during the adjustment of the modulator voltage changes the current flow in the diaphragm of the first anode. This current, passing through the common divider, changes the voltage distribution across its arms with resultant change in the potential of the

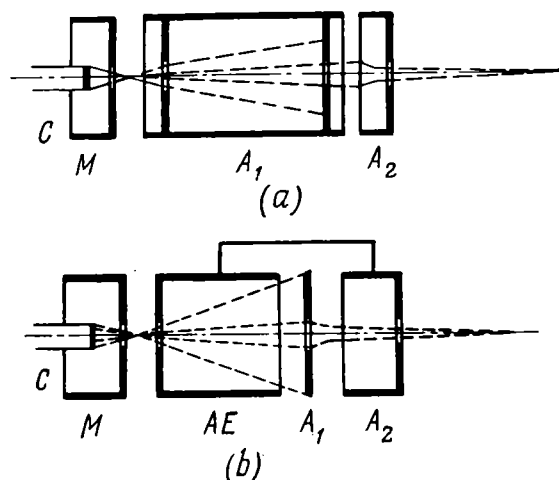


Fig. 4.23. Electrostatic focusing guns employing (a) immersion objective + immersion lens scheme; (b) immersion objective + single lens scheme

first anode, i.e. disturbs the focusing of the main lens. The adjustment of the focusing by varying the potential of the first anode also changes the modulator potential, i.e. causes brightness modulation.

Furthermore, in the guns employing the scheme shown in Fig. 4.23 special measures should be taken for catching the secondary electrons striken by the beam from the edges of the limiting diaphragms mounted in the cylinder of the first anode. The secondary electrons forced by the accelerating field in the gap between the first and the second anodes can fly to the screen.

Depending on the place of origination of secondary electrons and configuration of the field, the flow of secondary electrons can reach the screen in the form of a wide unfocused beam or in the form of a comparatively narrow beam whose least cross section does not coincide with the screen plane. In the former case the screen uniformity (weakly) emits light, and this impairs the contrast

(see Section 6.1). In the latter case the trace of the beam of secondary electrons is seen as a blurred spot large in diameter with resultant decrease in the resolving power. The simplest means for entrapping the beam of secondary electrons is a special diaphragm mounted in the cylinder of the second anode.

The gun employing the system shown in Fig. 4.23b is presently most common. It has a number of advantages as compared with the above-described gun. First the accelerating electrode located behind the modulator is at a constant potential and, therefore, serves as an electrostatic shield between the first and second lenses. The adjustment of the second (single) lens by varying the voltage of the first (focusing) anode does not affect the optical properties of the first lens. In the same manner, controlling of the brightness by varying the potential of the modulator has not effect on the second lens, i.e. such a gun features the absence of undesirable relationships between the lenses. Secondly the electron beam runs through the region of a high potential almost over the whole length of the gun which is good since the higher the electron velocity, the lower is the relative influence of the scatter of initial velocities and Coulomb repulsive forces. Furthermore, the higher the potential of the space in which the electrons move, the lower the relative influence of the external electrostatic and magnetic fields on the motion of electrons. The beam becomes more rigid with an increase in the potential.

In the gun under consideration the potential of the centre electrode of the single lens (first or focusing anode) is usually below the potential of the extreme electrodes (accelerating electrode and second anode). When $U_{a1} < U_{a2}$ all the electrodes of the gun are preferably fed with voltages from a common divider connected in parallel to the gun. Furthermore, when $U_{a1} < U_{a2}$, the cross section of the beam in the region of the second lens becomes less (see Section 1.8) and this reduces the aberrations of the second lens and, as a consequence, decreases the size of the spot.

Finally, the lower the potential of the central electrode, the lower is the relative influence of the instability of the focusing voltage on the quality of focusing. Therefore, it is expedient that the geometric sizes of the electrodes of the second lens are selected so that the required optical strength is obtained at the least possible values of U_{a1} . It is clear that within its range the value of U_{a1} may equal zero, i.e. the result is a single-potential lens with uncontrollable focal length. However, the use of single-potential systems in electron-beam devices is limited, since the focusing of such systems can be effected only by shifting the scene, the lens or the image plane along the axis, which cannot be practically accomplished. The lens electrodes having fixed geometrical dimensions require very accurate manufacture, precise finish and careful assembly of the system so

that they are inexpedient from the economic point of view. Consequently, the medium electrode potential is usually non-zero to provide adjustment of the second lens but, as a rule, $U_{a1} \ll U_{a2}$.

If a portion of the beam is cut off by the medium electrode of the second lens (first anode), a considerable current flows through its circuit and with a common divider any adjustment of the second lens redistributes the currents and voltages in the divider arms. This may result in an electrical coupling between the second and first lenses. This coupling may become still more complicated due to the secondary emission from the first anode surface.

The secondary electrons emerging from the edges of the diaphragm of the first anode are collected by the accelerating electrode and by the second anode which is at a higher potential, the result being a current drop in the first anode circuit. Should the secondary emission coefficient of the first anode surface be higher than unity, the current in the first anode circuit is reversed. Needless to say that the secondary electrons passing through the second anode diaphragm may cause parasitic illumination of the screen.

Thus the presence of an electric current in the first anode circuit is undesirable. Hence at the present time a gun with a zero current of the first anode is used in many types of electron-beam devices with electrostatic focusing. This gun features a special construction of the first (focusing) anode, which practically eliminates penetration of electrons to this electrode and, therefore, the current in the circuit of the first anode is equal to zero.

The absence of an electric current in the circuit of the first anode allows the electrodes of the optical system to be fed from a common voltage divider, in which case the variation of the voltage of the first anode during the focusing of the beam has no effect on the current distribution and, therefore, the distribution of voltages across separate parts of the divider, i.e. does not lead to maladjustment of the first lens. In the same manner, the brightness control does not cause maladjustment of the second lens. This is the main advantage of the gun with a zero current in the first anode.

Furthermore, in the gun with a zero current in the first anode the beam is limited by diaphragms mounted in the electrodes (accelerating electrode and second anode) having the highest potential. In this case there is no necessity of taking special measures (for example, mounting of additional diaphragms) for entrapping the secondary electrons, since the knock-on secondary electrons return back to the more positive electrodes.

A schematic diagram of the gun with a zero current in the first anode is shown in Fig. 4.24.

As shown in the drawing, from the optical point of view this gun is also a combination of an immersion objective and a single lens. In order that the centre electrode (first anode) does not entrap

the electrons, the diameter of the apertures made in it is 2 to 3 times the diameter of the apertures of the limiting diaphragms in the accelerating electrode and second anode.

In the system of the gun with a zero current of the first anode typical for many commercial oscillograph tubes the focusing potential of the first anode equals hundreds of volts at $U_{ae} = U_{a2}$ of

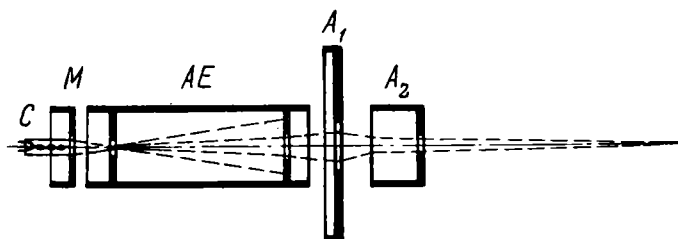


Fig. 4.24. Gun with zero current of the first anode

the order of 1-3 kV. As mentioned above, it is good practice to reduce the potential of the centre electrode of a single lens serving as the main projection lens of the gun. A decrease in the potential of the medium electrode of the single lens with a constant optical

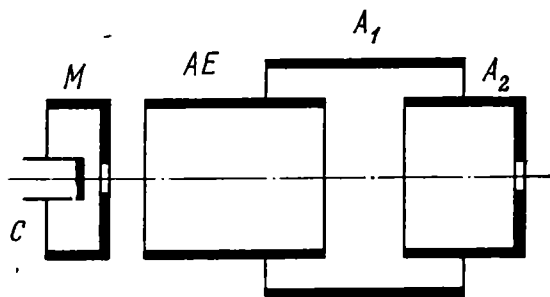


Fig. 4.25. Gun with focusing anode at zero potential

force providing for a crossover image in the screen plane can be effected with simultaneous change in the geometrical relationships of the electrode system forming the field of the lens. The reduction of the centre electrode potential is assisted by an increase in the diameter of the aperture of this electrode, a decrease in the diameters of the apertures of the lens and a shorter distance between the extreme electrodes of this lens. The realization of these possibilities has resulted in development of a gun with a focusing electrode at a zero potential (Fig. 4.25).

The first (focusing) electrode is made in the form of cylinder of a large diameter embracing the edges of the accelerating electrode and the second anode. Such a gun is actually a variation of the gun with a first anode at a zero potential since in this system the electrons can by no means penetrate to the external cylinder (focusing

electrode). Although it is possible to find such geometrical relationships of the system that the best focusing is obtained at $U_{a1} = 0$, in practice the potential of the focusing electrode is to be adjusted within a small range due to the fact that it is practically impossible to accurately follow the theoretical size of the electron-optical system, especially in mass production. The guns employing the above-given scheme find wide application in modern kinescopes.

The guns with electrostatic focusing employing the optical scheme "an immersion objective + a single lens" may be related to the triode-type guns, since their field in the near-cathode region does not depend on the potential of the first anode and is uniquely determined by the potentials, dimensions and mutual arrangement of the three electrodes: the cathode, modulator and accelerating electrode. The independence of the voltage acting in the modulator plane of the first anode potential is accounted for by a very low permeability of the accelerating electrode. The latter is usually made in the form of a comparatively long cylinder with limiting diaphragms mounted inside the electrode. Such an electrode is a reliable electrostatic shield preventing the field of the first anode from penetration into the near-cathode region. The use of the comparatively long cylinder of the accelerating electrode makes it possible to increase the distance from the crossover plane to the mid-plane of the main projection lens and this reduces the magnification of the main lens, i.e. reduces the radius of the spot and increases the resolving power.

As mentioned above, in the triode-system guns it is difficult to obtain small beam divergence angles beyond the crossover plane [see (4.19)] without considerable limitation of the beam by the diaphragm mounted in the accelerating electrode or the first anode. Furthermore, when using high accelerating voltages, it is difficult to obtain low (by absolute value) cutoff voltages. The cutoff voltage can be reduced by decreasing the diameter of the modulator aperture and reducing the distance between the modulator and the cathode. The former method results in restriction of the operating surface of the cathode and a sharp drop in the beam current, the latter method interferes with the assembly of the gun, since the modulator-cathode distance must be smaller than 0.1 mm and a small variation of this distance results in substantial scatter of the cutoff voltage. Therefore, alongside with triode guns, widely used in electron-optical systems are tetrode-type guns with electrostatic focusing, particularly in tubes operating at an accelerating voltage of 5-8 kV. The tetrode system, besides the above-said advantages (a low beam divergence angle, possibility to use small cut-off voltages), has still another advantage when using high accelerating voltages. In a triode-system gun there appears between the modulator and the accelerating electrode at a high potential a very high potential

gradient which can easily cause an electric breakdown. An additional electrode at a comparatively low (of the order of hundreds of volts) potential mounted between the modulator and the high-voltage accelerating electrode substantially increases the dielectric strength of the gun.

A schematic of a tetrode gun with electrostatic focusing is shown in Fig. 4.26.

The first accelerating electrode mounted near the modulator is made in the form of a short cylinder and is often equipped with

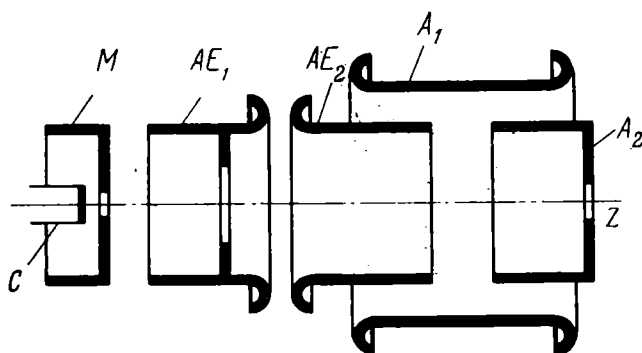


Fig. 4.26. Tetrode gun with electrostatic focusing

a limiting diaphragm. The diaphragm reduces the permeability of this electrode and allows for lower accelerating voltages. Mounted behind the first accelerating electrode is the second accelerating electrode at a high positive potential. The adjacent edges of the cylinders of the first and second accelerating electrodes are sometimes bent outside ("flared") for increasing the dielectric strength. Formed between the first and second accelerating electrodes is an immersion lens reducing the divergence angle of the beam emerging from the crossover plane. The main immersion lens in such guns is usually made in the form of a single lens at a zero potential of the focusing electrode. The tetrode-type guns are widely used in kinescopes with anode voltages higher than 10 kV.

4.6. Gun with Magnetic Focusing

The gun with magnetic focusing usually has an optical scheme in the form of an immersion objective (triode or tetrode) forming a crossover and a magnetic lens reflecting the crossover to the receiver plane. The magnetic lens of the gun is made in the form of a short coil often provided with a ferromagnetic shield and is put on the tube neck.

Until recently the gun with magnetic focusing was used in many types of cathode-ray tubes due to high quality of focusing. At the

present time, too, the guns with a magnetic coil serving as a second lens are preferable for devices with very high resolution.

The optical characteristics of a magnetic lens as compared with the electrostatic one are better due to lower aberrations, particularly spherical aberration. In the magnetic lens made in the form of a coil with a relatively large diameter (average diameter is of 45-50 mm) the field pierced by the electron flow occupies only a small near-axial region. Hence, for such lenses arranged outside the tube, the condition of paraxiality of an electron beam is well satisfied and, therefore, the aberrations can be much reduced. Furthermore, since the magnetic field does not change the energy of the electrons, the errors due to the scatter of the electron velocities are also reduced.

Of course, generally it is possible to develop an electrostatic lens with optical characteristics (magnitude of aberration) which are not worse than the characteristics of a specific magnetic lens. However, practical realization of electrostatic lenses with high optical characteristics for electron-beam devices is associated with some difficulties, while high-quality magnetic lenses can be made rather easily. The cause is in that the diameters of the electrodes of the electrostatic lens to be placed in bulb of an electron-beam device are limited by the size of this bulb. An increase in the diameter of the tube neck sharply reduces the efficiency of the beam deflection when using magnetic deflection systems (see Section 5.3). With an electrostatic deflection an increase in the beam aperture increases the deflection aberrations and thus eliminates the gain in the quality of focusing due to the increase in the diameter of the electrodes of the electrostatic lens.

As indicated above large beam apertures are allowable in guns with magnetic focusing. An increase in the aperture angle from the image side contributes to an increase in the resolving power [see (4.70)], therefore, in this respect too the gun with magnetic focusing has definite advantages. Furthermore, high aperture angles in the guns with magnetic focusing sometimes allow limiting diaphragms in the cylinders of the anode or accelerating electrode to be eliminated. The absence of diaphragms limiting the cross section of the beam somewhat improves the use of the cathode. In the guns with magnetic focusing without limiting diaphragms the beam current is practically equal to the cathode current, while in the guns with electrostatic focusing up to 80% of the cathode current is sometimes entrapped by the diaphragms with small apertures mounted inside the electrodes of the guns.

Good utilization of the cathode current results in that the steepness of the modulation characteristic in the magnetically focused guns is higher than in the guns with electrostatic focusing. High steepness of the modulation characteristic (low modulation value) is still another advantage of the guns with magnetic focusing.

Application of high apertures together with good focusing makes it possible to use magnetically focused electron guns with comparatively high beam currents, up to several milliamperes. At the same time, an increase of the beam section in the main lens area inevitably leads to a comparatively large diameter of the beam in the deflection

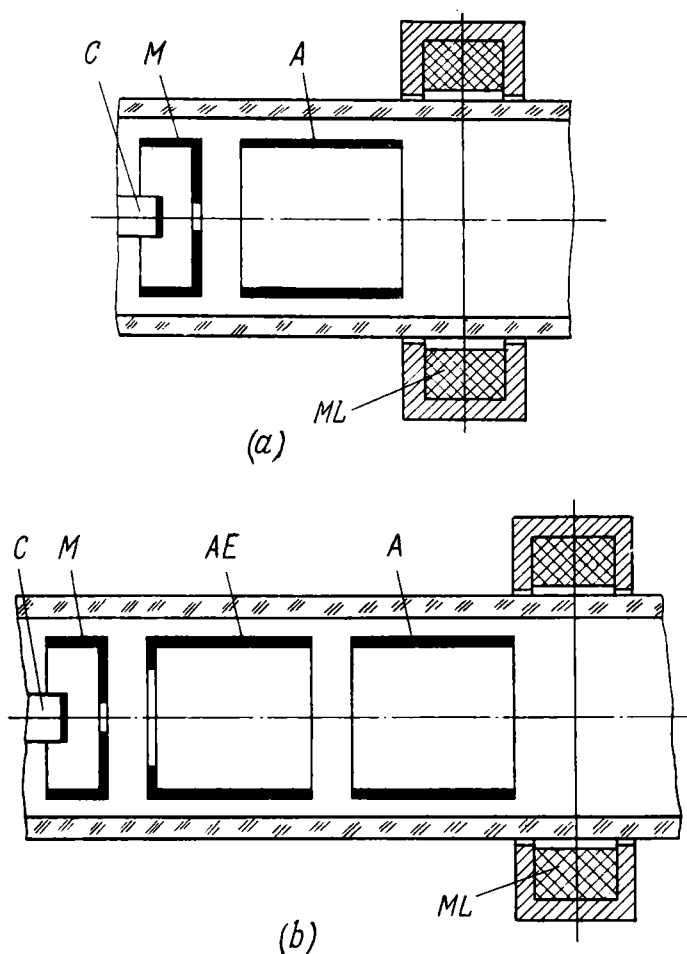


Fig. 4.27. Magnetic focusing gun
(a) triode; (b) tetrode

zone. In the case of electrostatic deflection this results in a sharp rise of the distortions with an increase of the beam deflection angle (see Section 5.4). Therefore, electrostatic deflection systems have found no practical application in cathode-ray tubes with magnetic focusing.

The electron guns with magnetic focusing employ both triode and tetrode schemes (Fig. 4.27).

A tetrode system is preferably used where a need arises for high resolution and low deflection distortions, since the additional electrostatic immersion lens reducing the divergence angle of the beam

emerging from the immersion objective causes a decrease in the beam cross-sectional area in the region of the main lens and of the deflection system. The tetrode gun is often used in high-voltage tubes, since the presence of an accelerating electrode with a comparatively low potential between the high-voltage anode and the modulator allow for low (in magnitude) values of the cutoff voltage. Furthermore, the required electric strength can easily be obtained in a tetrode system where the accelerating gap is subdivided into two stages.

The quality of focusing in a gun with a magnetic lens largely depends on the workmanship of the coil and the ferromagnetic shield. In addition to accurate geometric dimensions, the ferromagnetic envelope must have good magnetic symmetry. Hence, the shield of the magnetic lens must be made of highly uniform magnetically permeable materials. The resolving power also greatly depends on the accuracy of mounting the magnetic lens on the tube neck. Even insignificant misalignment or poor concentricity of the coil can disturb the axial symmetry of the magnetic field in the region of the electron beam and cause near-axial aberrations increasing the spot diameter.

From the optical point of view, the use of a magnetic coil as a main projection lens has a number of advantages. At the same time, in order to produce a required magnetic field, particularly when focusing high-speed electrons (such as accelerated by a potential difference of a few kilovolts), required is a relatively large magnetic force of several hundreds of ampere turns. Consequently, considerable electric power is required for feeding the magnetic lens, i.e., the magnetic lenses are less economical in operation. This disadvantage can be overcome by using permanent magnets for focusing purposes. However, the lenses formed by permanent magnets so far have not gained wide recognition due to difficulties associated with controlling their optical characteristics. It should be noted that the focusing is disturbed due to the nonuniformity of the equations of motion of electrons in a magnetic field (see Section 1.6) during the fluctuations of the supply voltage of the immersion objective and the magnetic lens. Hence, with electron guns using the magnetic focusing fluctuations of the supply voltages call for adjustment of the main lens and the use of stabilized power sources or special electronic circuit for stabilizing the focusing system.

Despite some disadvantages (low service economics, large weight and overall dimensions of the magnetic coils and necessity to stabilize the supply voltage), the high optical parameters of the guns with magnetic focusing systems account for their wide application, particularly in the radar cathode-ray tubes and some special-purpose electron-beam devices.

Where the resolving power plays the decisive part, while the beam current can be of a small (few microamperes) value, use is

made of a gun with artificial crossover, which is sometimes referred to as a gun with uniform density of the current in the spot (Fig. 4.28).

As shown in the drawing, mounted between the modulator and the accelerating electrode (anode) approximately in the crossover plane is a diaphragm with a very small aperture. If the diaphragm aperture is much less than the calculated crossover section, the greater portion of the peripheral trajectories of the electrons is cut off by the diaphragm and the beam passing through the diaphragm

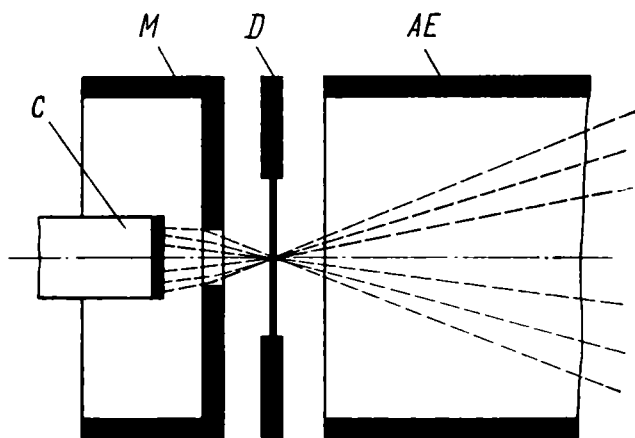


Fig. 4.28. Artificial crossover gun

has approximately the same current density over its whole cross-sectional area. The second lens is adjusted so that the image of the diaphragm "illuminated" by electrons is obtained in the receiver plane. As a result, the spot serving as an image of artificial crossover also has uniform current distribution and a sharper boundary. Of course, higher resolving power, the spot diameter of the order of a hundredth fraction of one millimetre, can be obtained only at low beam currents and with the screen plane at high potentials, i.e., at a perveance of the spot not higher than 10^{-9} A/V, otherwise the Coulomb repulsive forces in the spot plane displace the electrons from the central area, the uniform distribution of current density is disturbed and the spot diameter increases.

With the artificial crossover guns use is frequently made of a magnetic projection lens but, since the aperture angles in such a gun are small, satisfactory results are obtained when employing an electrostatic lens with low aberrations as the main projection lens.

Deflection Systems

5.1. General

Deflection systems are designed for deflection (spatial displacement) of an electron beam collimated by a gun. In many electron-beam devices the electron beam must be deflected to any point on the electron receiver surface. Owing to this fact the electron-beam device, as a rule, has two deflecting elements shifting the beam in two directions at square angles to each other. In some cases use is made of the so-called circular sweep, in which one deflecting element forces the beam to inscribe a circle (or a spiral line) on the screen, while the other element displaces the beam in a radial direction, i.e., normally to the scanning trace.

Regardless of the type and construction features, the deflection systems must satisfy the following requirements:

(1) The system must have adequate deflection sensitivity, i.e., the deflection of the beam by a specified value (angular or linear displacement in the receiver plane) must take place at a minimum deflecting factor (voltage or current);

(2) the system must be linear, i.e. the beam deflection (displacement in the receiver plane) must be proportional to the magnitude of the deflecting factor at any value of the deflection angle admissible for the given device or at any part of the screen surface;

(3) the system must not substantially disturb the beam focusing, i.e. the focused beam must be deflected by the deflection system as an integral part, the shape and value of the spot remaining approximately invariant at any portion of the screen surface.

Furthermore, the deflection systems of some specific types of electron-beam devices must meet specific requirements. For example, the deflecting systems of kinescopes must provide for large (up to 60°) deflection angles while maintaining linearity and focusing. The deflection systems of high-frequency oscillograph tubes must have the least possible persistence, which, in turn, calls for small values of the capacitances, inductances, transit time of electrons, etc.

As will be shown below, neither of these requirements can be satisfied completely, since during the deflection of the beam by an angle higher than $10\text{--}15^\circ$ the deflection aberrations become noticeable,

i.e., the focusing of the spot on the screen is deteriorated. The deflection systems are not strictly linear elements. Therefore, at large deflection angles the proportionality between the values of deflection and the signal fed to the system is disturbed. The deflection sensitivity also cannot practically be sufficiently high and this in many cases does not allow the signal being analyzed to be fed directly to the deflection system without preliminary amplification.

From the optical point of view the deflection systems are electron prisms. As indicated in Section 1.2, the effect similar to refraction of a light beam during the passage through a prism takes place when an electron beam passes through a transverse electric or magnetic field. The deflection angle of an electron beam is determined by the following formulas:

During the passage through a uniform transverse electric field [see (1.9)]

$$\tan \alpha_e = \frac{eEl}{mv_z^2}$$

During the passage through a uniform transverse magnetic field [see (1.15)]

$$\tan \alpha_m = \frac{eBl}{mv_z}$$

In these expressions E is electric field strength, B is magnetic induction, l is the length of regions of transverse field, v_z is velocity of an electron entering the transverse field.

Express the electron velocity through the potential difference U_a between the cathode and exit electrode of the gun and substitute the obtained value into the above equations. Then

$$\tan \alpha_e = \frac{El}{2U_a} \quad (5.1)$$

$$\tan \alpha_m = \sqrt{\frac{e}{2m}} \times \frac{Bl}{\sqrt{U_a}} \quad (5.2)$$

At a distance from the centre of deflection to the electron receiver (screen) equal to L the magnitude of the linear displacement (h) of the spot on a flat screen is as follows: $h = L \tan \alpha$. The beam, as a rule, is deflected by means of electrostatic and magnetic fields which are close to uniform. In this case E and B in equations (5.1) and (5.2) may be presented in the form

$$E = \kappa U_{def}$$

$$B = \chi n I_{def}$$

where κ and χ are constant factors; U_{def} is the deflection voltage fed to an electrostatic deflecting system; nI_{def} is the magnetizing force (I_{def} is the deflection current in the deflecting coil having n turns).

Thus the absolute value of the deflection (displacement) of the spot on a flat screen is:

With electrostatic deflection

$$h_e = \frac{\chi l L}{2U_a} U_{def} \quad (5.3)$$

With magnetic deflection

$$h_m = \sqrt{\frac{e}{2m}} \times \frac{\chi l L}{\sqrt{U_a}} n I_{def} \quad (5.4)$$

From equations (5.3) and (5.4) one can easily obtain an expression for the deflection sensitivity by dividing the absolute value of the deflection h by the value of the deflecting factor U_{def} or nI_{def} . Then the deflection sensitivity is:

With electrostatic deflection

$$\varepsilon_e = \frac{h_e}{U_{def}} = \frac{\chi l L}{2U_a} \text{ [mm/V]} \quad (5.5)$$

With magnetic deflection

$$\varepsilon_m = \frac{h_m}{nI_{def}} = \sqrt{\frac{e}{m}} \times \frac{\chi l L}{\sqrt{2U_a}} \text{ [mm/A} \times \text{turn]} \quad (5.6)$$

Sometimes, instead of the deflection sensitivity, use is made of a quantity which is the reciprocal of ε or the so-called *deflection factor*. This is a factor (in volts or ampere-turns) required to displace the spot on the screen for a distance of 1 mm.

The parameters of an electron-beam device sometimes include the so-called specific sensitivity defined as the ratio of the deflection sensitivity to the spot diameter or a specific deflection factor having the sense of a deflecting force required for displacing the beam on the screen over a distance equal to the diameter of the spot. Since the concept of the spot diameter is arbitrary, the specific parameters are also arbitrary and characterize not only the deflection system but also the quality of the gun collimating the electron beam. The quality of the deflection system can also be estimated by the value of the sensitivity ε' defined as the ratio of deflection sensitivity to the anode (accelerating) voltage

$$\varepsilon'_e = \frac{\varepsilon_e}{U_a} = \frac{\chi l L}{2U_a^2} \quad (5.7)$$

$$\varepsilon'_m = \frac{\varepsilon_m}{U_a} = \sqrt{\frac{e}{2m}} \times \frac{\chi l L}{U_a^{3/2}} \quad (5.8)$$

Of course, expressions (5.3) to (5.6) should be regarded only as a first approximation, since the original equations (1.9) and (1.15) were derived assuming that the deflection angles are small and the

deflecting fields are strictly uniform in the region l and are zero outside this region. None the less, the analysis of equations (5.3) to (5.6) makes it possible to correctly estimate the basic parameters of the deflection systems. From these equations it follows that in the first approximation with both the electrostatic and magnetic deflection systems the deflection of the beam is linear. Thus, there is direct proportionality between the values of displacement of the spot and the deflecting signal, i.e., the deflection sensitivity is constant for the given configurations and deflection systems and accelerating voltage and is independent of the deflection angle (magnitude of the deflection factor).

From equations (5.5) and (5.6) it follows that in the case of magnetic deflection a change in the accelerating voltage (U_a) has a lower effect on the sensitivity, since $\varepsilon_m \sim 1/\sqrt{U_a}$, while $\varepsilon_0 \sim 1/U_a$. In other words at magnetic deflection the factor ε' decreases slower with an increase in the gun voltage than at electrostatic deflection ($\varepsilon'_m \sim 1/U_a^{3/2}$; $\varepsilon'_e \sim 1/U_a^2$). This gives explanation of the wide use of magnetic deflection in high-voltage devices. Furthermore, when electrostatic deflection is employed at a high accelerating voltage, the required amplitude of the deflecting voltage can be so high that it would be difficult to provide for adequate dielectric strength of the deflection system itself and, particularly, of the electronic circuits generating the deflecting voltages.

Magnetic deflection is advantageous in lower aberrations as compared to the electrostatic deflection (see Section 5.4). In the case of electrostatic deflection the spot is markedly defocused at deflection angles higher than $15\text{--}20^\circ$, while the magnetic deflection allows the beam to be deflected for $50\text{--}60^\circ$ with adequate resolution. It may be said that the specific sensitivity with magnetic deflection features much lower dependence on the deflection angle than in the case of electrostatic deflection. In particular, this explains wide application of magnetic deflection systems in TV kinescopes with a total beam deflection angle of up to 120° *.

Comparing electrostatic and magnetic deflection systems, one can note that the former is much more economical. Indeed, an electrostatic field is produced in a space without power consumption, while energy is required for producing a magnetic field. Assuming that in the deflection zone there is stored the same energy both in the electrostatic and magnetic fields, i.e., if $\varepsilon_0 E^2/2 = B^2/2\mu_0$, the beam is deflected in the electrostatic field to a larger extent. This follows from the equality $h_m/h_e = Bv_z/E$ (v_z is the longitudinal component of the velocity of the electrons). When the deflecting

* In TV engineering the deflection angle is usually referred to the total span angle of the electron beam, i.e. the double value of the maximum deflection angle.

energies are the same, $B = \sqrt{\epsilon_0 \mu_0} E$. Then

$$\frac{h_m}{h_e} = \sqrt{\epsilon_0 \mu_0} v_z = \frac{v_z}{c} \quad (5.9)$$

Accelerating voltages in electron-beam devices are seldom higher than 15-20 kV. Consequently, the ratio h_m/h_e is a value of the order of 0.2, i.e., even in the case of comparatively high accelerating voltages the electrostatic deflection is several times that of the magnetic one. In low-voltage devices this difference is still more noticeable.

It should be noted that electrostatic deflection systems feature much lower persistence than the magnetic systems, since the capacitances and inductances of the deflecting plates can be very small, while the distributed capacitance and inductance of deflecting coils cannot be small, particularly with a large number of coil turns. Therefore, electrostatic deflection permits deviation of the deflecting voltage at frequencies up to tens and hundreds of MHz, while in the case of ordinary deflecting coils the system manifests persistence at frequencies of a few dozens of kHz.

Thus both types of deflection have their advantages and disadvantages and in the general case one cannot give any preference to electrostatic or magnetic deflection. Oscillograph tubes are usually based on the electrostatic deflection, while kinescopes are provided with magnetic deflection systems, yet there may appear a necessity of making a special oscillograph tube with magnetic deflection or an economical kinescope with an electrostatic deflection system. Finally, there may be devices in which the beam is deflected in one direction by means of electrostatic deflecting plates and in the other direction (usually at a right angle to the first one), by means of magnetic coils.

Depending on the deflection systems employed there are electron-beam devices with electrostatic, magnetic and combined deflection of the beam. It should be noted that high quality of magnetic focusing can be impaired by the electrostatic deflection. Consequently, the tubes with magnetic focusing are as a rule equipped with a magnetic deflection system, while the tubes with electrostatic focusing may employ both electrostatic and magnetic deflection.

Electrostatic systems deflecting the beam in two directions at right angles to each other are arranged along the path of the beam in series and, as a rule, are carefully shielded from each other. Combining the two electrostatic systems in the space is inexpedient for the following reasons: (1) an increase in the distance between the plates is associated with a decrease in the deflection sensitivity; (2) interference of the fields of both systems considerably distorts the beam path during the deflection; (3) combination of the two systems is associated with a considerable increase in the parasitic

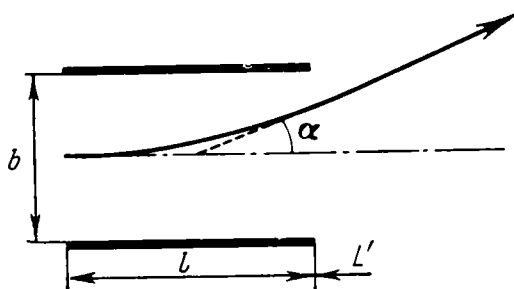
capacitive couplings limiting the utilization of the tube at high frequencies.

The magnetic deflection systems are usually arranged in the tube space so that with the coils in a strictly symmetrical position the total magnetic flux of one pair of the coils piercing the second pair is equal to zero and the change in the magnetic field deflecting the beam in one direction has no effect on the magnetic field of the other pair of coils deflecting the beam in the perpendicular direction. Thus, there is no interconnection of deflecting fields in correctly designed magnetic deflection systems and spatial coincidence of the magnetic systems deflecting the beam in two directions at right angles to each other is quite admissible and expedient.

5.2. Electrostatic Deflection Systems

The simplest electrostatic deflection system is a capacitor consisting of two parallel plates (Fig. 5.1).

The magnitude of the deflection (displacement of the spot on the screen) h when applying a deflecting voltage U_{def} to the plates of



the capacitor can be defined by equation (5.3). Designate the length of the plates as l , the distance between the plates b and the distance from the outlet edge of the plate to the receiver plane (screen) as L' . Assuming that the field between the plate is uniform, replace the coefficient κ in equation (5.3) by l/b . Then the displacement of the spot on the screen is

$$h = \frac{lU_{def}}{2bU_a} (l' + L') = \frac{lLU_{def}}{2bU_a} \quad (5.10)$$

where $l' + L' = L$ is the distance from the screen to the centre of deflection.

One can easily see that the tangent to the parabolic path of the electrons constructed from the point of intersection of the parabola with the plane passing through the outlet edges of the plates intersects the axis at a distance of $l/2$ from the edges of the capacitor. Thus,

in the case of plane-parallel deflecting plates the deflection centre coincides with the geometric centre of the deflection system.

The deflection sensitivity of the given system is equal to

$$\varepsilon = \frac{k}{U_{def}} = \frac{lL}{2bU_a} \text{ [mm/V]} \quad (5.11)$$

In fact the deflection sensitivity of a plane-parallel system is somewhat higher than the theoretical value, since outside the plate edges the field decreases gradually. The presence of stray fields outside the edges of the plates produces an additional deflecting force. The action of this force results in an increase in the beam deflection angle at the same deflecting voltage, i.e., the deflection sensitivity rises up so that practically we have

$$\varepsilon = k \frac{lL}{2bU_a} \quad (5.12)$$

where k is the coefficient taking into account the action of the stray fields ($k > 1$). The value of k depends on the geometric relationships of the system and its location with respect to the other electrodes and the electrically conductive coating on the tube neck. Theoretical

calculation of k is difficult but experimental estimations give $k=1.10-1.15$.

However, even taking into account the action of the stray fields, the sensitivity of a plane-parallel deflection system of a practically acceptable size and at a moderate anode voltage is low. Consequently, the electrostatic deflection systems formed by plane-parallel plates are presently out of use.

As seen from formula (5.11), the sensitivity can be increased by reducing the distance b between the plates. It should be noted, however, that a decrease of the gap between the parallel plates results in a decreased deflection angle due to possible cutoff

of the electron beam by the edge of the plate. Nevertheless, some shortening of this gap at the inlet side is quite possible. In this case we obtain a system of non-parallel ("tilted") plates (Fig. 5.2).

The deflection of the electron beam by the field of such plates can approximately be calculated ignoring the stray fields outside the edges of the plates and the axial component of the field intensity between the plates. Under these assumptions the field intensity between the plates at a distance z from the entry edge of the plates is equal to

$$E_y = \frac{U_{def} l}{b_1 l + (b_2 - b_1) z} \quad (5.12a)$$

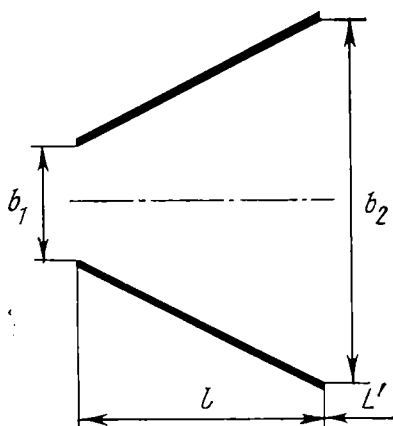


Fig. 5.2. Deflection system formed by tilted plates

where l is the length of the system; b_1 and b_2 are respectively the entry and exit spans between the plates.

The transverse component of the velocity acquired by the electrons at the exit from the deflection system is expressed by

$$\begin{aligned} v_y &= \frac{e}{m} \int_0^l E_y dt = \frac{e}{m} \int_0^l E_y \frac{1}{v_z} dz = \frac{e}{m} \times \frac{U_{def}}{v_z} \int_0^l \frac{dz}{b_1 + \left(\frac{b_2 - b_1}{l}\right) z} = \\ &= \frac{e}{m} \times \frac{U_{def}}{v_z} \times \frac{l}{b_2 - b_1} \ln \frac{b_2}{b_1} \quad (5.13) \end{aligned}$$

The angle of inclination of the beam to the axis at the system exit can be determined by the ratio of the transverse velocity component (acquired in the deflection system) to the longitudinal velocity component of the electrons

$$\tan \alpha = \frac{v_y}{v_z} = \frac{e}{m} \times \frac{U_{def}}{v_z^2} \left(\frac{l}{b_2 - b_1} \right) \ln \frac{b_2}{b_1} \quad (5.14)$$

Expressing v_z through the gun accelerating voltage U_{ac} , we obtain a formula for the deflection of the beam on the screen at a distance L' from the deflection system to the screen

$$h = (L' + l') \tan \alpha = \frac{U_{def}}{2U_{ac}} \left(\frac{l}{b_2 - b_1} \ln \frac{b_2}{b_1} \right) (L' + l') \quad (5.15)$$

where l' is the distance from the deflection centre to the exit edge of the plates.

This distance can be estimated by the maximum deflection $\alpha_{\max}$ for the given system

$$l' = b_2 \cot \alpha_{\max} \quad (5.16)$$

In the case of non-parallel plates we have $l' > \frac{l}{2}$, i.e., the deflection centre of the beam does not coincide with the centre of the system. Denoting the distance from the deflection centre to the screen as $(l' + L') = L$, determine the deflection sensitivity of the system formed by non-parallel plates

$$\varepsilon = \frac{h}{U_{def}} = \frac{1}{2U_{ac}} \times \frac{lL}{b_2 - b_1} \ln \frac{b_2}{b_1} \quad (5.16a)$$

Comparison of expressions (5.16a) and (5.11) shows that with the same overall dimensions, i.e., with the same values of l and b_2 in (5.16) equal to b in (5.11), the sensitivity of the system formed by non-parallel plates is approximately 1.5 times that of the plane-parallel system.

It is evident that the maximum possible deflecting action of the electrostatic field on the electron beam takes place when the force acting on the electron is perpendicular to the direction of its motion

in the whole region of deflection. Such an ideal electrostatic system (Fig. 5.3) is formed by two bent plates, where the distance b_1 between the plates at the beam entry may be equal to the diameter of the electron beam.

At the maximum deflection angle the beam as if slid over the surface of the plate without touching it. Since the electrically conductive plate is an equipotential surface, the field lines of force orthogonal to the plate are always perpendicular to the electron beam, i.e., the above-mentioned maximum possible deflecting action is provided.

Analytical calculations show that the curve of bending such optimum deflecting plates is described by the exponential function.

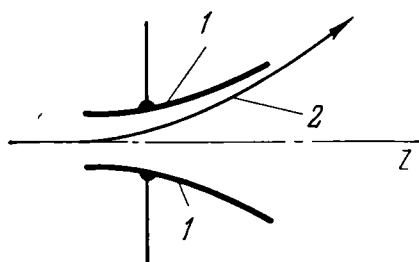


Fig. 5.3. Optimum-shape deflecting plates
1—deflecting plates; 2—electron beam

In this case the beam deflection centre does not coincide with the centre of the system but is somewhat displaced aside from the entry edge of the plate. The deflection sensitivity of the given system is approximately twice that of the system formed by plane-parallel plates having the same overall dimensions.

In spite of high sensitivity, the systems formed by optimum bent plates have not gained wide recognition, mainly due to the difficulties associated with accurate making and assembly. The ideal deflection system is technologically inexpedient, particularly during mass production. However, if the smooth curve describing the centre of the optimum plate is replaced by a broken line consisting of several lengths of a straight line so that the bend angles lie on the optimum curve, one can obtain a deflection system convenient in making and assembly and having almost the same characteristic as the ideal system. Of course, the nearer the broken line to the optimum curve, i.e., the greater the number of bends of the plates, the closer is the sensitivity to the optimum value. On the other hand, the lower the number of bends of the plates, the better the technological characteristics of the system. Calculations and experiments have shown that the sensitivity of the system having even a single bend is close to the ideal value. Consequently, deflection systems formed by single-bend plates have found wide application in electron-optical

devices (in Fig. 5.4 the dash line shows the shape of the optimum plates).

Of course, the sensitivity of such a system depends on the relationship between the lengths of both plane sections of the plates (on the position of the bending angle). Analytical calculation shows that the sensitivity diagram has a mildly sloping maximum, the position of this maximum varying depending on the value of the selected maximum deflection angle. In any case the maximum sensitivity is obtained when $l_1 < l_2$. The deflection sensitivity of the given system is

$$\varepsilon = \frac{1}{2U_{ac}} \left[\frac{l_1 \ln \frac{b_2}{b_1}}{b_2 - b_1} + \frac{l_2 \ln \frac{b_3}{b_2}}{b_3 - b_2} \right] L \quad (5.17)$$

where $L = l' + L'$ is the distance from the screen to the deflection centre, in which case the beam deflection centre does not coincide with the system centre, $l' > (l_1 + l_2)/2$.

Widely used in commercial oscillograph tubes are deflection systems formed by single-bend plates consisting of a parallel and tilted parts (Fig. 5.5). These systems are very convenient in manufacture.

The angle of inclination of the beam passing through such a system can be roughly calculated if, as the case is with tilted plates, the axial component of the field strength and the stray fields are neglected. Then the transverse velocity component v_y acquired by the electrons of the beam passing through the system is equal to

$$v_y = v_{y1} + v_{y2} \quad (5.18)$$

where v_{y1} is the transverse velocity component of the electrons passing through the section l_1 between the parallel plates; v_{y2} is the transverse velocity component of the electrons passing through the section l_2 between the tilted plates.

The value v_{y2} is determined by equation (5.13), while the value v_{y1} can be found from formula (5.1):

$$v_{y1} = v_z \tan \alpha_e = \frac{e}{m} \times \frac{U_{def}}{b_1} \times \frac{l_1}{v_z} \quad (5.19)$$

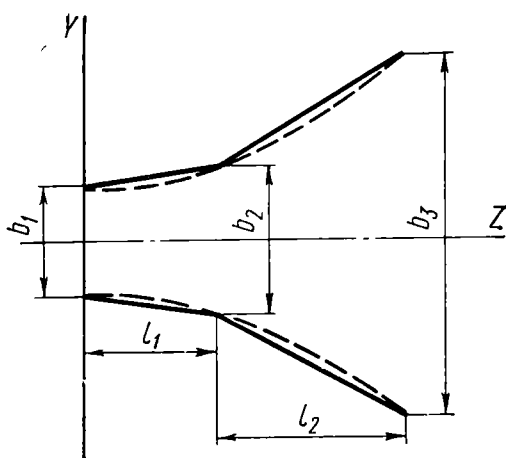


Fig. 5.4. Single-bend deflecting plates close to optimal

The transverse velocity component at the deflection system exit according to (5.13), (5.18) and (5.19) is as follows:

$$v_y = \frac{e}{m} \times \frac{U_{def}}{v_z} \left(\frac{l_1}{b_1} + \frac{l_2}{b_2 - b_1} \ln \frac{b_2}{b_1} \right) \quad (5.20)$$

while the value of deflection of the beam on the screen is

$$h = (L' + l') \frac{v_y}{v_z} = \frac{U_{def}}{2U_{ac}} \left(\frac{l_1}{b_1} + \frac{l_2}{b_2 - b_1} \ln \frac{b_2}{b_1} \right) L \quad (5.21)$$

where $L = L' + l'$ is the same value as in (5.15).

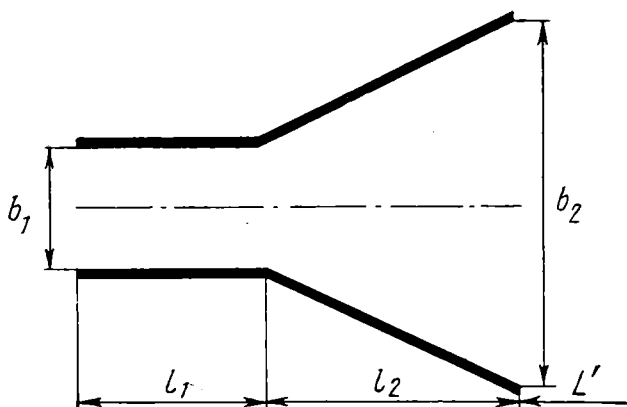


Fig. 5.5. Single-bend deflecting plates with parallel portion

The beam deflection centre, as the case is with the non-parallel plates, does not coincide with the centre of the system, $l' < (l_1 + l_2)/2$. The sensitivity

$$\varepsilon = \frac{1}{2U_{ac}} \left(\frac{l_1}{b_1} + \frac{l_2}{b_2 - b_1} \ln \frac{b_2}{b_1} \right) L \quad (5.22)$$

The maximum sensitivity is obtained at l_1 somewhat less than l_2 . As compared with a plane-parallel system of the same dimensions [$(l_1 + l_2) = l_{\text{=}}$, $b_2 = b_{\text{=}}$], the sensitivity of the given system is higher than that of the plane-parallel system approximately by a factor of 1.8.

Nevertheless, it should be noted that the deflection sensitivity of electrostatic deflection systems when the dimensions of the system and the device as a whole (L') are practicable and a practicable value of accelerating voltage seldom exceeds 1 mm/V.

Since the electrostatic systems deflecting the beam in two directions at right angles to each other cannot be aligned in space, the deflection sensitivity of this system is different in the vertical and horizontal directions due to the difference in L' —the distance between the plates and the screen. It is clear that the plates located nearer to the electron gun (lower plates) have a higher sensitivity.

Electrostatic deflection is often used in oscillograph tubes. If that is the case, the requirement of high sensitivity is placed only on one pair of the plates to which the signal being analyzed is applied. The lower plates having a higher sensitivity must be selected as the "signal" plates. Furthermore, sometimes the sensitivity of the lower plates may be increased by increasing their length without changing the overall dimensions of the device itself. If that is the case, the length of the upper plates located nearer to the screen must be reduced. Thus, the increase in the sensitivity of the lower "signal" plates is obtained at the expense of reducing the sensitivity of the upper plates which are usually employed in oscillographs for sweeping the beam over the screen, a saw-tooth voltage being applied to the latter plates. Since the amplitude value of the sweeping voltage can be sufficiently high, while high sensitivity of the signal plates is often a predominant requirement, the above-described method is quite practicable.

5.3. Magnetic Deflection Systems

To deflect an electron beam by a magnetic field use is made of deflection systems consisting of current coils mounted on the neck of the cathode-ray tube. The linearity and minimum defocusing are provided on condition that the magnetic field at any moment of time is close to uniform in the entire region of beam deflection. Therefore, in order to deflect the beam in one direction, it is necessary to have two coils symmetrically arranged on the tube neck. Since the beam is usually deflected in two directions at right angles to each other, the magnetic deflection system, as a rule, has four coils producing approximately uniform mutually perpendicular magnetic fields. In the case of a uniform field the displacement of the beam on a screen placed at a distance L' from the edge of the area of action of a magnetic field with induction B can be determined according to (5.2) as follows:

$$h_m = (l' + L') \tan \alpha_m = \sqrt{\frac{e}{m}} \times \frac{BlL}{\sqrt{2U_{ac}}} \quad (5.23)$$

where l is the length of the deflecting magnetic field along the axis of the device; $L = l' + L'$ is the distance from the deflection centre to the screen.

With a very limited magnetic field the deflection centre approximately coincides with the centre of the deflection system, i.e., $l' \approx l/2$.

The deflection sensitivity can be found from equation (5.23) as the ratio h/B . However, definition of the sensitivity in mm per gauss is inconvenient for practical purposes, since the deflecting signal is introduced into the system as an electric current flowing

in coils having a certain number of turns. Taking into account that the magnetic induction is proportional to the magnetizing force (ampere-turns), it is expedient to calculate the sensitivity in mm per ampere-turn.

Assuming that $B = \chi n I_{def}$, where n is the number of turns, I_{def} is the deflecting current and χ is a constant coefficient depending on the geometric size of the coils, their mutual position and the presence of ferromagnetic cores, we obtain the following expression for the deflection sensitivity:

$$\varepsilon_m = \sqrt{\frac{2e}{m}} \times \chi \frac{lL}{\sqrt{U_{ac}}} \quad (5.24)$$

Substituting the numerical values of e and m and denoting $\chi l = \xi$, we have

$$\varepsilon = 0.3\xi \frac{L}{\sqrt{U_{ac}}} [\text{mm/A} \cdot \text{turn}] \quad (5.25)$$

where U_{ac} is in volts and L is in millimetres. The quantity ξ is constant for the given deflection system and can be found experimentally. For example, with circular coils without a ferromagnetic core having the diameter and spaced at a distance l from each other, the coefficient ξ is equal to $(0.4-0.5) \text{ V}^{1/2} \text{A} \times \text{turns}$.

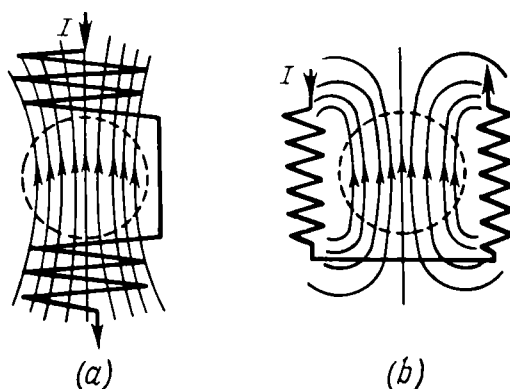


Fig. 5.6. Magnetic deflection system with external magnetic circuit
(a) with series summed magnetic fluxes; (b) with parallel summed magnetic fluxes

The constant sensitivity (linear deflection) can be obtained only till the electron beam remains inside a uniform magnetic field at any possible deflection angles. Therefore, during the development of magnetic deflection systems the coils are constructed so that the field is close to uniform within the greatest possible region. Two types of magnetic deflection systems are presently employed in electron-beam devices: systems with series summed magnetic fluxes and systems with parallel summed magnetic fluxes (Fig. 5.6).

The systems with series summed magnetic fluxes are more economical, since in this case a greater portion of the magnetic energy stored in the coils is used for deflection of the beam. In the systems with parallel summed magnetic fluxes, the area of deflection is pierced only by the stray field, while the greatest portion of the energy stored inside the coils is not used for deflection of the beam. However, the parallel summation of the fluxes provides comparatively simple means for obtaining an approximately uniform field in a larger area.

Better economics of the deflection performance and, therefore, a gain in the sensitivity under otherwise equal conditions can be obtained by using magnetic circuits of ferromagnetic materials concentrating magnetic energy. Therefore, where the deflection systems operate at relatively low frequencies their coils are usually equipped with ferromagnetic cores and shields. The coils of the systems with parallel summed fluxes are mounted on a core made

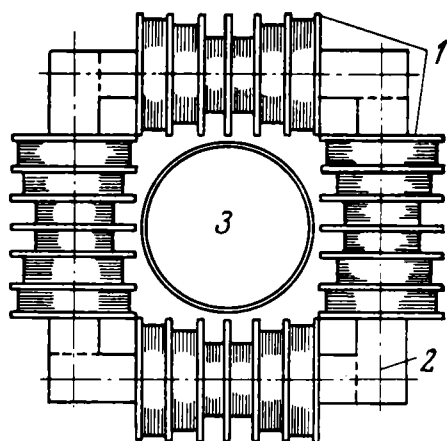


Fig. 5.7. Magnetic deflection system with ferromagnetic core
1—coils; 2—core; 3—core neck

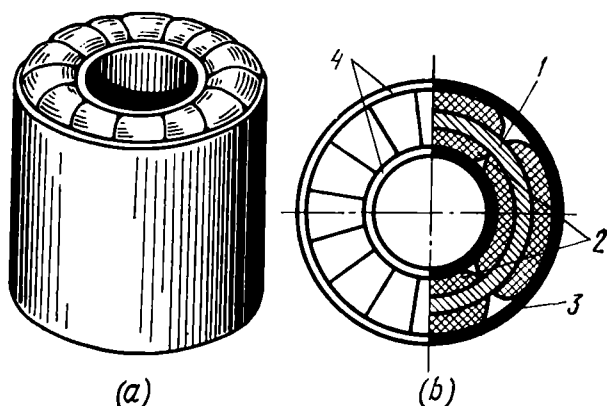


Fig. 5.8. Magnetic deflection system with internal magnetic circuit
(a) general view; (b) section view; 1 — magnetic circuit; 2—horizontal deflection coils; 3—vertical deflection coils; 4—plastic sleeves

of a high-quality high-permeability magnetic material. An example of such a deflection system whose magnetic circuit is assembled of L-shaped laminations is given in Fig. 5.7.

The windings of the coils are usually made sectional, since a sufficiently uniform field is obtained by selecting the number of

turns in each section. Furthermore, the sectionalization contributes to reduction of distributed capacitance.

To reduce the external size of the system, the magnetic circuit is given a cylindrical shape (Fig. 5.8). Since in the systems with parallel summed fluxes the magnetic circuit is placed inside the coils, such systems are commonly referred to as *systems with internal magnetic circuits*.

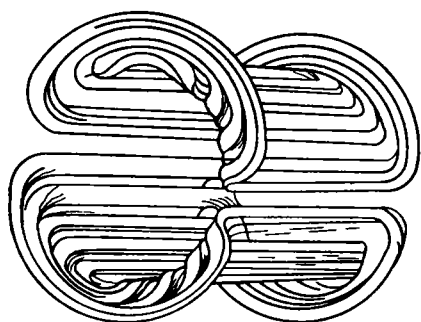


Fig. 5.9. Sectioned deflecting coils

The systems with series summed fluxes usually have no internal magnetic circuits and are formed by coils directly adjoining the tube neck. The coils are saddle-shaped so that the turns of the winding embrace the tube neck. The ends of the coils are bent outside to reduce the stray fields beyond the deflection system. The coils are

sometimes assembled of separate sections for reducing the distributed capacitance. The coils for horizontal deflection of the beam are shown in Fig. 5.9.

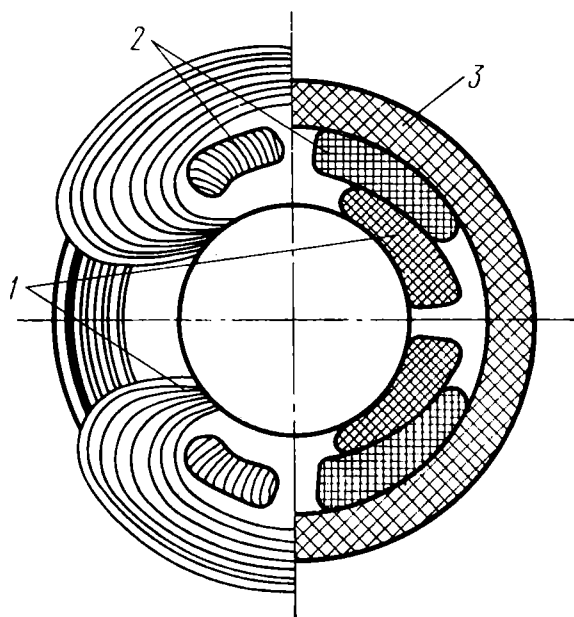


Fig. 5.10. Magnetic deflection system with external magnetic circuit
1—horizontal deflection coils; 2—vertical deflection coils; 3—magnetic circuit

The systems with series summed fluxes are usually enclosed in a cylindrical case of a ferromagnetic material. This case is used for shorting the magnetic lines of force emerging outside the coils. The bent edges of the coils are located outside the case. In such

a system the magnetic circuit is placed outside the coils. Therefore, the systems with series summed fluxes are often called the *systems with external magnetic circuits*. A sectional view of such a system is shown in Fig. 5.10. The systems with an external magnetic circuit are economical, compact and with correctly selected geometric relations allow for obtaining sufficiently large deflection angles at good linearity and comparatively small defocusing of the beam. The systems with an external magnetic circuit are widely used as deflection systems for kinescopes.

The magnetic deflection systems are often classified by their construction features: systems without magnetic circuits, systems with external magnetic circuits and systems with internal magnetic circuits (toroidal type). Modern kinescopes with large beam deflection angles are equipped with combined systems—one pair of coils has an internal magnetic circuit which simultaneously serves as an external magnetic circuit of the second pair of coils. Also known in the art of electron optics are stator-type systems the construction of which is similar to the stator of an electric motor. Such systems are featured by high efficiency, but due to the presence of salient teeth of magnetic circuit the field is nonuniform at large deflection angles. In addition, such systems are complicated in manufacture. Owing to these disadvantages, the stator-type systems have not gained wide recognition.

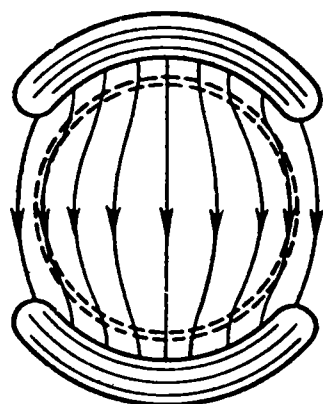


Fig. 5.11. Barrel-shaped field produced by uniformly wound coil turns

As mentioned above, in order to provide constant sensitivity of a magnetic deflection system, i.e., linear deflection of the beam, the field must be approximately uniform within the whole region of the deflection of the beam. One can easily see that with uniform distribution on the turns on the deflecting coils the field in the deflection region will be barrel-shaped (Fig. 5.11), i.e., substantially nonuniform.

Simple calculations allow one to find such distribution of the turns with which it is possible to obtain an adequately uniform deflecting field.

Consider the coils with series summed fluxes without iron cores. The coils are bent in such a way that the system embraces the tube neck (Fig. 5.12).

If the internal radius of curvature of the coil is designated through r , the length of the line of force acting on the turn disposed at an angle α to the mid-plane of the system will be equal to (see Fig. 5.12)

$$l_{\alpha} = 2r \sin \alpha$$

Using the full current law, we obtain for a uniform field

$$\oint_{l_\alpha} B \, dl = 2Br \sin \alpha = \mu_0 (nI)_\alpha \quad (5.27)$$

where (nI) is the number of ampere turns within the angle α .

The function of distribution of the turns F_α can be found taking into account the requirement for uniformity of the field ($B = \text{const}$)

$$2 \int_0^\alpha F(\alpha) \, d\alpha = (nI)_\alpha \quad (5.28)$$

where the multiplier 2 makes account for the action of the line of force on the turns of both coils. Taking into account (5.27), find

$$2 \int F(\alpha) \, d\alpha = \frac{2Br \sin \alpha}{\mu_0} \quad (5.29)$$

from which

$$F(\alpha) = \frac{Br}{\mu_0} \cos \alpha \quad (5.30)$$

Equation (5.30) shows that a uniform field is produced when the density of winding of the turns is maximum near the edges and

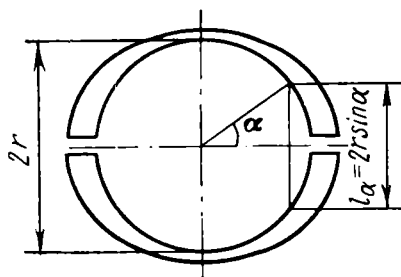


Fig. 5.12. On calculation of function of coil turns distribution

drops towards the centre by the cosine law. Practically it is difficult to make coils with permanently varying number of turns; however, deflecting coils of separate sections with different number of turns satisfy the requirement placed by equation (5.30). The use of an external magnetic circuit also contributes to levelling the field.

During the discussion of electrostatic deflection it was shown that an increase in the sensitivity could be obtained by reducing the distance between the deflecting plates. The minimum distance between the plates at the beam side is limited only by the cross section of the beam in the space of the deflection system. The magnetic deflecting coils are located outside the vacuum envelope of the device. They are mounted on the tube neck. Consequently, the gap in which the deflecting magnetic field is concentrated is limited

by the diameter of the tube neck rather than by the parameters of the electron beam. It is clear that the lower the distance between the coils (the smaller the gap), the higher magnetic energy can be concentrated in the deflection space with the same magnetizing force (constant ampere turns). Calculations show that proportional reduction of all geometric sizes of a magnetic deflection system with a fixed number of ampere turns results in the same increase in the magnetic induction in the deflection space. Since the deflecting coils are usually arranged directly on the tube neck, the sensitivity is preferably increased by reducing the neck diameter. This accounts for a comparatively small diameter of the necks of electron-beam tubes with the magnetic deflection of beam.

Modern magnetic deflection systems allow for obtaining deflection sensitivities up to 1 mm/ampere turn. In the case of electrostatic deflection the deflecting plates placed into the tube are usually assembled together with an electron gun, and the quality of the electrostatic system largely depends on the accuracy of assembly of the internal elements of the device. A magnetic deflection system mounted outside the tube is not mechanically connected with the elements of the electron gun. Therefore, mounting of the deflection system is an independent operation. Selection of the position of the system on the tube neck and the accuracy of its adjustment are the only factors which govern the deflection parameters. In particular, displacement of the deflection system along the tube axis may result in limitation of the beam deflection angle due to the cutoff of the beam by the adapter portion of the bulb (between the neck and cone), while misalignment of the deflection system may cause additional aberrations during the deflection with resultant decrease in the resolving power and distortion of the raster, when sweeping the beam on the screen.

5.4. Distortion in Electron-beam Deflection

Our discussion of deflection systems was based on an assumption that equations (5.1) and (5.2) derived on the basis of small deflection angles also hold for comparatively large deflection angles. Strict fulfilment of these equations means that the systems are ideal, i.e., the deflection linearly depends on the value of a deflecting voltage (or current) and the electron beam formed by the gun is deflected by the system as a whole without disturbing the focusing.

However, experimental study of deflection systems shows that disturbance of the linearity and distortion of the spot shape on the screen, i.e. defocusing, start being noticeable even at comparatively small deflection angles of 5-8°. These distortions called the aberrations of deflection systems may be subdivided into two groups:

- (1) Disturbances to the linearity observed as a distorted raster on the screen when sweeping the beam;
- (2) Disturbances to the focusing observed as a distorted shape of the spot on the screen.

Let us consider the distortion of the shape of the raster during the electrostatic deflection. It is evident that the potential in the

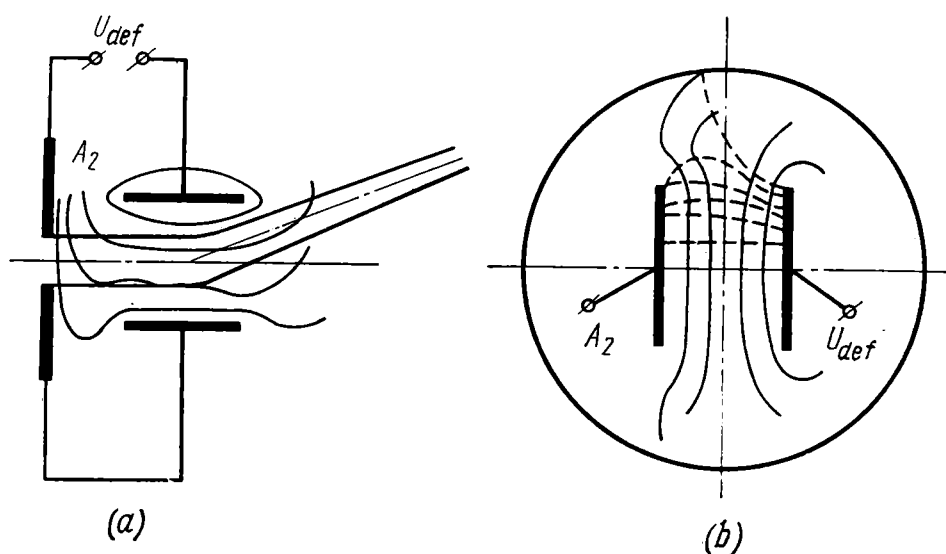


Fig. 5.13. Asymmetric connection of deflecting plates
(a) field in longitudinal plane; (b) field in transverse plane

deflection region should be close to that of the exit electrode of the gun, otherwise the energy of the electrons of the beam passing the deflection system would be varied with resultant disturbance to the focusing. One can easily see, however, that the equality of these potentials within the entire deflection region is impossible, as there is no deflecting force in an equipotential space. Thus the beam electrons passing through the electrostatic deflection system inevitably fly through a region at a potential differing from the potential of the exit electrode of the gun, i.e. electrostatic systems, besides deflecting the beam, always change the energy (velocity) of the electrons.

With the so-called asymmetric connection one plate of the deflection system is directly connected to the exit electrode of the gun. In this case the field is asymmetric with respect to the centre-plane (Fig. 5.13).

The potential in the centre-plane differs from the anode voltage of the gun by a value $\pm U_{def}/2$. Thus, in the case of asymmetric connection the sensitivity ε' for small deflection angles can be

calculated by the formula

$$\epsilon' = \frac{lL}{2b \left(U_{ac} \pm \frac{U_{def}}{2} \right)} = \frac{\epsilon}{1 \pm \frac{U_{def}}{2U_{ac}}} \quad (5.31)$$

From expression (5.31) it follows that the deflection sensitivity depends on the deflecting voltage, i.e., the linearity of the deflection is disturbed. Equation (5.31) also shows that the violation of the linearity is the higher, the greater is the value of U_{def} , i.e., the nonlinear distortions increase with the beam deflection angle.

With asymmetric connection of both pairs of deflecting plates the deflection of the beam by the second (upper) pair of plates depends on the amount of beam deflection by the first (lower) pair of plates. As shown in Fig. 5.13*b* when use is made of the asymmetric connection, the field is drastically distorted at the edges of the plate, that is not connected to the anode of the gun, in which case some lines of force (dash lines on the figure) are shorted not to the adjacent plate but to the electrically conductive inner coating of the tube neck which is at the potential of the gun anode. In this case the electrostatic field at the edges of the plates has a component parallel to the plate plane. This component is responsible for a force deflecting the beam near the edge of the plate not connected to the gun anode, in a direction at right angle to the main direction of deflection of the beam by the given pair of plates. In other words, the horizontal deflection plates connected asymmetrically cause additional deflection of the beam in a vertical direction. Additional deflection takes place only when the beam passes by the edges of the plate, since in the centre-plane the lines of force of the field are at right angle to the surface of the deflecting plates and the horizontal-deflection plates deflect the beam only in a horizontal direction. Thus, the additional deflection of the beam is observed only if the beam has been predeflected by the lower pair of plates before entering the space between the upper plates. Since the lines of force are markedly curved only near the plate, which is not connected to the gun anode, the additional deflection in a vertical direction will be the greater, the nearer the beam is to the plane of this plate.

The presence of additional deflection may formally be considered as a change in the deflection sensitivity of the lower pair of plates depending on the angle of deflection of the beam by the upper pair of plates. Since the vertical deflecting force is directed downwards at the upper edge of the plate and upwards at the lower edge of the plate (Fig. 5.14), the apparent sensitivity of the lower pair of plates is reduced when the beam approaches the surface of the upper plate, which is not connected to the gun anode.

As a result, when the beam is swept on the screen surface by a saw-tooth voltage with a constant amplitude, the raster on the screen is not rectangular but trapezoidal with a lower base at the side of the plate, which is not connected to the anode. By the shape of the raster this distortion is usually called trapezoidal.

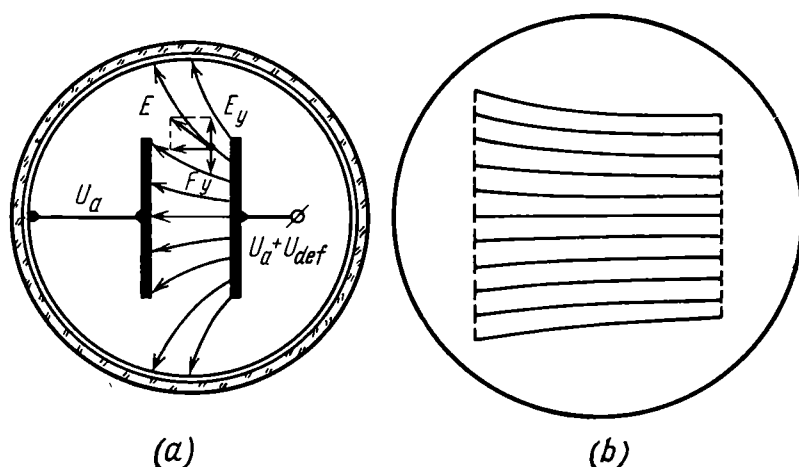


Fig. 5.14. Trapezoidal raster distortion
(a) field picture; (b) raster shape

The linearity is also perceptibly disturbed due to the presence of stray fields outside the edges of the deflecting plates. The stray fields between the deflection system and the exit electrode of the

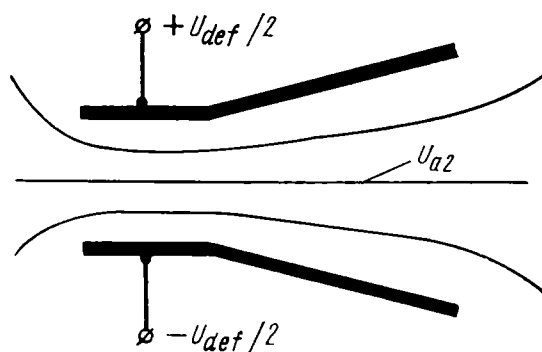


Fig. 5.15. Symmetric connection of deflecting plates

gun, as well as between the two pairs of plates and the shield located between the horizontal-deflection and vertical-deflection plates, produce in these regions cylindrical lenses causing additional deflection of the beam.

The distortions can be reduced materially by using the so-called symmetric connection of the deflecting plates (Fig. 5.15).

In this case both plates of each pair are interconnected through a voltage divider composed of two identical high-ohmic (a few megohms) resistors, while the mid-point of the divider is connected to the gun anode. With symmetric connection the fields in the deflection space are symmetric with respect to the centre-plane of the deflection system and the trapezoidal distortions become practically eliminated. In the same manner, the electron velocity variation due to nonequipotentiality of the deflection space is lower and the deflection sensitivity features smaller dependence on the deflecting voltage. Finally, the cylindrical lenses formed by the stray fields also become symmetric and their action manifests itself to a smaller extent. Consequently, the symmetric connection of deflecting plates is predominant in the field of electron optics.

When the plates cannot be connected symmetrically for any reason, the trapezoidal distortion can be reduced considerably by using the upper plates of special shape (Fig. 5.16).

As seen from the drawing, the plate, which is not connected to the anode of the gun, has bent edges so that the field near the edge of the plates is more uniform and the beam near the plate edge is not subjected to additional vertical deflection.

The distortions can be reduced by selecting the shape of the slots in the shields located between the vertical-deflection plates and horizontal-deflection plates, as well as between the gun anode and the deflection system. The harmful effect of the cylindrical lens formed between the gun anode and the inlet edge of the deflection system can sometimes be compensated for by slightly varying the mean potential between the deflecting plates with respect to the gun anode.

It should be noted that the nonequipotentiality of the deflection space is a principal disadvantage of electrostatic deflection. This disadvantage is particularly perceptible at comparatively great beam deflection angles and cannot be eliminated. Therefore, electrostatic systems cannot be strictly linear. The degree of nonlinearity can be estimated by the nonlinearity factor q defined as a ratio of the difference of displacement of the spot on a screen of an actual tube (h') and displacement of the spot at deflection by the ideal system

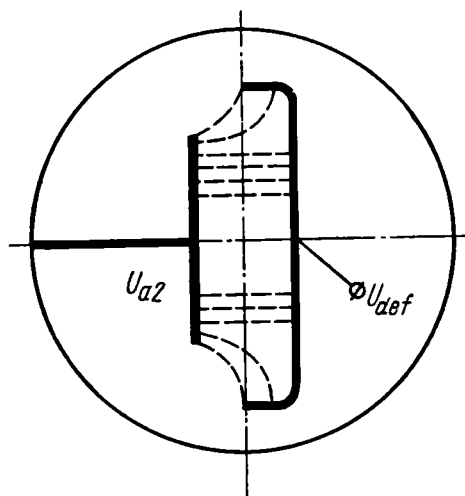


Fig. 5.16. Shape of upper pair of plates of system with reduced trapezoidal distortion

(h) to the total value of displacement

$$q = \frac{h' - h}{h} = \frac{\Delta h}{h} \quad (5.32)$$

The nonlinearity factor rises with the beam-deflection angle. A considerable value of q at beam deflection angles exceeding $15-20^\circ$ is one of the factors limiting the use of electrostatic systems at rather large deflection angles. The nonlinearity factor of electrostatic systems of modern oscillograph tubes is approximately equal to 0.5-2%.

Magnetic deflection offers smaller disturbance to the linearity than the electrostatic deflection at the same deflection angles. This is mainly due to the fact that the magnetic deflecting force (Lorentz force) is always at right angle to the direction of the velocity of electrons and this velocity (electron energy) is constant at magnetic deflection. Nevertheless, analysis of magnetic deflection shows that the linearity is disturbed even in the case of perfectly uniform magnetic fields.

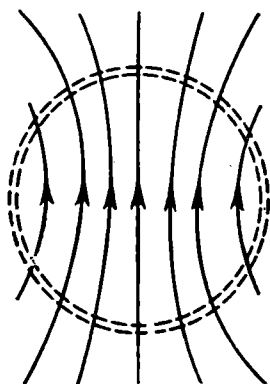


Fig. 5.17. Pillow-shaped deflecting field

Assume that both magnetic deflection systems are fed with currents varying with time by the saw-tooth law (ordinary TV sweeping). Also assume that at any moment of time the deflecting fields of both systems are strictly uniform within the whole deflection zone and that these fields are at right angles to each other. It is clear that in this case a rectangular raster appears on the screen, i.e., the image is

not distorted. However, the time of occurrence of the electrons in the field of one system depends on the magnitude of deflection of the beam by the second system (the fields of the systems are supposed to be coincident in space as it usually takes place in practice). For example, the time of occurrence of the electrons in the field of the vertical-deflection coils is the greater, the higher the angle of deflection of the beam due to the action of the field of the horizontal-deflection system, since in this case the electron path is prolonged. Consequently, the horizontal pulse transmitted to the electrons of the beam at a constant induction of the horizontal deflecting field will be the larger, the greater is the vertical deflection angle.

This affects the linearity: the raster on the screen becomes pillow-shaped even with ideally uniform deflecting fields. The aberration can be much reduced by using deflecting fields of a slight pillow shape (Fig. 5.17).

In this case, with great deflection angles, the electrons are admitted into the zone having a lower induction than the zone near the axis and the pin-pillow distortion of the raster is reduced due to a decrease in the deflecting force despite of greater time of electrons residence in the field. This correction is practically effected by shaping the bent edges of the coils for the purpose. This method gives good results only at comparatively low deflection angles. When the beam deflection angle exceeds 45° , it is good practice to use small permanent magnets arranged behind the exit edge of the deflection system parallel to the direction of sweeping. The permanent magnets produce a field opposite to the field of the deflection system. In this case the deflecting force produced by the field of the deflection system is somewhat reduced as the beam approaches the edge of the coil (the field of the deflection system is compensated for by oppositely directed field of the permanent magnet) and the pillow distortion of the raster is perceptibly reduced.

In addition to the above-considered distortion of the raster, because of the magnetic deflection due considerations should be made of the deflection nonlinearity caused by the nonuniformity variations of the deflecting fields. Practically it is very difficult to produce a uniform (or slightly pillow-like) field in a large region, which is necessary in the case of large deflection angles (higher than 30°). One should also take into account the action of the stray fields which are always present outside the planes limiting the entry and exit ends of the deflection system.

If the field in the deflection region is barrel-shaped (Fig. 5.18), it results in pillow distortion of the raster. This distortion appears due to the fact that the Lorentz force causing the deflection is not parallel to the axis OX and at any point is directed along the tangent to the curve intersecting the lines of force at right angles.

Furthermore, since the magnetic induction is higher near the edges of the deflecting field (the lines of force are thickened to the edges), the horizontal deflection is greater for the beam shifted vertically. Thus, the vertical edges of the raster are also distorted. On the contrary, in the case of a pillow deflecting field (Fig. 5.18b), the Lorentz force is directed along the barrel curve. Due to the looseness of the lines of force to the edges of the field, the amount of horizontal deflection is dependent on the vertical displacement of the beam and drops with an increase in the distance from the centre plane. If that is the case, the raster becomes barrel-shaped.

The magnitude of this raster distortion can be estimated by the sag of the line (Δy in Fig. 5.18). Calculations show that the sag largely depends on the beam-deflection angle: it is approximately proportional to the third power of the value of the deflection angle

$$\Delta y = k\alpha^3 \quad (5.33)$$

Thus, even a small barrel configuration of the deflecting field can result in considerable distortion of the raster at great deflection angles. At the same time, a pillow shape of the deflecting magnetic field may prove to be useful, since in this case, as has been shown above, the raster distortion due to the use of a strictly uniform deflecting field is compensated for.

The second group of aberrations of deflection systems distorting the shape of the spot on the screen is caused mainly by a final cross-sectional area of the beam in the deflection space. These aberrations

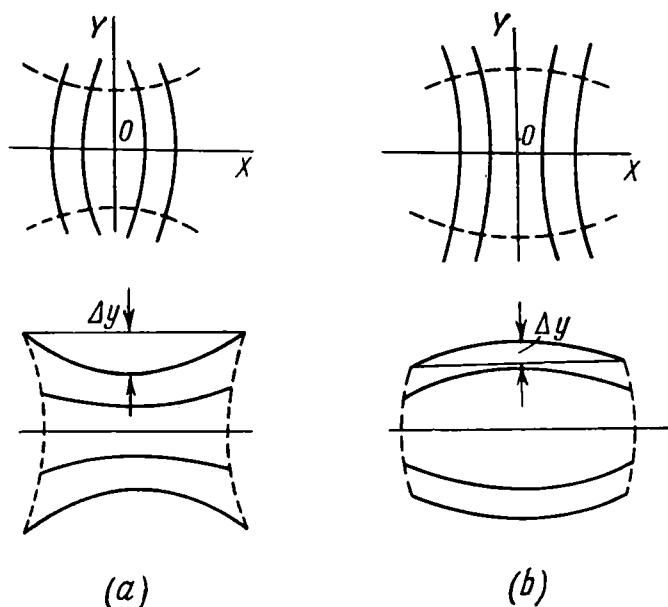


Fig. 5.18. Raster distortion with pillow-shaped (a) and barrel-shaped (b) deflecting fields

are practically present at all times, since the beam at the exit of the gun cannot be made infinitely thin. Furthermore, it is often expedient to have not too small aperture angle from the screen side (see Section 3.1).

Figure 5.19 illustrates defocusing (astigmatism) when use is made of the electrostatic deflection.

Assume that a circular beam is deflected by a uniform field of plane-parallel plates of an electrostatic deflection system. The undeflected beam (dash lines in Fig. 5.19) are focused into a point in the screen plane. Now assume that the deflecting field abruptly drops to zero beyond the plates. Finally, assume that the plates are connected symmetrically, i.e. the potential in the centre-plane of the deflection system is equal to the potential of the exit electrode of the gun. It is clear that the electrons moving along the paths lying in the so-called sagittal plane, i.e. in the plane perpendicular to the beam-deflection plane (Fig. 5.19, plane *ab*), are subjected

to the same action of the deflecting field and remain in the plane for the same time regardless of the fact whether the path passes near point a or b . At the same time, the beams, which before the deflection occupy the meridian plane (Fig. 5.19, plane cd), are deflected differently, depending on their original position, i.e., depending on whether they were near the point c or d . Indeed, the electrons moving in the meridian plane near the point c have somewhat lower velocity than the electrons moving near the point d , since the potential at the point c is somewhat lower and the potential at the point d is a bit higher than the potential in the centre-plane. It is evident that this potential difference and, therefore, the velocity difference are the greater, the higher is the deflecting voltage. Because of

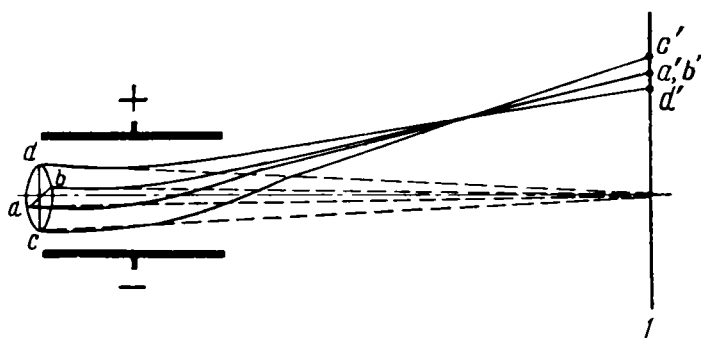


Fig. 5.19. Astigmatism of electrostatic deflection system
1—screen

the difference in the velocities, the electrons moving near point c are subjected to the action of the deflecting field longer and are more deflected than the faster electrons flying near point d .

Thus, the electrons moving along the paths lying in the meridian plane are intersected not in the screen plane but closer to the deflection system and formed on the screen, instead of a point (circular spot), is a vertical line (ellipse). This distortion of the spot shape is known as *astigmatism of deflection systems*, since the aberration figure is similar to that obtained at astigmatism of the focusing lens of the gun.

The deflection system astigmatism depends on the cross section of the beam and on its deflection angle. One may approximately consider that the size of the aberration figure (the larger axis of the ellipse) depends linearly on the aperture angle and, quadratically on the deflection angle.

In actual deflection systems the defocusing at great deflection angles is caused by one more factor. The stray fields outside the plate edges generally form a cylindrical lens whose axis lies in the sagittal plane. This lens causes focusing effect on the beams moving near the meridian plane and these beams intersect before the screen

plane. At the same time, the cylindrical lens does not act on the sagittal beams and they produce a crossover image in the screen plane. Due to the action of the cylindrical lens the spot on the screen is distorted—it acquires an elliptical shape.

The astigmatism caused by the focusing action of the cylindrical lens occurs even at a small deflection angle and drastically increases with this angle. This is accounted for by the fact that the potential gradient between the deflection system and the second anode of the gun (or the shield at the potential of the gun exit electrode) rises up with the deflecting voltage thereby increasing the optical strength of the cylindrical lens.

The astigmatism of electrostatic deflection systems associated with the final thickness of the beam in the deflection space can be reduced to some extent by using narrow plates with a width slightly exceeding the beam diameter or a special-shape plate with projections (Fig. 5.20).

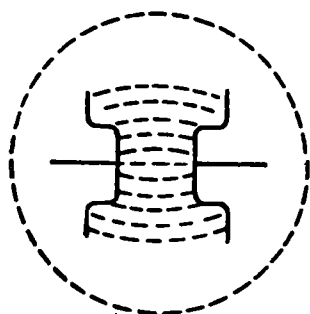


Fig. 5.20. Electrostatic deflection system with reduced astigmatism

When using such plates, the deflecting field assumes a pillow shape in the plane at right angle to the axis. This nonuniformity of the deflecting field to some extent compensates for the excessive deflection of the meridian beams due to the fact that, as the beam deflection angle increases, the beam passes to the region of lower field intensity.

The astigmatism caused by the focusing action of cylindrical lenses is much less when the plates are connected symmetrically. Furthermore, it can markedly be reduced by selecting the most advantageous position of the shields between both the pairs of deflecting plates and at the exit of the system, as well as by varying the shape of the slits in these shields. A good effect is also obtained by some shift of the mean potential in the deflection space with respect to the second anode potential.

Since the deflection astigmatism substantially increases with an increase in the deflection angle, the use of electrostatic deflection systems is usually associated with relatively small beam deflection angles (up to 15-20°).

Magnetic deflection systems are also characterized by astigmatism. It is known that a uniform transverse magnetic field performs focusing action. Thus, the magnetic system not only deflects the beam but also focuses it. This additional focusing action cannot be compensated for by adjusting the gun lens, since the optical strength of the additional lens is not constant and varies in proportion to the square of the deflection angle. Since the additional focusing

action is applied only to the cathode rays lying in the deflection plane, the beam having a circular section at the entry of the deflection system becomes elliptical in the screen plane and an aberration figure characteristic of astigmatism appears on the screen. If the deflecting field is not uniform, this also results in defocusing.

Figure 5.21 illustrates the cross section of the electron beam in the region of a nonuniform magnetic field.

One can easily see that due to the curvature of magnetic lines of force the Lorentz force has not only a horizontal component deflecting the beam in a specified direction but also a vertical component whose magnitude and direction are different at different points of the beam section. The presence of this component causes a change in the shape of the beam cross section, i.e. astigmatism.

Since the astigmatism occurs even when deflecting the beam by a uniform field, the attempts to find a nonuniform field, in which astigmatism is at its minimum, are quite natural. Such a field can be calculated analytically. It has been found that the minimum astigmatism is obtained when using slightly pillow-shaped fields for deflecting the beam.

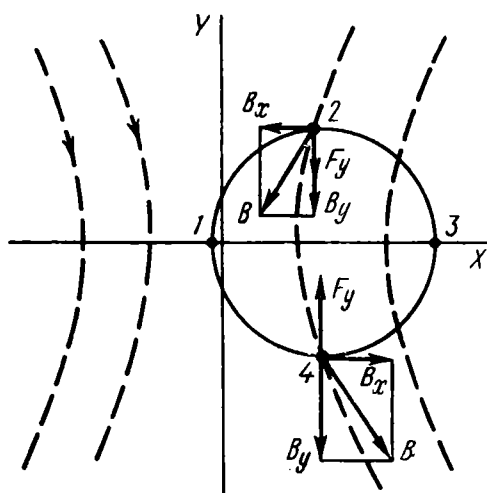


Fig. 5.21. Astigmatism of magnetic deflection system

It should be noted that the defocusing is much lower with magnetic deflection than with electrostatic deflection and this is mainly due to the constant velocity of the electrons moving in a magnetic field. Higher linearity and lower distortion of the spot shape (lower defocusing) allow use of magnetic deflection systems for deflecting the beam through angles up to 60° .

It should be noted that it is practically impossible to satisfy both requirements—high linearity and high resolving power (good focusing)—at large deflection angles. Indeed, in order to provide for linear deflection, the screen must be flat, while in order to maintain good focusing, its surface must be a part of a sphere with a centre coinciding with the centre of the beam deflection. It is clear that these requirements cannot be satisfied simultaneously. Therefore, some disturbance to the linearity or focusing in deflecting the beam through not a small angle will take place even with ideal deflection systems free of aberrations.

In the presence of astigmatism of the deflection system, one can select such a curved surface of the screen at which the astigmatic ellipse transforms itself into a circle. This surface of the best resolution is neither a plane nor a part of a sphere with a centre coinciding with the deflection centre. For example, during the magnetic deflection the least astigmatism is obtained on a screen surface close to spherical but with a radius which is considerably lower than distance from the screen to the centre of deflection. Of course, such a screen having a large convexity at the observer's side cannot be used due to high optical distortions. Therefore, selection of the best shape of screen is rather a sophisticated problem and, besides the above factors, requires that the mechanical strength of the screen be taken into account. In practice the screens of small electron-beam devices often have a flat surface, while larger screens have a spherical surface with a radius exceeding the distance from the screen to the deflection centre.

In conclusion it should be noted that the distortions occurring in the deflection of electron beams may generally be regarded as aberrations of electron prisms represented by the deflection systems from the optics point of view. Under such an assumption the distortions of deflection systems should be considered as image errors of the third kind: astigmatism and distortion of the image field, coma and optical distortion. In most cases the most important are astigmatism causing defocusing and the optical distortion affecting the raster.

It is possible to calculate the coefficients of the above-said kinds of aberrations and to try to minimize these coefficients by correspondingly varying the deflecting fields and the screen shape. However, simultaneous elimination of all the aberrations, i.e. achievement of zero values of all the aberration coefficients, is impossible. Consequently, during the calculation of deflection systems the engineer tries first of all to eliminate one of the errors, preferably astigmatism. The aberration coma figure often lies within an astigmatic ellipse and, therefore, in this case the coma plays no substantial role. The coma may be important when using wide beams in the deflection. The correction of this distortion is a very difficult problem.

When it is necessary to have high resolution over the entire screen surface, particularly at large deflection angles, the above-described methods of reducing the distortions prove to be insufficient and some additional devices for correcting the distortions are required. These devices can be made in the form of four- or eight-pole magnetic lenses arranged between the deflection system and the screen. By selecting the value of the current in the coils of these lenses, one can produce additional fields allowing the deflection distortions to be considerably reduced.

A note should be made of the so-called dynamic trimming focusing employed in some types of television tubes. This focusing consists in changing within certain limits the focusing coil current (or the focusing anode voltage) of the projection lens in step with changing the deflecting signal. If that is the case, the law of variation of the focusing coil current is selected so that the crossover image coincides with the screen surface at any beam deflection angle of the given device. It is clear that in this case the defocusing of the deflecting beam is minimum.

5.5. Deflection Systems of Microwave Electron-beam Devices

The above study of the beam deflection was based on an assumption that the deflecting voltage (current) varies with time at a slow rate so that during the residence of the electrons in the deflection space the deflecting fields might be regarded as constant in time. Experiments show, however, that during fast electrical processes or super-high frequency periodic processes the curve traced on the screen by the electron beam produces a distorted signal, the error increasing with the signal frequency.

These distortions are caused by several factors. One of the factors limiting the upper frequency of the signal analyzed by means of an ordinary cathode-ray oscilloscope is a final transit time of the electrons in the deflection system.

Consider the influence of the transit time of electrons on the sensitivity of an electrostatic deflection system (magnetic deflection systems cannot operate at a microwaves (super-high frequency) due to high inductance and distributed capacitance of the deflecting coils).

Assume that an electron flies in a space between two parallel deflecting plates having length l and located at distance b from each other, at velocity v_z directed along the axis. A. c. deflecting voltage U_{def} is applied to the plates. Then the instantaneous value of the deflection field intensity $E \approx U_{def} / b$ and the deflection angle according to (1.9) are as follows:

$$\alpha \approx \tan \alpha = \frac{eU_{def}}{mb} \times \frac{z}{v_z^2} \quad (5.33a)$$

If during the transit of the electron flying between the plates the value of U_{def} is not perceptibly changed, replacement of z by the length l results in that the above expression (5.33a) is in agreement with the earlier expression (5.1).

Assume now that U_{def} varies markedly during the transit of the electron. In this case we may estimate the change in the angle α over a small distance along the axis OZ (over the section dz)

$$d\alpha = \frac{e}{m} \times \frac{U_{def}}{bv_z^2} dz \quad (5.34)$$

To find the final deflection angle, expression (5.34) must be integrated within the limits from 0 to 1. Since the law of U_{def} variation is usually specified with respect to time and not to the length of the plates, the integration with respect to z is preferably replaced by integration with respect to t (using $dz = v_z dt$)

$$\alpha = \int_0^{\tau} \frac{e}{m} \times \frac{U_{def}(t)}{bv_z} dt \quad (5.35)$$

where $\tau = l/v_z$ is the transit time of electron.

Assume that the deflecting voltage varies by the sine law

$$U_{def} = U_m \sin \omega t \quad (5.36)$$

In the general case the electron can enter the deflection field in any phase of the deflecting voltage. Therefore, the integration should be performed from a certain value of t to $t + \tau$

$$\begin{aligned} \alpha &= \frac{e}{m} \times \frac{U_m}{bv_z} \int_t^{t+\tau} \sin \omega t dt = \frac{e}{m} \times \frac{U_m}{bv_z \omega} \times [\cos \omega t - \cos \omega(t + \tau)] = \\ &= \frac{2e}{m} \times \frac{U_m}{bv_z \omega} \sin \omega \left(t + \frac{\tau}{2} \right) \sin \frac{\omega \tau}{2} \end{aligned} \quad (5.37)$$

The first multiplier in (5.37) can be transformed by replacing v_z by l/τ :

$$\alpha = \frac{e}{m} \times \frac{l}{bv_z^2} U_m \sin \omega \left(t + \frac{\tau}{2} \right) \frac{\sin \frac{\omega \tau}{2}}{\frac{\omega \tau}{2}} \quad (5.38)$$

If the distance from the deflection system to the screen is equal to L' , then the beam deflection on the screen is given by

$$h = \frac{lL'}{2bU_{ac}} U_m \sin \omega \left(t + \frac{\tau}{2} \right) \frac{\sin \frac{\omega \tau}{2}}{\frac{\omega \tau}{2}} \quad (5.39)$$

(here the value v_z is expressed through the accelerating voltage U_{ac}).

The deflection sensitivity is found by dividing h by the deflecting voltage

$$\varepsilon_d = \frac{lL'}{2bU_{ac}} \times \frac{\sin \frac{\omega \tau}{2}}{\frac{\omega \tau}{2}} \quad (5.40)$$

The sensitivity determined by expression (5.40) is called the *dynamic sensitivity*, since this expression is obtained with due account for the transit time of the electrons

The first multiplier in expression (5.40) is conventional static sensitivity [confer to (5.11)]. Consequently, equation (5.40) may be presented in the form

$$\varepsilon_d = \varepsilon \frac{\sin \frac{\omega\tau}{2}}{\frac{\omega\tau}{2}} \quad (5.41)$$

Expressions (5.39) and (5.41) show that when the transit time of the electron flying between the plates and the period of the a-c. voltage under study is commensurable, the curve described by the

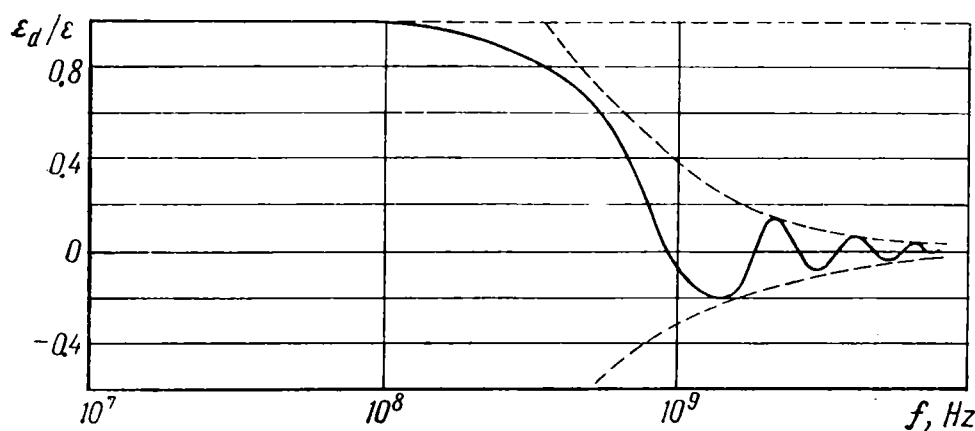


Fig. 5.22. Dynamic sensitivity *versus* frequency

electron beam on the screen shows the a-c voltage being analyzed with its amplitude and phase distortions.

In the case of sinusoidal deflecting voltage, the shape of the curve is not distorted. The phase shift, $\varphi = \frac{\omega\tau}{2}$ according to (5.39) depends on the frequency and the transit time and decreases with decrease in the values of ω and τ .

When the frequency varies, the dynamic sensitivity may go to zero (at $\frac{\omega\tau}{2} = 2\pi n$, where $n = 1, 2, 3 \dots$) and change its sign. The ε_d versus the frequency is given in Fig. 5.22.

The expression for (5.41) may be written as follows:

$$\varepsilon_d = \zeta \varepsilon \quad (5.42)$$

where $\zeta = \frac{\varepsilon_d}{\varepsilon} = \frac{\sin \frac{\omega\tau}{2}}{\frac{\omega\tau}{2}}$ is a measure of the amplitude error.

The closer ζ to unity, the higher is the accuracy of the display of the high-frequency signal being analyzed. In the given analysis of the deflection errors due to the final transit time of the electrons

the influence of the stray fields beyond the edges of the deflecting plates is neglected. Computation of these fields greatly complicates the analysis. Assuming that the field beyond the edges of the plates drops down linearly and goes to zero at a distance l' from the edges of the plates, we have

$$\zeta = \frac{\sin \frac{\omega\tau}{2}}{\frac{\omega\tau}{2}} \times \frac{\sin \frac{\omega(\tau+\tau')}{2}}{\frac{\omega(\tau+\tau')}{2}} \quad (5.43)$$

where $\tau' = \frac{2l'}{v_2}$.

The amplitude error is a function of the frequency of the oscillations under study (f), the length of the deflection system (l) and

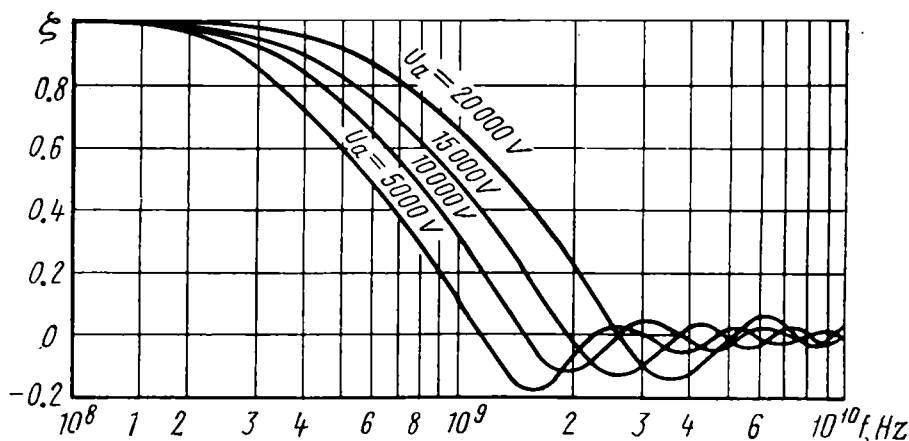


Fig. 5.23. Amplitude error versus frequency

the accelerating voltage (U_a). The amplitude error versus the frequency at several values of U_a is shown in Fig. 5.23.

One can easily see that even at high accelerating voltages (20 kV) the dynamic sensitivity markedly decreases at frequencies higher than 1000 MHz.

While in the case of high-frequency sinusoidal oscillations the deflection sensitivity decreases without affecting the shape of the curve, nonsinusoidal processes are associated with more complex distortions. Presenting a nonsinusoidal curve in the form of a Fourier series, one can easily make certain that the amplitude error ζ has a different value for each harmonic component. In the same manner, the phase shift $\omega\tau/2$ is different for each harmonic. Therefore, in the general case nonharmonic processes will be shown on the screen with large distortions which in some cases do not allow one to have correct impression of the character of the signal being analyzed.

The second important factor causing perceptible distortions to a high-frequency signal fed to the deflecting plates is the final

value of the deflection system capacitance. This capacitance is a load at the end of the line feeding the signal to the deflecting plates. Since this line has a certain characteristic impedance Z , the amplitude of the voltage applied to the deflecting plates is lower than that at the input of the line due to the presence of a load at the end of this line. This decrease in the amplitude is equal to the sensitivity drop. The higher the characteristic impedance of the supply line, the greater the sensitivity drop. Figure 5.24 illustrates

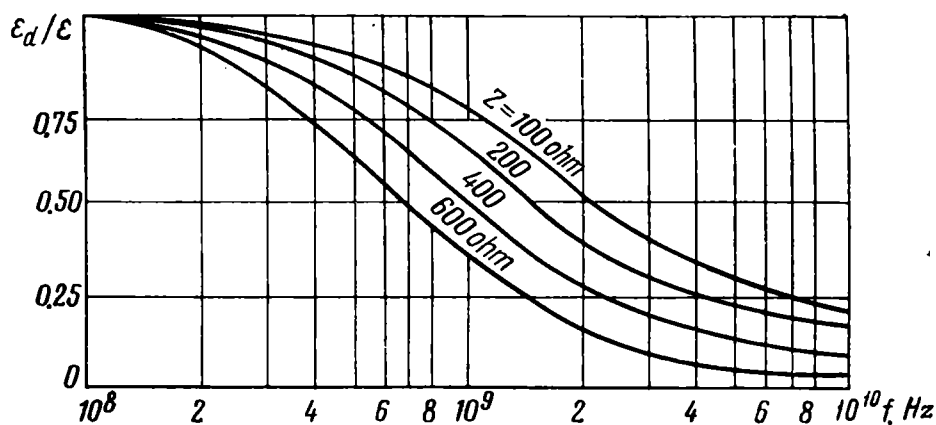


Fig. 5.24. Dynamic sensitivity *versus* characteristic impedance of feeding line

the curves of the dynamic sensitivity of the deflection system *versus* the frequencies for different characteristic impedances of the feeding line.

Figure 5.24 shows that the sensitivity drops down considerably at frequencies higher than 1000 MHz even for a system with a small capacitance.

Thus, both of the above factors (transit time and deflection system capacitance) are of importance when using a high-frequency deflecting voltage. The transit time of the electrons and the capacitance of the deflection system can be decreased by reducing the length of the deflecting plates. However, shorter plates have a lower sensitivity so that decreasing their length can be used only to certain limits. Making the plates shorter than 10 mm is bad practice since in this case the area of the stray fields near the edges of the plates becomes commensurable with their length and this causes additional distortions. The capacitance of the deflection system is determined by the plates area; therefore, to reduce this capacitance, one should decrease not only the length of the plates but also their width. Besides, the capacitance and inductance of the conductors feeding a high-frequency signal to the deflecting plates can be reduced by making the terminals of the deflection system in high-frequency

tubes in the form of short wire sections passing through the tube neck where the deflecting plates are located.

Nevertheless, even by using short and narrow plates and short terminals, one cannot eliminate amplitude and phase distortions at frequencies higher than 1000 MHz. These distortions limit application of ordinary deflection systems in the microwaves range.

The recent years have been marked by the use of new deflection systems for investigation of microwave processes. These include travelling-wave systems and self-scanning systems. In the former case the beam is deflected by a field of a high-frequency wave travelling along a spiral deflection system, i.e. there is used the same principle of wave delay that is typical for travelling-wave microwave amplifying tubes. In the latter case the deflection of the electron beam is effected through a field produced by a microwave voltage standing wave.

The travelling-wave deflection system is made in the form of a spiral placed into a casing at the potential of the second anode of the gun. When the microwave voltage spreads through the spiral turns at the velocity of light, a wave travels along the spiral at the phase velocity which is many times less than the velocity of light as the pitch of the spiral is less than the length of its turn. Using a spiral with a small pitch and a comparatively large diameter, one can easily obtain a phase velocity (leading-edge velocity) many times less than the velocity of light. For example, at the ratio of the turn length to the spiral pitch equal to 10 the phase velocity is equal to $3 \cdot 10^7$ m/s. Such a velocity is acquired by the electrons having been subjected to an accelerating potential of about 2.5 kV. Thus, an electron flow having a velocity equal to the phase velocity of microwaves can be obtained even at comparatively low accelerating voltages. It is clear that in this case the electrons of the beam will always be in one phase of the deflecting voltage.

It is good practice to build up the deflecting field pierced by the electron beam in the space between the spiral and casing in the form of a cylinder surrounding the spiral. The deflecting field strength and, therefore, the force acting on the electron are the higher, the smaller is the distance between the spiral and the casing. On the other hand, the transmitted pulse is the greater, the longer the spiral.

Let us consider a spiral deflection system consisting of n turns with a pitch Λ , a turn width l' and a distance l'' between the turns (Fig. 5.25).

The equation of motion of the electron in the direction of the axis OY may be written in the form

$$m \times \frac{dv_y}{dt} = -eE_y \quad (5.44)$$

The field between the spiral turn and the casing may approximately be considered uniform. Then, if the deflecting voltage is sinusoidal, we have

$$E_y = \frac{U_{def}}{b} = \frac{U_m}{b} \sin \omega t \quad (5.45)$$

where b is the distance between the spiral turns and the casing.

Substitute the expression for E from (5.45) into (5.44):

$$\frac{dv_y}{dt} = -\frac{e}{m} \times \frac{U_m}{b} \sin \omega t \quad (5.46)$$

and integrate in the limits from t to $t + \tau'$ (where τ' is the transit

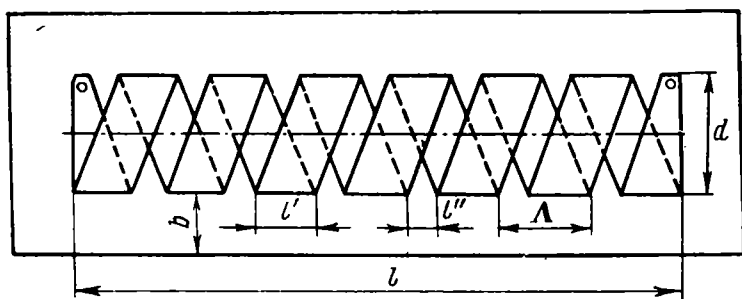


Fig. 5.25. Spiral deflection system

time of the electron flying under the spiral turn):

$$\begin{aligned} v'_y &= -\frac{e}{m} \times \frac{U_m}{b\omega} \int_t^{t+\tau} \sin \omega t dt = \frac{e}{m} \times \frac{U_m}{b\omega} [\cos \omega(t + \tau') - \cos \omega t] = \\ &= \frac{2e}{m} \times \frac{U}{b\omega} \sin \omega(t + \tau'/2) \sin \frac{\omega\tau'}{2} \end{aligned} \quad (5.47)$$

Multiplying and dividing (5.47) by $\tau' = l'/v_z$, we obtain

$$v'_y = \frac{e}{m} \times \frac{U_m l'}{b v_z} \sin \omega \left(t + \frac{\tau'}{2} \right) \frac{\sin \frac{\omega\tau'}{2}}{\frac{\omega\tau'}{2}} \quad (5.48)$$

where the last term $\sin \frac{\omega\tau'}{2} / \frac{\omega\tau'}{2} = \xi$ is the coefficient of the sensitivity decrease.

It is clear that the maximum value of the velocity $v_{y \max}$ takes place when $\left(\omega t + \frac{\omega\tau'}{2} \right) = \frac{\pi}{2} n$ where $n = 1, 3, 5 \dots$:

$$v_{y \max} = \frac{e}{m} \frac{U_m l'}{b v_z} \xi \quad (5.49)$$

When the electron flies between the spiral turns (where the field is assumed to be zero), its velocity v'_y is not changed. The electron

enters the field again when it flies under the next turn. One can easily find that when the longitudinal velocity of the electron is equal to the phase velocity of the wave, the transverse velocity of the electrons acquired under the second turn is equal to the velocity acquired under the first turn and has the same direction. To this end, it is required that the electron enter the space under the second turn in the same phase as under the first turn.

The change in the phase of the electron can be presented by the transit angle

$$\Delta\varphi_e = \omega(\tau' + \tau'') = \omega \frac{l' + l''}{v_z} = \omega \frac{\Lambda}{v_z} \quad (5.50)$$

During the same time $\tau' + \tau''$ the wave phase is varied by

$$\Delta\varphi_w = \omega \frac{\Lambda}{v_{ph}} \quad (5.51)$$

where v_{ph} is the phase velocity of the wave.

When $v_z = v_{ph}$, $\Delta\varphi_e = \Delta\varphi_w$, i.e. the necessary condition for further accumulation of electrons is satisfied.

Since, when the electron flies under each regular turn, the same conditions are satisfied as for the second turn, the velocity in the direction of the axis OY , after the electron has travelled through the entire spiral, is equal to the sum of velocities acquired during the passage under each turn

$$v_y = \sum_1^n v'_y = \frac{e}{m} \times \frac{U_m}{bv_z} \zeta nl' \quad (5.52)$$

where $nl' = l_1$ is the length occupied by all the turns of the spiral (reduced length) differing from the geometric length of the spiral by the sum of the gaps between the turns.

In accordance with expression (5.52), let us define the deflection sensitivity of a travelling-wave tube

$$\varepsilon = \frac{L'v_y}{v_z U_m} = \frac{l_1 L'}{2bU_{ac}} \zeta \quad (5.53)$$

Expression (5.53) shows that the sensitivity of a travelling-wave tube is close to that of an ordinary oscillograph tube since the term $\zeta = \sin \frac{\omega\tau'}{2} / \frac{\omega\tau'}{2}$ taking into account the amplitude error when the electron flies near one turn of the spiral can be approximated to unity by reducing the turn width l' and the transit time at the expense of a sufficiently high value of U_{ac} .

Of course, high sensitivity can be attained only with strict equality of the longitudinal velocity v_z of the electron and the phase velocity v_{ph} of the wave. Since the physical size of the spiral determining the deceleration of the wave cannot be changed during the operation of a travelling-wave deflection system, the equality $v_z = v_{ph}$ is

provided by controlling the accelerating voltage. The sensitivity of the deflection system has a pronounced maximum at $v_z = v_{ph}$ (Fig. 5.26).

The given calculation of the sensitivity is approximate, since it does not take into account the influence of the stray fields between the turns. Experimental study of travelling-wave deflection systems shows that the approximate calculation gives a value of sensitivity exceeding the experimental value by 20-30 %. Consequently, when calculating the travelling-wave systems, the value of sensitivity must be corrected correspondingly.

The cylindrical spiral in a cylindrical casing is not the only possible construction of a travelling-wave deflection system. Known in the field of

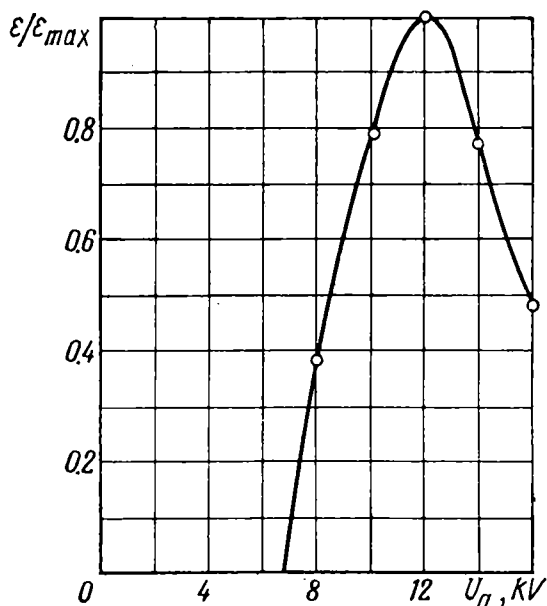


Fig. 5.26. Sensitivity versus accelerating voltage

electron optics are systems formed by two zig-zag wires along which propagates a high-frequency wave. In other systems the beam travels inside a cylindrical spiral between the latter and an internal cylindrical shield. More uniform field can be obtained by using a spiral with rectilinear portions of the turns placed into a casing having a flat wall from one side. Typical constructions of travelling-wave deflection systems are shown in Fig. 5.27.

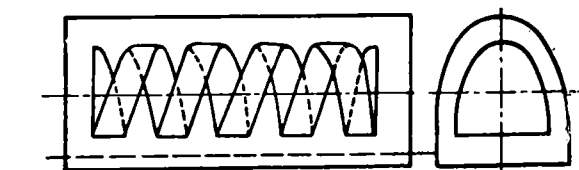


Fig. 5.27. Constructions of travelling-wave deflection systems

In 1961 Yu. Shamaev suggested a self-scanning deflection system for oscillography of microwave processes. This system consists of a length of a two-conductor line twisted into a coil (Fig. 5.28).

When a high-frequency voltage is fed to the input ends of the coil (with open output ends), a standing voltage wave is produced

in it. The pulsating field in this system can be presented as a sum of two revolving fields having the same amplitude but opposite polarity. In such a revolving field an electron is entrained by the field; the force acting on the electron is maximum when the transit time is equal to the period of the high-frequency voltage. Therefore, by selecting the pitch of the spiral and the accelerating voltage value, the time of interaction of the electron with the field can be maximized, i.e. be made equal to the transit time. In this case the deflection sensitivity can be not lower than during the oscillography of low-frequency processes.

Let us consider the operation of a spiral deflection system. The motion of the electron in a plane perpendicular to the system axis

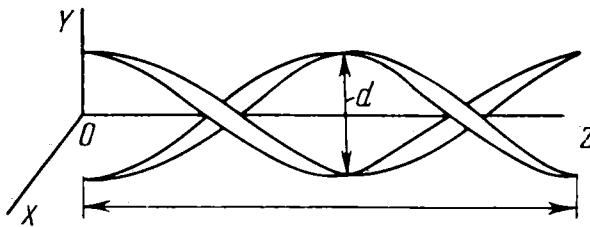


Fig. 5.28. Self-sweeping deflection system

is determined by the electric field intensity vector which turns in the space together with the coil provided that the stray field at the coil ends is ignored.

In this case the deflection of the electron from the axis is conveniently characterized by a complex number whose absolute value determines the deflection while the argument determines the direction. Let us select the axes of the coordinates OX and OY in a plane at right angle to the system axis, while the axis OZ is aligned with the system axis. In a first approximation the field inside the coil may be considered purely transverse, i.e. it is assumed that $E_z = 0$.

In this case the equation of motion of the electron is written in the following form:

$$\left. \begin{aligned} \frac{dv_x}{dt} &= -\frac{e}{m} E_x \\ \frac{dv_y}{dt} &= -\frac{e}{m} E_y \\ \frac{dv_z}{dt} &= 0, \quad v_z = \text{const} = \sqrt{\frac{2e}{m} U_{ac}} \end{aligned} \right\} \quad (5.54)$$

From the last equation of the system it follows directly that

$$z = v_z t = \sqrt{\frac{2e}{m} U_{ac}} t \quad (5.55)$$

Let us write the equation of motion of the electron in a projection on the plane XOY:

$$\frac{dv_{\perp}}{dt} = -\frac{e}{m} E \quad (5.56)$$

where in the complex form

$$\begin{aligned} v_{\perp} &= v_x + jv_y \\ E &= E_x + jE_y \end{aligned}$$

Assume that the coil has only one turn, i.e. its length is equal to the pitch Λ (Fig. 5.28). If the wavelength of the microwaves signal under study is much greater than the length (and pitch) of the coil, in a first approximation we may neglect the variation of the field intensity along the axis. In this case the transverse velocity component is

$$E \approx E(t) e^{jk \frac{2\pi z}{\Lambda}} \quad (5.57)$$

If the oscillographed voltage is sinusoidal, we have

$$E(t) = E_m \sin(\omega t + \varphi)$$

The pulsating field is conveniently presented as a sum of two fields rotating in opposite directions by the known Euler formula

$$\sin(\omega t + \varphi) = \frac{e^{j(\omega t + \varphi)} - e^{-j(\omega t + \varphi)}}{2j}$$

Then the equation of motion can be presented in the form

$$\frac{dv_{\perp}}{dt} = -\frac{e}{2m} \times \frac{E_m}{j} \{e^{j[(\omega_c + \omega)t - \varphi]} - e^{-j[(\omega_c + \omega)t + \varphi]}\} \quad (5.58)$$

where

$$\omega_c = \frac{2\pi z}{\Lambda t} = \frac{2\pi v_z}{\Lambda}$$

Assume also that the electron enters the deflection system at an instant $t=0$, at a velocity $v_z = \sqrt{\frac{2e}{m}} U_{ac}$ and in the phase with the deflecting voltage φ (φ is the initial phase). If the coil has a length Λ , the transit time is

$$\tau = \frac{\Lambda}{v_z} = \frac{\Lambda}{\sqrt{\frac{2e}{m}} U_{ac}}$$

To find the transverse velocity component of the electron at the output of the deflection system, we must integrate equation (5.58)

within the limits from 0 to τ

$$v_{\perp} = -\frac{eE_m}{2m} \times \frac{e^{j(\omega_c - \omega)\tau} - 1}{\omega_c - \omega} e^{-j\varphi} + \frac{eE_m}{2m} \times \frac{e^{j(\omega_c + \omega)\tau} - 1}{\omega_c + \omega} e^{j\varphi} \quad (5.59)$$

In expression (5.59) all quantities but φ are constant. Thus the end of the vector $\mathbf{v}$ will describe an ellipse in the space, since the vector $\mathbf{v}$ is a sum of two vectors revolving in opposite directions.

If the distance from the deflection system to the screen is equal to L' , the value of deflection of the beam (spot) on the screen is

$$h = L' \tan \alpha = L' \frac{v_{\perp}}{v_z}$$

or

$$h = -\frac{e}{2m} \times \frac{E_m L'}{v_z} \times \frac{e^{j(\omega_c - \omega)\tau} - 1}{\omega_c - \omega} e^{-j\varphi} + \frac{e}{2m} \times \frac{E_m L'}{v_z} \times \frac{e^{j(\omega_c + \omega)\tau} - 1}{\omega_c + \omega} e^{j\varphi} \quad (5.60)$$

Denote the coefficients before $e^{-j\varphi}$ and $e^{j\varphi}$ through A and B respectively. Then the value of deflection is

$$h = A e^{-j\varphi} + B e^{j\varphi} \quad (5.61)$$

The semiaxes of the ellipse described by the beam on the screen are equal to

$$a = |A| + |B|, \quad b = |A| - |B| \quad (5.62)$$

Let us determine the absolute values of the coefficients A and B

$$\begin{aligned} |A| &= \frac{e}{2m} \times \frac{E_m L'}{v_z} \times \frac{\sqrt{[e^{j(\omega_c - \omega)\tau} - 1][e^{-j(\omega_c - \omega)\tau} - 1]}}{\omega_c - \omega} = \\ &= \frac{e}{2m} \times \frac{E_m L' \tau}{v_z} \left| \frac{\sin x_1}{x_1} \right| \\ |B| &= \frac{e}{2m} \times \frac{E_m L'}{v_z} \times \frac{\sqrt{[e^{j(\omega_c + \omega)\tau} - 1][e^{-j(\omega_c + \omega)\tau} - 1]}}{\omega_c + \omega} = \\ &= \frac{e}{2m} \times \frac{E_m L' \tau}{v_z} \left| \frac{\sin x_2}{x_2} \right| \end{aligned} \quad (5.63)$$

where

$$\begin{aligned} x_1 &= \frac{\omega_c - \omega}{2} \tau = \frac{\omega_c - \omega}{2v_z} \Lambda \\ x_2 &= \frac{\omega_c + \omega}{2} \tau = \frac{\omega_c + \omega}{2v_z} \Lambda \end{aligned}$$

Then the semiaxes of the ellipse are equal to

$$\begin{aligned} a &= \frac{E_m L' \Lambda}{4U_{ac}} \left(\left| \frac{\sin x_1}{x_1} \right| + \left| \frac{\sin x_2}{x_2} \right| \right) \\ b &= \frac{E_m L' \Lambda}{4U_{ac}} \left(\left| \frac{\sin x_1}{x_1} \right| - \left| \frac{\sin x_2}{x_2} \right| \right) \end{aligned} \quad (5.64)$$

The image on the screen depends on the ratio of the semiaxes b/a . The deflection on the screen can be presented as a sum

$$h = Ae^{-j\varphi} + Be^{j\varphi}$$

If A and B go to zero, the beam will describe a circle on the screen, the amplitude of the second component being constant. The amplitude will evidently be constant when the electron is always in the same phase. This means that the transit time of one turn of the coil is equal to the period of the high-frequency voltage. This condition can be written as follows:

$$\omega_c = 2\pi \frac{v_z}{\Lambda} \omega \quad \text{or} \quad v_z = \Lambda f \quad (5.65)$$

In this case the second component of the displacement relative to the electron will revolve at a doubled frequency. Write the condition which allows the second component to go to zero

$$e^{2j\omega_c\tau} - 1 = 0$$

which takes place at $2\omega_c = 2k\pi$, where $k = 1, 2, 3 \dots$. This condition is satisfied when $\tau = k \frac{T}{2}$.

The expression for x_1 and x_3 can be transformed. Taking into account that $\omega_c = 2\pi \frac{v_z}{\Lambda}$ and $\omega = 2\pi f = 2\pi \frac{1}{T}$, we obtain

$$x_1 = \frac{\omega_c - \omega}{2} \tau = \frac{\pi v_z \tau}{\Lambda} - \pi f \tau = \frac{k\pi}{2} - \frac{\pi\tau}{T}$$

$$x_2 = \frac{\omega_c + \omega}{2} \tau = \frac{\pi v_z \tau}{\Lambda} + \pi f \tau = \frac{k\pi}{2} + \frac{\pi\tau}{T}$$

and

$$\begin{aligned} \sin x_1 &= \sin \frac{k\pi}{2} \cos \frac{\pi\tau}{T} - \cos \frac{k\pi}{2} \sin \frac{\pi\tau}{T} \\ \sin x_2 &= \sin \frac{k\pi}{2} \cos \frac{\pi\tau}{T} + \cos \frac{k\pi}{2} \sin \frac{\pi\tau}{T} \end{aligned} \quad (5.66)$$

It is clear that with $k = 2, 4, 6, 8 \dots$ $\sin \frac{k\pi}{2} = 0$ and

$$\begin{aligned} |\sin x_1| &= \left| \sin \frac{\pi\tau}{T} \right| \\ |\sin x_2| &= \left| \sin \frac{\pi\tau}{T} \right| \end{aligned} \quad (5.67)$$

When $k = 1, 3, 5, 7$ $\cos \frac{k\pi}{2} = 0$ and

$$\begin{aligned} |\sin x_1| &= \left| \cos \frac{\pi\tau}{T} \right| \\ |\sin x_2| &= \left| \cos \frac{\pi\tau}{T} \right| \end{aligned} \quad (5.68)$$

Consequently, for any whole-number values k ($k = 1, 2, 3 \dots$) the ratio of the semiaxes of the ellipse is expressed as

$$\frac{b}{a} = \frac{||x_2| - |x_1||}{|x_2| + |x_1|} \quad (5.69)$$

Thus, with a constant value of accelerating voltage ($U_{ac} = \text{const}$) we can study the ratio b/a versus the frequency ω of the accelerating voltage. Here three cases are possible.

(1) The period of the high-frequency voltage is higher than the transit time*

$$\begin{aligned} \frac{k\pi}{2} &> \pi \frac{\tau}{T} \\ \frac{b}{a} &= \frac{\omega\tau}{k\pi} \end{aligned} \quad (5.70)$$

(2) The period of the high-frequency voltage is equal to the transit time

$$\frac{k\pi}{2} = \pi \frac{\tau}{T}$$

and

$$\frac{b}{a} = 1 \text{ (circle)} \quad (5.71)$$

(3) The transit time exceeds the period of the high-frequency voltage

$$\frac{k\pi}{2} < \pi \frac{\tau}{T}$$

and

$$\frac{b}{a} = \frac{k\pi}{\omega\tau} \quad (5.72)$$

When the transit time is equal to the period of the high-frequency voltage and we have a circle on the screen, peculiar resonance takes place. Let us use a concept of a resonance frequency f_r determined by the equality of the transit time and the period of the high-frequency voltage

$$f_r = \frac{1}{\tau} \quad \text{or} \quad f_r = \frac{v_z}{\Lambda}$$

Then expressions (5.70) to (5.72) can be transformed by using the equalities

$$\tau = \frac{1}{f_r}, \quad \omega = 2\pi f$$

Consequently,

$$\begin{aligned} \text{prior to the resonance } b/a &= f/f_r \\ \text{during the resonance } b/a &= 1 \\ \text{after the resonance } b/a &= f_r/f. \end{aligned} \quad (5.73)$$

* Here and further by the transit time is meant the time taken by the electron to pass one turn of the coil.

Since the transit time depends on the pitch of the coil and the accelerating voltage $\tau = \Lambda / \sqrt{\frac{2e}{m} U_{ac}}$, it is clear that at a constant value of the high-frequency voltage the image depends on the value of the accelerating voltage. A similar analysis gives the following dependencies:

$$\begin{aligned} \text{prior to the resonance } b/a &= \sqrt{U_{ac}/U_e} \\ \text{during the resonance } b/a &= 1 \\ \text{after the resonance } b/a &= \sqrt{U_e/U_{ac}} \end{aligned} \quad (5.74)$$

Here U_e is the accelerating voltage providing for the equality of the time required by the electron for passing one turn of the coil and the period of the high-frequency voltage.

The curves illustrating the ratio b/a versus the frequency of high-frequency voltage and accelerating voltage are given in Fig. 5.29.

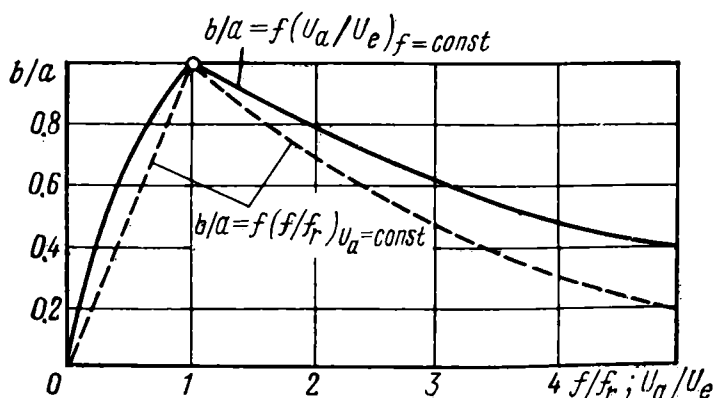


Fig. 5.29. Ellipse semiaxes ratio versus frequency and accelerating voltage

It is shown that the ratio b/a features much higher dependence on the frequency of the oscillographed voltage than on the accelerating voltage.

Varying the accelerating voltage we can obtain circular sweep within a certain range of frequencies of the high-frequency voltage under analysis. Varying the physical size of the coil, one can widen the range of frequencies which can be analyzed by means of technically available accelerating voltages.

The sensitivity of the coil deflection system can be estimated as the ratio of the diameter of the circle obtained on the screen to the amplitude value of the voltage under study. Generally we shall have an ellipse rather than a circle on the screen. If that is the case, the average size of the image is

$$h_{zv} = \frac{2a + 2b}{2} \quad (5.75)$$

The value h_{av} according to (5.61) is expressed by

$$h_{av} = 2 |A|$$

while the absolute value $|A|$ [see (5.63)] is expressed by

$$|A| = \frac{E_m L' \Lambda}{4U_{ac}} \left| \frac{\sin \frac{\omega_c - \omega}{2} \tau}{\frac{\omega_c - \omega}{2} \tau} \right| \quad (5.76)$$

The amplitude of the field strength can approximately be estimated as U_m/d , where d is the diameter of the coil. Then

$$h_{av} \approx \frac{U_m L' \Lambda}{2U_{ac} d} \left| \frac{\sin \frac{\omega_c - \omega}{2} \tau}{\frac{\omega_c - \omega}{2} \tau} \right| = U_m \varepsilon \left| \frac{\sin \frac{\omega_c - \omega}{2} \tau}{\frac{\omega_c - \omega}{2} \tau} \right| \quad (5.77)$$

while the sensitivity of the coil deflection system is

$$\varepsilon_c = \frac{h_{av}}{U_m} = \varepsilon \left| \frac{\sin \frac{\omega_c - \omega}{2} \tau}{\frac{\omega_c - \omega}{2} \tau} \right| = \varepsilon \xi \quad (5.78)$$

where ε is the sensitivity of an ordinary deflection system of two parallel plates having length Λ and distance d between the plates; ξ is the coefficient playing the same role as the amplitude error coefficient in an ordinary deflection system.

One can easily see that at the resonance ($\tau = T$) $\xi = 1$, i.e. the sensitivity of the coil deflection system is equal to that of an ordinary deflection system at low frequencies. It should be noted that the sensitivity of the coil deflection system at low frequencies and at voltages providing good focusing and luminance of the screen the electron velocity is very low. Thus, the use of a self-scanning coil deflection system is expedient only at sufficiently high frequencies of the accelerating voltage starting approximately with 100-200 MHz.

It should be noted that the upper frequency limit is also restricted, since, in order to obtain a maximum sensitivity (resonance) at very high frequencies, we must use very high accelerating voltages, which is associated with engineering difficulties. However, if a spiral deflection system is used under the travelling-wave conditions, the upper frequency limit of the displayed voltage can be materially increased. To provide for the travelling-wave conditions, the output ends of the spiral must be loaded with matched resistance.

The coil deflection system provides for good analysis of microwave signals. Self-scanning is an advantage of such a system, since it requires no time sweep generator. Moreover at low frequencies

the coil deflection system is featured by high sensitivity comparable with the sensitivity of ordinary deflection systems. Finally, the coil deflection system has a simple construction.

However, the spiral deflection systems have a significant disadvantage consisting in that the studied signal can be observed only in the form of the Lissajous figure. The analysis of purely sinusoidal frequency or amplitude modulated processes, as well as sinusoidal waves, is not very complicated. The character of time variations of the studied signal can accurately be determined by the appearance and size of the oscillograms. The analysis of nonsinusoidal high-frequency signals is much more complex. The analysis of ultradynamic Lissajous figures presents many difficulties and the problem of the character of time variation of a high-frequency signal in many cases cannot be solved at all. The spiral deflection system is also disadvantageous in that it is a comparatively narrow-band system, since, in order to obtain sufficient sensitivity, it is used at frequencies close to the resonance ones.

Luminescent Screens

6.1. Cathodoluminescence.

General Requirements for Screens

One of the basic elements of many electron-beam devices (oscillograph tubes kinescopes, radar tubes, photoelectric image intensifiers) is a luminescent screen converting the energy of the electron beam incident to the screen into light energy, usually in the visible zone of the spectrum. The screen of an electron-beam device is a layer of material glowing under the action of electron bombardment. It is deposited on a substrate, usually the bottom of a bulb of cathode-ray tube. The screen luminescence is usually observed at the side opposite to that subjected to the electron bombardment, i.e. through the transparent substrate. Only in a few rare occasions, when the luminescence is observed at the side of incidence of the electron beam, the substrate can be nontransparent, for example, be made as a thin metal layer.

The properties of the screen are determined substantially by the material converting the energy of the electron beam into light. However, the type of substrate, the method of applying the phosphor material, its structure, the presence of admixtures and additional coatings, thermal treatment, and some other factors can considerably change the properties of the screen. In the same manner, the parameters of the electron beam exciting the luminescence, for example, excitation of the screen by a stationary or travelling beam of the same power, influence the screen characteristics.

The phenomenon of luminescence under the effect of electron bombardment was noted as far as at the end of the nineteenth century. It is intrinsic in many natural substances, as well as in synthetic inorganic and organic compounds. The phenomenon of glowing known as *cathodoluminescence* is used in luminescent screens. The substances or materials capable of glowing under the action of electron bombardment are called *cathode luminophors* or *phosphors*. It should be noted that of a great variety of substances having luminophor properties, the phosphors are only those materials which have adequate practical efficiency, i.e. which glow with a required brightness at low energy consumption.

In spite of the fact that the phenomenon of luminescence was known long ago, the theory of cathodoluminescence was developed only in the 30-40 s of our century. Even now some problems of this theory require additional study. Academician S. Vavilov and his followers made a considerable contribution to the development of the modern theory of cathodoluminescence.

According to the up-to-date concepts the phenomenon of cathodoluminescence is caused by the transition of electrons from higher energy levels to any of allowed lower levels. During such a transition

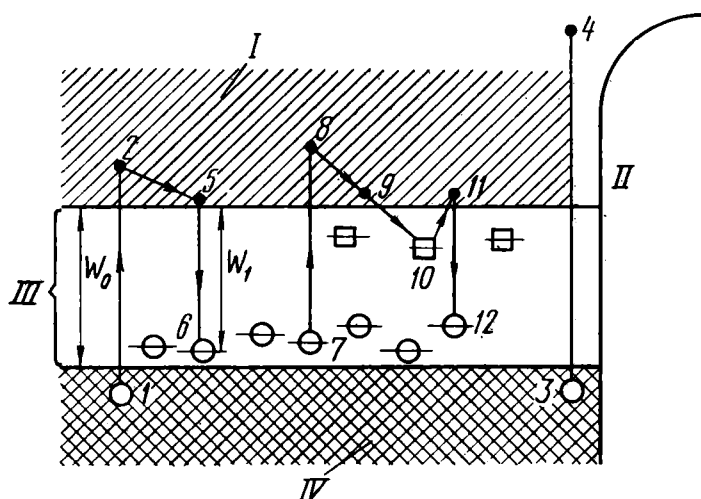


Fig. 6.1. Origination of cathodoluminescence

I—conduction band; *II*—vacuum; *III*—forbidden band; *IV*—valence band

produced is a photon with an energy determined by the differential energy of the upper and lower energy levels. Most of commercially used phosphors are crystal substances with very low electrical conductivity (specific resistance is 10^{12} - 10^{14} ohm·cm), however, these materials cannot be regarded as dielectrics. Given in Fig. 6.1 is a diagram of energy levels typical for many phosphors.

As is seen from the drawing, there are local allowed levels within a fairly wide (1.5-2 eV) forbidden band W_0 which occur due to impurities and defects of the crystal lattice. Thus the phosphors are close to extrinsic semiconductors by their physical properties, and the presence of complementary levels in the forbidden band determining their semiconductor properties plays an important role in the process of luminescence.

When the phosphor crystals are bombarded by the electrons of the beam exciting the luminescence, a part of the electrons can be transferred from the valence band to the conduction band (transitions 1-2, 3-4, 7-8 in Fig. 6.1). If in this case the electrons will be shifted to the conduction band at levels lying above the potential

barrier (transition 3-4), they can leave the crystal thus producing secondary electron emission. The secondary electron emission of phosphor materials plays a great role in the operation of electron-beam devices (see Section 6.5). The electrons remaining in the conduction band increase the electrical conductivity of the crystals. This phenomenon known as *induced conductivity* is used in some types of memory tubes (see Chapter 11). However, only a small fraction of secondary electrons emerges from the screen. A significant portion of secondary electrons interacts with the phosphor crystals causing new acts of excitation or emission of tertiary electrons which, in turn, can excite the phosphor. The acts of excitation by the secondary or tertiary electrons are even more probable than the direct excitation by the fast (primary) electrons.

The electrons transferred to the conduction band but not outside the crystal move quickly to the free lower levels of the conduction band (transitions 2-5, 8-9). These electrons have two possibilities of recombination with the hole: either direct transition to the valence band or transition to one of the local levels (transition 5-6). According to theoretical calculations, the first case is hardly probable (such transition would be accompanied by separation of a quantum with energy $h\nu_0 = W_0$). The transition to one of the local levels accompanied by separation of a quantum with energy $h\nu_1 = W_1 < W_0$ is more probable. Since real crystals have quite a number of local levels, the radiation spectrum of the phosphor usually occupies a certain band with a pronounced maximum corresponding to the transitions of the electrons from the lower level of the conduction band to the local bands which are most abundant in the given crystal. Moreover, the erosion of the radiation spectrum is largely caused by heat vibrations of the atoms of the crystal lattice of the phosphor.

The above-discussed mechanism of appearance of cathodoluminescence allows us to assume that the radiation spectrum is defined solely by the character of the phosphor and is independent of the parameters of the beam exciting the luminescence (current density, electron energy, speed of motion of the beam on the screen). However, it was found experimentally that in some cases a change of the beam parameters affects the colour of luminescence of the screen. This phenomenon is particularly noticeable in the presence of several bands in the radiation spectrum. The conditions of excitation are not the same for different bands: short-wave radiation is largely excited by the high-speed (primary) electrons, while the long-wave radiation is excited by the low-velocity (secondary) and the tertiary electrons. Hence, when changing, for example, the accelerating voltage, the intensity of short-wave radiation increases with a rise of the energy of the electrons and this is observed as a change in the colour of the screen during its operation.

The transitions of type 1-2-5-6 are very rapid processes (10^{-8} to 10^{-9} s) therefore, the rise and drop of the intensity of glowing of the phosphor should be practically instantaneous. Experiments show, however, that in most cases the luminescence rises up during a very short time, while the drop of luminescence after removing the electron bombardment of the phosphor sometimes takes a few seconds and even minutes. The long afterglow is due to the delay of the electrons in the so-called centres (electron traps)—local defects of the crystal lattice collecting the electrons from the conduction band and holding them during a long time. The electron can pass from the trap to the conduction band due to heat excitation and therefrom to one of the local levels with isolation of a photon (quantum of light). Such a transition with a delay in the trap is given in Fig. 6.1 in the form of a chain 7-8-9-10-11-12.

As has been mentioned above, adequately efficient conversion of the energy of the electron beam into the phosphor luminescence can be obtained only in the presence of defects of the crystal lattice, i.e. the presence of local levels in the forbidden band. In some cases such defects can appear during the heat treatment of the phosphor. For example, when heating zinc oxide, the crystal lattice has excessive atoms of zinc, i.e. a disturbance in the stoichiometric regularity. The same disturbances are observed in some tungstenates such as CaWO_4 used as phosphor. The phosphor is usually activated for obtaining a required brightness of luminescence, i.e. a small amount of donors called activators is added to the principal material, for example, zinc sulphide or silicate. Heavy metals such as Ag, Cu, Mn are also often employed as activators. The concentration of the donor activator is usually selected experimentally. In many cases an amount of activator equal to fractions of one per cent of the principal material is sufficient. It should be noted that some additions to phosphor materials do not increase their brightness and can even cancel the luminescence. Such admixtures "poisoning" many phosphors are iron and cobalt.

Since many properties of the screen depend on the cathode phosphor, the requirements placed on the screens of cathode-ray tubes are determinative in the selection of phosphor. The first and most important requirement is high efficiency of the conversion of electron energy into light energy. Therefore, the phosphor must have high efficiency or energy output calculated as the ratio of the energy of radiation of the phosphor to the energy delivered to the screen by the electron beam exciting the luminescence. The energy output of the phosphor is low (8-10 %). The most efficient sulphide phosphors have an output of 15-20 %. The second, also very important requirement, is the colour of luminescence. A definite character of attenuation of the luminescence or the persistence of afterglow (luminescence after the electron bombardment has been ceased) are also very

important. In some cases it is necessary to have a constant radiation spectrum during variations of the parameters of the exciting electron beam. The secondary emissive properties of the screen play an important role, since the removal of an electric charge from an electrically nonconductive screen in many types of devices can be made only at the expense of secondary emission.

High physical and chemical resistance is a general requirement placed on a phosphor. The phosphor must be moisture-proof, since otherwise it would be inconvenient in storage and application on a substrate. The phosphor properties must not suffer changes when heated to a temperature of 400-450°C, since the bulbs of electron-beam devices are heated to such a temperature in the process of welding and degassing. The phosphor must not change its parameters when ground to a grain size of several microns. Sometimes it must have a high chemical resistance to aggressive media, for example, vapours of alkali metals, which are introduced into the device when making photocathodes. Finally, the phosphor must be a good vacuum material, i.e. to be easily degassed and evolve no gases or vapours during the operation in a vacuum device.

We may see that the requirements placed on phosphor materials are quite severe and to meet them is rather a sophisticated problem.

6.2. Characteristics and Parameters of Screens

The selection of a screen and comparative estimation of various screens are expediently made by using a concept of characteristics and parameters of these screens. The screen characteristics are curves expressing the dependence of any value defining the screen properties (for example, brightness, secondary emission coefficient, etc.) on another value associated with the parameters of the electron beam exciting the luminescence such as wavelength of the emitted light, time, etc. The screen parameters are numerical values characterizing the properties of the screen. The characteristics and parameters of the screen are largely determined by the type of phosphor, however, the parameters of the phosphor and screen should not be identified, since the screen properties can change significantly as a function of the type of substrate, the degree of disintegration and the method of application of the phosphor, the presence of additional coatings, etc.

One of the basic parameters of the screen is the *luminous efficiency* defined as the ratio of the luminous flux emitted at right angle to the screen surface to the radiant flux produced by the electron beam exciting the luminescence

$$\eta = \frac{J_{light}}{P_{el}} \text{ [c/W]} \quad (6.1)$$

It is clear that the light output, as well as the energy output, characterizes the efficiency of conversion of the energy of the electron beam into the energy emitted by the phosphor. In most cases the phosphor radiation is within the visible spectrum, since both the light output and energy output give equally complete characteristics of the screen. In practice light output is preferable since it gives a clear characteristic of the screen. The light output of a luminescent screen varies from a few tenths of a candle per watt to 8-12 cd/W depending on the type of phosphor, the grain size, the methods of application, etc. The light output depends on the parameters of the beam exciting the luminescence—the energy (velocity) of the

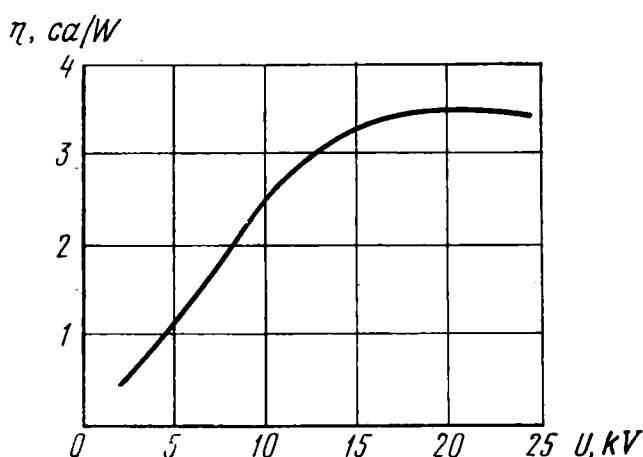


Fig. 6.2. Luminous efficiency *versus* accelerating voltage

electrons and, to a lesser extent, the beam current density. The velocity of the electrons determines the depth of their penetration into the phosphor layer: at a low velocity the depth of penetration of the electrons into the phosphor is small, the emission occurs near the surface bombarded by the electrons and a significant amount of light is absorbed or scattered in the phosphor layer. For an observer seeing the screen at the side opposite to that subjected to the electron bombardment the luminous efficiency output is low. As the electron velocity rises, the luminescence is excited in the region nearer to the observer, the portion of the absorbed light is reduced while the luminous efficiency increases. However, when the velocity of the electrons is so high that a considerable portion thereof is shot through the phosphor layer, the luminous efficiency drops down, since the conditions of energy transfer from the electrons to the phosphor are unfavourable. The characteristic expressing the dependence of the light output of the accelerating voltage (Fig. 6.2) has a mildly sloping maximum shifting to the higher thicknesses of the phosphor layers with a rise of the accelerating voltage. The

luminous efficiency also somewhat decreases with an increase in the current density due to the luminance saturation (see Fig. 6.3).

During testing and service of electron-beam devices, it is the screen luminance, rather than the luminous efficiency, that is measured. The *luminance* is defined as the intensity of light emitted by one square metre of a luminescent surface in the direction of the observer. A uniformly luminous surface of 1 m^2 radiating light with intensity of 1 cd in the direction of the observer has luminance of $1 \text{ nit (cd/m}^2\text{)}$. The luminance of the screens of electron-beam devices must

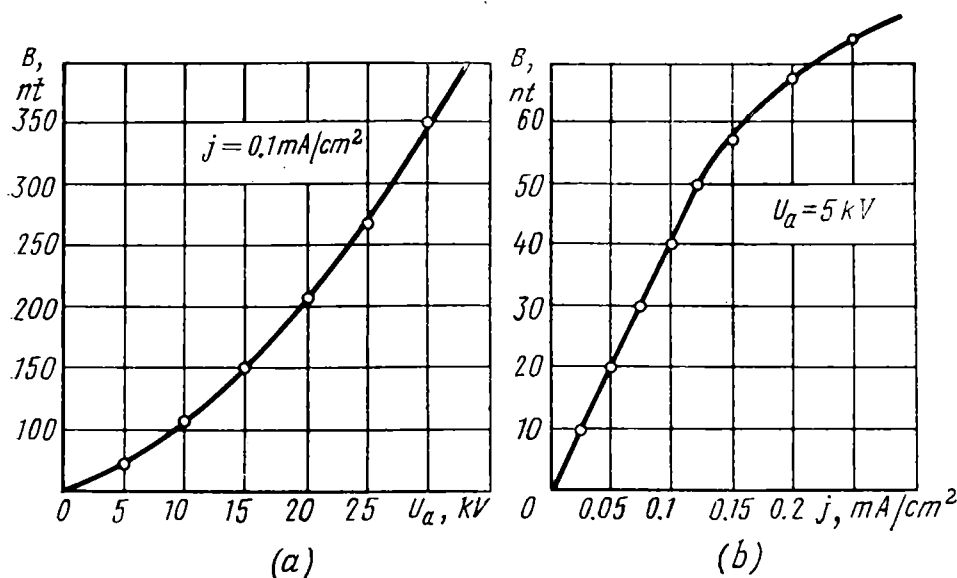


Fig. 6.3. Luminance versus accelerating voltage (a) and beam current density (b)

be not less than a few dozens nits to allow the observer to distinguish small details of the image; some types of these devices, such as projection kinescopes, have screens with a brightness of thousands and even tens of thousands of nits.

Since the luminance is determined by the light intensity of the screen which, in its turn, depends on the power of the exciting electron beam, the luminance also depends on the power, i.e. on the current and accelerating voltage determining the velocity of the electrons incident on the screen. The theoretical calculations and experimental studies have shown that the dependence of the luminance on the parameters of the electron beam can be presented with sufficient accuracy by the expression

$$B = Aj (U_{ac} - U_0)^n \quad (6.2)$$

where A is the coefficient characterizing the phosphor; j is beam current density; U_{ac} is the accelerating voltage; U_0 is the initial ("dead") potential defined as the minimum accelerating voltage causing luminescence; n is the exponent varying within the range of 1 to 2.5 for different phosphors.

Strictly speaking, expression (6.2) gives the value for luminance of the phosphor rather than of the screen. However, it adequately describes the luminance of the screen when a correction is introduced into the coefficient A , and this expression is usually used for calculation of the luminance of the screen. The characteristics of the screen luminance obtained experimentally (Fig. 6.3) show that the curve showing the luminance versus the accelerating voltage is a parabolic, i.e., it is in good agreement with equation (6.2) in a wide range of accelerating voltages (data are available for $U_{ac} = 200$ kV). At the same time, the linear character of the dependence of the brightness on the current density is preserved only for small values, $j = 10^{-5}$ to 10^{-4} A/cm². At higher current densities there is observed a noticeable deviation from the linear law. Then, we have saturation of the luminance and further increase of the current results in heating of the screen and destruction of the phosphor. The above-given dependence shows that it is expedient to increase the luminance by raising the accelerating voltage at a low beam current density. Moreover, an increase in the beam current density is not good practice for another reason: the greater the current density, the lower the screen resolution. When the current density corresponds to the saturation, the normal distribution of brightness in the spot is disturbed, the medium portion of the distribution curve becomes flatter (Fig. 6.4), the apparent radius of the spot increases and this results in a decrease of the resolving power.

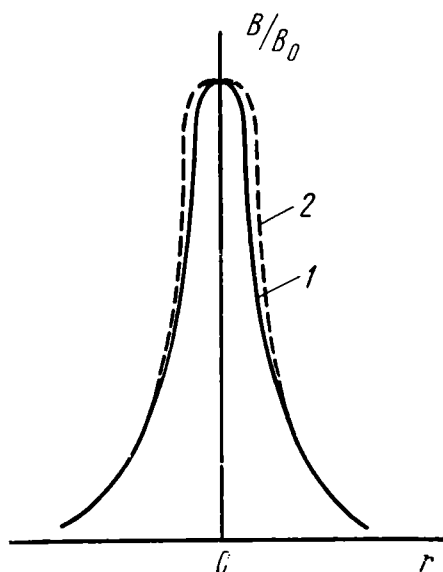


Fig. 6.4. Increase in spot radius with rise in current density

1— $j = 0.1$ mA/cm²;
2— $j = 0.25$ mA/cm²

It should be remembered that in expression (6.2) the value U_{ac} is a measure of the energy of the electrons bombarding the screen. The energy of the electrons transferred to the screen is determined not by the accelerating voltage of the gun but by the true potential of the screen relative to the gun cathode. The screen potential (see Section 6.5) may sometimes differ from the potential of the last electrode. Consequently, generally U_{ac} in formula (6.2) should be replaced by the true value of the screen potential.

The initial potential U_0 for different phosphors varies from 30 to 300 V. Its direct measurement by the extinction of the screen is incorrect, since due to the accumulation of a negative charge on

a screen having a non-conductive backing at a low accelerating voltage (see Section 6.5) the screen potential is much lower than the potential of the accelerating electrode of the gun and the experimentally found value of the initial potential may prove to be several times higher than the true potential. The value of U_0 can be determined more accurately by studying a screen with a metal backing electrically connected to the last electrode of the gun, i.e. when the screen potential is kept equal to the accelerating voltage. The initial potential value can also be found by extrapolating the logarithmic curve of the dependence of the brightness on the accelerating voltage to intersection with the abscissa axis.

There is no theoretical explanation for the presence of initial potential of tens and hundreds of volts, since the width of the forbidden band of the phosphor crystals does not exceed a few electronvolts. The existence of such a value of U_0 is probably associated with the presence of inactive surface layer on the phosphor crystals. In order to penetrate through this inactive layer, the electrons of the beam must have energy not below eU_0 .

The exponent n in expression (6.2) is close to 2 for many practically investigated phosphors. This value can be calculated for a uniform phosphor. Providing that the number of excitations is proportional to the length of the electron path in the crystal lattice of the phosphor, the brightness of luminescence must linearly depend on the depth of penetration of the electron into the phosphor layer. The depth of penetration of the electron into a dense uniform layer of phosphor, in turn, can rather accurately be determined by the formula

$$x = Cv^4$$

where C is a constant; v is the electron velocity.

Since for nonrelativistic electrons v is similar to $\sqrt{U_{ac}}$, under the above assumptions we should suggest that the luminance is proportional to the square of the accelerating voltage, i.e. $n = 2$. However, the experimental data indicate that in many cases $n < 2$. The exponent can approximately be determined by the slope of an experimentally plotted (in a logarithmic scale) curve of dependence of the luminance on the accelerating voltage.

6.3. Spectral Characteristics of Screens

As has been mentioned above, a definite colour of luminescence of a screen is one of the requirements placed thereon. The visible (apparent) colour of a screen is uniquely determined by the position and intensity of the emission band of the phosphor (it is assumed that the transparent backing of the screen has no selective absorption properties), i.e. its spectral characteristic. At the same time, many

spectral characteristics may correspond to a specified colour of luminescence. The screen colour can be determined most accurately by the colour coordinates of the emission bands on a colour triangle*. The majority of phosphors used for making screens of electron-beam devices have one maximum on the spectral characteristic, the position of which largely determines the visible colour of the screen.

The screen colour is selected depending on the field of application of the device. The screens intended for direct visual observation must have a spectral characteristic close to the curve of spectral sensitivity of the human eye. In this case the image on the screen will appear as the most bright one under otherwise equal conditions

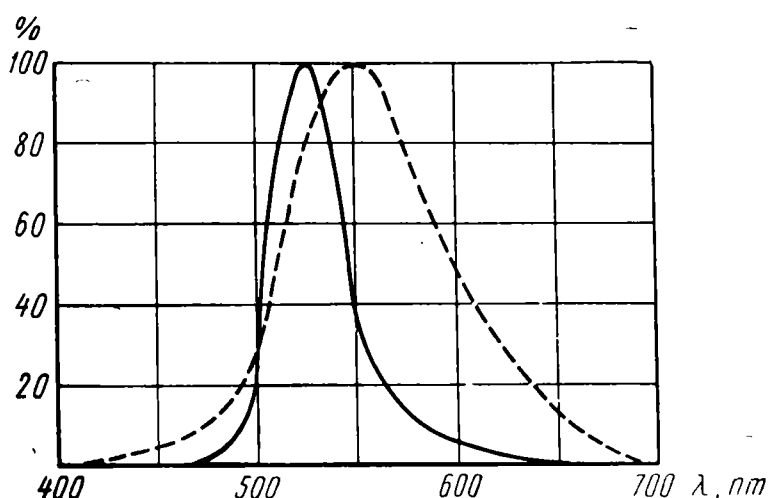


Fig. 6.5. Spectral characteristic of willemite and curve of spectral sensitivity of human eye (dash lines)

(the same light energy radiated). Since the maximum of the eye sensitivity lies in the yellow-green part of the spectrum, it is expedient that the screen radiation is concentrated in this part of the spectrum, when the device is used for visual observation. As an example, Fig. 6.5 gives a spectral characteristic of one of the best known phosphors—zinc orthosilicate activated with manganese (willemite). Presented in the same figure by a dash line is the diagram of spectral sensitivity of the human eye. It is seen that both these curves are adequately close to each other.

When the picture on the screen is to be photographed, it is expedient to have a screen with a shorter wave of radiation, since the photographic emulsion is more sensitive to the short-wave region of the spectrum. In principle, it is possible to obtain a phosphor with

* The colour triangle is discussed in the book of M. S. Kayver "Fundamentals of Colour Television".

the maximum radiation in the ultraviolet region, however, such phosphors are impracticable due to heavy absorption of ultraviolet rays by the glass of the bulb bottom and the optical system of the camera. Manufacture of bulbs and objectives of photographic cameras of special glass passing ultraviolet rays is economically inexpedient. Furthermore, preliminary observation and photography of invisible oscillograms are associated with considerable difficulties. Consequently the screen designed for photography usually has a blue or blue-violet colour of luminescence. As an example, Fig. 6.6 gives a spectral characteristic of the screen coated with silver-activated zinc sulphide (screen "A").

The TV tubes for monochrome television receivers provide the best performance from the aesthetic point of view with a white

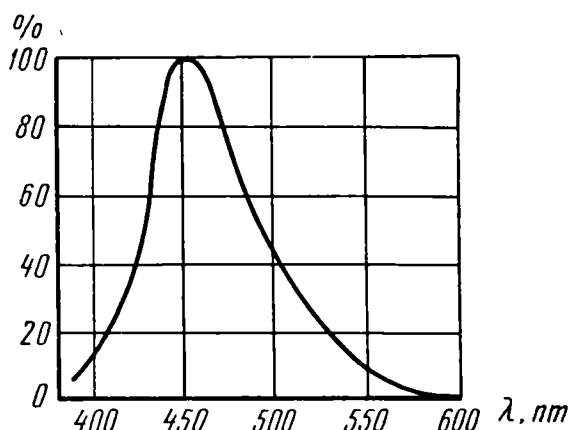


Fig. 6.6. Spectral characteristic of screen "A"

colour of luminescence of the screen, i.e. the colour corresponding to the medium area of the colour triangle.

The physical nature of cathodoluminescence does not allow development of a uniform phosphor radiating a continuous (white) spectrum. Hence, the screens of modern TV tubes are coated with a mixture of two (sometimes three) phosphors producing complementary colours. Figure 6.7 illustrates a summary spectral characteristic of a white screen whose active layer consists of a mixture of zinc sulphide and zinc-cadmium sulphide activated with silver (screen "B").

The position of the maximums on the spectral characteristics and, therefore, the visible colour of radiation of the screen can be varied in a wide range by changing the composition of the two-component phosphor. Thus, for example, widely used zinc-cadmium sulphide (solid solution of zinc and cadmium sulphides) activated with silver may have maximum radiation at any part of the visible spectrum depending on percentage of ZnS and CdS (Fig. 6.8).

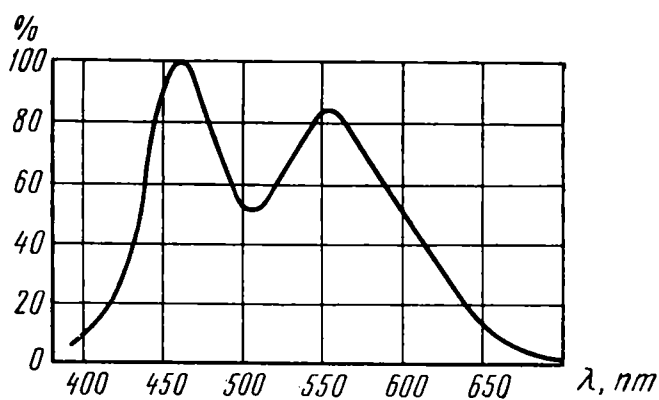


Fig. 6.7. Spectral characteristic of screen "B"

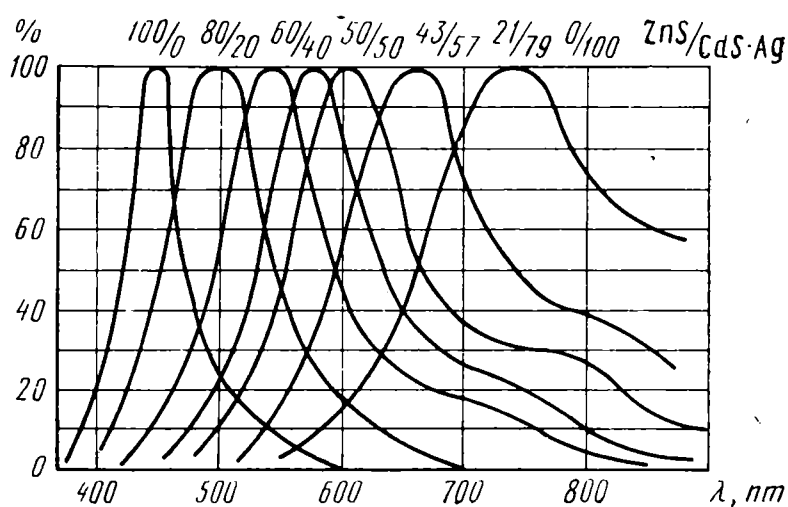


Fig. 6.8. Spectral characteristics of zinc-cadmium sulphide phosphor

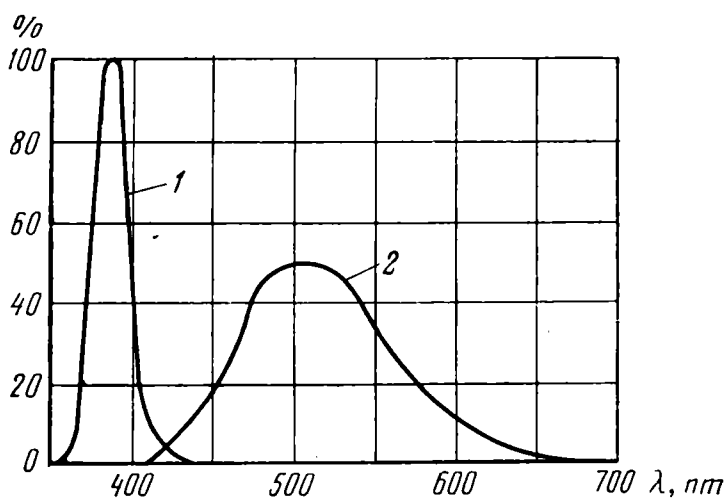


Fig. 6.9. Spectral characteristics of zinc oxide calcinated in air and hydrogen media

1—air; 2—hydrogen

Some variation of colour can be obtained by selecting the activator. Sometimes, a required colour of luminescence is obtained by incorporating two activators. The spectral characteristics of some phosphors can be changed markedly by means of heat treatment. Figure 6.9 illustrates the spectral characteristics of zinc oxide calcinated at a temperature of 1000°C in the air and in a reducing medium (hydrogen). In the first case the phosphor has violet luminescence, in the second—green.

The colour of luminescence of phosphors having the same emission band practically does not depend on the parameters (current and accelerating voltage) of the exciting electron beam. The visible colour of luminescence of two-component phosphors, as well as of single-component phosphors having one of several emission bands, can vary considerably with a change in the electron velocity (accelerating voltage) and the beam current density, since the dependence of the brightness (light output) on the accelerating voltage and the beam current density is different for different phosphors. This difference is lower in different emission bands of a single phosphor, but in this case, too, the observed colour varies its shade when the accelerating voltage changes.

6.4. Cathode-ray Excitation and Screen Persistence

The time of excitation of screen luminescence is determined by the quantum transitions in the cathodoluminophor and is very low, as it was indicated above. Therefore, the characteristics of cathode-ray excitation of the screen are usually unimportant for the operation of electron-beam devices. When a luminescent screen is excited by very short current pulses, the level of intensity of its glow is close to the level of intensity at continuous excitation. If the radiation receiver features integrating properties (for example, a human eye or photographic emulsion), even when the duration of the exciting pulse is lower than the time of raising the luminescence, the total effect observed practically does not depend on the steepness of the front of the curve of raising of luminescence.

Figure 6.10 shows the characteristics of luminescence of two screens with different time of raising the luminescence.

Assume that both screens have the same luminous efficiency. Then during continuous excitation after a time period exceeding the time of excitation of the screen having a higher inertia, both screens will have the same luminance. Assume now that both screens are excited by short current pulses whose duration t_1 is lower than the time of excitation of luminescence of both of the screens.

The energy of an electron beam, which can be transformed into radiant energy ($P_1 t_1$) by the phosphor, is presented in Fig. 6.10 by a rectangle $OCDB$. During the pulse time t_1 radiated is only

a fraction of this energy presented in the drawing by a part of the triangle OAB lying under the luminescence rise-up curve. The fraction of this energy presented by the $OCDA$ part of the rectangle above the luminescence rise-up curve is stored by the phosphor and is radiated after the excitation has ceased (afterglow). It is clear that the area $OCDA$ is equal to the area BAE , i.e. the entire radiant energy is equal to the fraction of the electron-beam energy transformed into radiation. The same is true for the second screen having greater inertia. Thus, the resultant effect observed for both screens will be the same. When the screen luminescence is perceived by eye, the above assumption is valid also in the case when the resultant

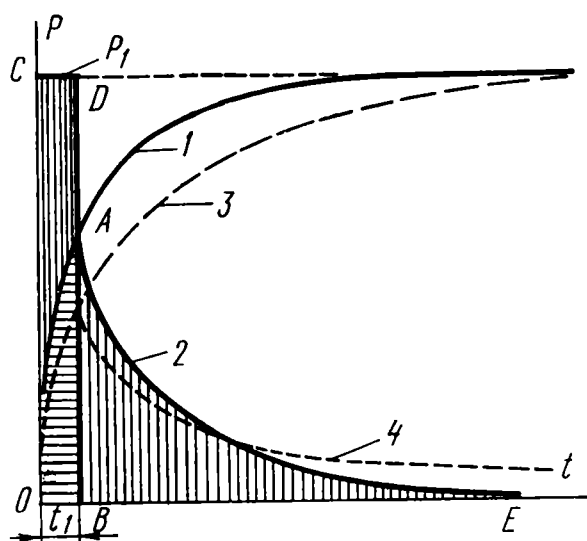


Fig. 6.10. Excitation and luminescence of screens

time of exciting the luminescence and the persistence does not exceed a few hundredths of a second, i.e. until the integrating capacity of the eye is preserved. The blackening of a photoemulsion is proportional to the illumination and the time of exposure; therefore, in this case, too, the final effect in a first approximation does not depend on the law of excitation of the phosphor.

If the time of raising the luminescence and the type of an excitation curve are of no practical importance, the persistence time and the shape of the curve of decay of the luminescence after the excitation of the phosphor by the electron beam has been ceased are often very essential in the choice of phosphors for various cathode-ray screens. For example, for television kinescopes it is expedient to have a screen with persistence of the order of a few hundredths of a second, since in this case the flickering due to the frame succession at a frequency of 50 Hz is less noticeable. A higher persistence time will result in appearance of "tails" behind the quickly moving

details of the image. The screen of radar tubes used in circular-scanning indicators must have sufficiently long persistence in the order of several seconds. An example of a screen having very short afterglow (microseconds) is that used in a travelling-beam transmitting tube. Thus, the persistence must be varied within a wide range—from microseconds to several seconds and even minutes—depending on the field of application of electron-beam devices.

From the point of view of afterglow period the screens are divided into five groups: (1) screens with very short persistence, less than 10^{-5} s; (2) short-persistence screens with afterglow time of 10^{-5} to 10^{-2} s; (3) medium-persistence screens with afterglow time of 10^{-2} to 10^{-1} s; (4) long-persistence screens having afterglow time of 10^{-1} to 16 s; (5) screens having very long persistence of higher than 16 s.

Since the screen luminescence decays gradually, precise measurement of the total persistence time—from the moment of ceasing the excitation to the zero luminance—presents some difficulties associated with quantitative estimation of very small luminance values of the screen at the end of the process of luminescence. In order to overcome this difficulty, the screen persistence is taken as a time interval from the moment of ceasing the excitation to the moment when the screen brilliance is equal to 1% of the initial luminance. It is clear that luminance of one hundredth of the initial one can be measured with a sufficient accuracy.

The characteristics of decay of the screen luminescence are determined mainly by the phosphor properties. The experimental curves obtained for simple (homogeneous) phosphors can be approximated by using the exponential or hyperbolic laws.

For phosphors whose luminescence is caused only by the internal transitions in the atoms, the rate v (velocity) of disappearance of excited states is proportional to the number of these states, N :

$$v = -\frac{dN}{dt} = kN \quad (6.4)$$

The intensity of luminescence I is, in its turn, proportional to the velocity of disappearance of excited states

$$I = \kappa N \quad (6.5)$$

where κ is the constant characterizing the phosphor.

From (6.4) and (6.5) we obtain the equation of the phosphor decay

$$\frac{dI}{dt} = -kI \quad (6.6)$$

whose solution is the exponent

$$I = I_0 e^{-\frac{t}{\tau}} \quad (6.7)$$

where I_0 is the luminescence intensity at the moment of excitation ($t = 0$); τ is the decay constant characterizing the phosphor.

It is clear that the higher the value of τ , the higher the persistence. The persistence time, i.e. the time of decay of the luminescence intensity to $0.01 I_0$, is uniquely determined by the decay constant:

$$t_{per} = (\ln 100) \tau \approx 4.6\tau$$

For phosphors, in which the fluorescence can be excited not only by the interatomic transitions but due to the transitions inside the crystal lattice (bimolecular transitions), the rate of disappearance of excited states is proportional to the number of excited electrons and vacant places at the lower energy levels:

$$v = -\frac{dN}{dt} = kN^2 \quad (6.8)$$

In this case the equation of decay of the phosphor takes the form

$$dI = -kI^{3/2} \quad (6.9)$$

and its solution is written as follows:

$$I = \frac{I_0}{\left(\frac{t}{\tau} + 1\right)^2} \quad (6.10)$$

i.e. the decay characteristic is a hyperbola. In this case the persistence time is also uniquely determined by the decay constant: $t_{per} = 9\tau$.

The decay constant in equations (6.7) and (6.10) must not depend on the parameters of the exciting beam. Experimental study shows that for homogeneous phosphors this assertion is valid in a wide range of change of the exciting beam current density and the accelerating voltage. Experimental verification of formula (6.10) shows that for some phosphors the decay curve is close to a hyperbola but the exponent is somewhat smaller than two.

The attenuation characteristics of multicomponent phosphors and of those with several excitation bands usually cannot be described by equations (6.7) and (6.10). These curves are often approximated by selecting exponents with different values of τ at different sections or the characteristic is approximated by one or two exponents and a hyperbola. In this case the concept of "decay constant" loses its sense. In the same manner, for compound phosphors the persistence can markedly depend on the parameters of the exciting beam. An increase in the current density and the accelerating voltage, as a rule, increases the persistence.

Most of commercial phosphors have rather a low persistence (10^{-6} to 10^{-2} s). Much higher persistence (up to several minutes) is obtained by exciting the luminescence not by an electron beam but by short-wave light. In this case we apparently deal with

photoluminescence rather than with cathodoluminescence. The long-persistence screens often have a two-layer coating, the luminescence of the second layer being excited by the light of the first layer bombarded by an electron beam. A characteristic feature of long-persistent screens is that they are capable of accumulating the excitation.

When the phosphor is excited by several pulses following one another at intervals shorter than the persistence time, we observe

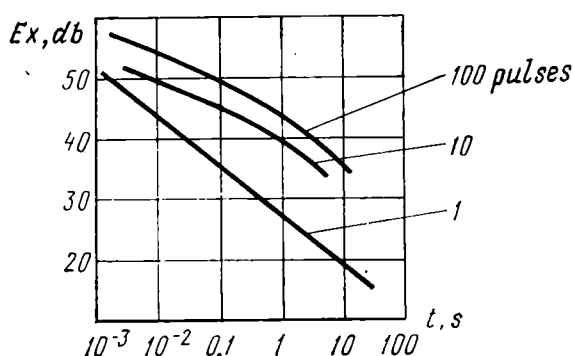


Fig. 6.11. Persistence *versus* number of exciting pulses

an increase in the brilliance of the screen. This increase is due to the summation of the energy stored by the phosphor during the excitation of the preceding pulse (and radiated during the afterglow) and the energy obtained during the following exciting pulse. As an example, Fig. 6.11 shows the persistence characteristics of copper-activated zinc-cadmium sulphide excited by one pulse, ten pulses and a hundred pulses following at an interval of 1 s. The accumulation of excitation has found its practical application in radar tubes.

6.5. Secondary Emission and Potential of Screen

Discussing the screen properties in the preceding Sections, we assumed that the energy of the electrons bombarding the screen is uniquely determined by the value of accelerating voltage of the gun. This assumption holds when the phosphor layer is applied to an electrically conductive substrate or when a conductive coating is applied to the phosphor layer, the electrically conductive substrate or coating being connected to last anode of the gun, i.e. when the screen potential (relative to the gun cathode) is equal to that of the gun anode. In many types of electron-beam devices a phosphor layer having specific resistance of 10^{12} - 10^{14} ohm·cm is applied to the glass bottom of the bulb. The specific resistance of technical glasses used for making bulbs of electron-beam devices is also within the range of 10^{11} to 10^{13} ohm·cm. Thus the screen as a whole is

a dielectric. The potential of a nonconductive screen can generally be substantially different from the potential of the gun anode.

When an electron beam impinges a nonconductive screen, the negative charge carried by the electrons is stored on the screen and the potential of its surface drops down approaching the potential of the gun cathode. It is clear that in this case the beam is deflected from the screen, i.e. the screen luminescence is ceased. Thus, normal

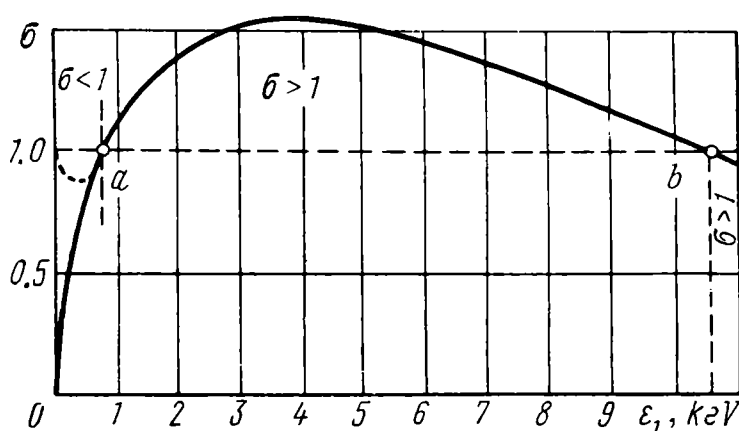


Fig. 6.12. Secondary emission coefficient *versus* energy of primary electrons

operation of a nonconductive screen is possible only if the electric charge carried by the electron beam is withdrawn from the screen.

The only practical method of withdrawal of the charge from a dielectric surface is the use of secondary electron emission. The energy of the electrons striking the screen is consumed for excitation of luminescence, heating of the screen, excitation of X-ray radiation (at high accelerating voltages) and excitation of secondary electron emission. The fraction of the energy of the electron beam consumed for excitation of secondary electron emission depends on the properties of the phosphor layer and on the velocity (energy) of the beam electrons. The dependence of the secondary emission coefficient $\sigma = I_2/I_1$ (where I_1 is the current of the primary electrons and I_2 is the current of the secondary electrons) on the energy of the primary electrons is expressed by the known curve with a mildly sloping maximum (Fig. 6.12).

The maximum value of the secondary emission coefficient exceeds unity for the majority of solid bodies including phosphor crystals. Only at two points of the characteristic (*a* and *b* in Fig. 6.12) $\sigma = 1$.

Assume that a dielectric screen receives a flow of electrons accelerated by a potential difference lower than the potential corresponding to the point *a* (Fig. 6.12). It is clear that in this case the amount of electrons transferred by the beam to the screen surface is larger than the amount of secondary electrons emitted from the screen ($\sigma < 1$),

a negative charge is accumulated on the screen and a field decelerating the beam electrons is built up near the screen surface. The screen potential, in the limit, tends to zero (the potential of the gun cathode). The beam electrons start being reflected from the screen due to the retarding field and the "apparent" secondary emission coefficient tends to unity (a dash line in Fig. 6.12), which corresponds to reflection of all the electrons, directed to the screen, back to the gun. Thus, when the energy of the beam electrons is lower than eU_1 , the screen potential approaches zero, the electrons do not reach the screen, the luminescence is ceased. Strictly speaking, the luminescence is ceased before the moment when the screen surface acquires a "dead" potential U_0 [see formula (6.2)]. The potential U_1 corresponding to the point a on the curve shown in Fig. 6.12 is called *the first critical potential* $U_{cr 1}$.

When the screen is bombarded by electrons accelerated at a sufficiently high potential difference (higher than the first critical potential), the number of secondary electrons leaving the screen becomes higher than the number of primary electrons ($\sigma > 1$). In this case the screen starts being charged positively with respect to the gun anode.

However, there is no significant excess of the screen potential over the gun anode potential, since some of the secondary electrons return back to the screen charged positively with respect to the gun anode. The secondary electrons returning to the screen reduce the screen potential. The equilibrium is set with a screen potential approximately equal to the potential of the gun anode. In this case the number of secondary electrons emerging from the screen is equal to the number of electrons transmitted by the beam to the screen, i.e. the "apparent" secondary emission coefficient is equal to unity (although the true value of $\sigma > 1$). Thus, in the region of the energy of the beam electrons eU_1 to eU_2 (the points a and b on the curve in Fig. 6.12) corresponding to $\sigma \geq 1$, the screen potential can with adequate accuracy be considered equal to the potential of the gun anode. The measurement of the screen potential shows that, depending on the beam current density and the conditions of withdrawal of secondary electrons from the screen, the true screen potential (at $\sigma \geq 1$) can differ from the anode potential by a few volts in either direction. For example, with a high current density and poor withdrawal of secondary electrons, at the screen surface there can form a negative space charge so that the screen potential becomes somewhat lower than the potential of the gun anode. On the contrary, at low current density and good withdrawal of secondary electrons the screen potential can exceed the potential of the gun anode by a few volts.

When the energy of the beam electrons corresponds to the section of the curve to the right from the point b in Fig. 6.12, the secondary emission coefficient of the screen becomes less than unity. A negative

charge starts accumulating on the screen, since the number of electrons emerging from the screen is lower than the number of electrons transferred to the screen by the beam. The accumulation of the negative charge on the screen reduces the potential on its surface, and developed near the screen is a field decelerating the electrons of the beam. It is clear that the screen potential will decrease continuously until the secondary emission coefficient is equal to unity, i.e. to the potential corresponding to the point *b* on the curve shown in Fig. 6.12. When the screen potential reaches the point *b*, the screen ceases to accumulate the charge. An equilibrium sets up between the number of electrons carried by the beam and the number of electrons leaving the screen due to the secondary emission.

The screen potential $U_{cr\ 2}$ corresponding to the point *b* on the curve shown in Fig. 6.12 is called *the second critical potential* or *limiting potential* and is one of the important parameters of the screen. The value of this potential determines the maximum energy of the beam electrons or maximum accelerating voltage of the gun which should be selected for tubes of the given type. In fact, there is no need in increasing the gun accelerating voltage to values higher than $U_{cr\ 2}$, since the screen potential determining the brightness in this case will not exceed $U_{cr\ 2}$, i.e. an increase in the accelerating voltage will not result in an increase in the screen brilliance. The magnitude of the second critical potential for different screens is usually within the range of 5 to 25 kV; for some special types of screens having a mixture of two or several phosphors it rises up to ~ 40 kV. In the process of the screen service with the use of high values of $U_{cr\ 2}$, the second critical potential sometimes tends to drop down which practically means a decrease in the screen brilliance at constant parameters of the electron beam. Sometimes the screens having a thin or nonuniform phosphor layer manifest an abnormally low value of the second critical potential—about 3 kV. This phenomenon is observed due to the fact that the thin phosphor layer is shot through by the electrons of the beam or the electrons penetrate to the bulb face in-between the grains of a loose phosphor layer. In this case the secondary emissive properties of the screen are determined by the dependence of the secondary emission coefficient on the energy of the incident electrons, the glass of the bulb face and not on the phosphor. The secondary critical potential of most of technical glasses is equal to about 3 kV and this value determines the “apparent” secondary critical potential of the screen. Therefore, dealing with accelerating voltages close to $U_{cr\ 2}$, one should use sufficiently thick and uniformly applied layers of phosphor eliminating a possibility of penetration of electrons to the bulb face.

The above assumptions of a steady-state potential hold only for an ideal dielectric, when prior to the electron bombardment the true potential of the phosphor surface is close to the accelerating

voltage of the gun, otherwise, for example with a screen potential, before the electron beam strikes the screen, close to zero (gun cathode potential), regardless of the value of the accelerating voltage of the gun, the equilibrium value of the screen potential will be close to zero and the electrons will be reflected from the screen surface, i.e. there will be no luminescence. However, the experience shows that the luminescence takes place almost instantly as soon as the electrons reach the screen surface, independently of the true potential of the screen before switching the tube on (it is assumed that for the given energy of the electrons of the beam $\sigma \gg 1$). This apparent contradiction is due to the small finite conductivity of the glass of the wall and face of the bulb, and of the phosphor layer. When the high voltage is turned on a charge wave propagates along the glass and phosphor layer from the conductive coating of the internal surface of the bulb connected to the gun anode. This wave brings the screen potential (before it is acted on by the beam) to the potential of the conductive coating. Therefore, in the absence of electron bombardment, for example when the beam is cut off by the negative bias of the modulator, the screen potential is practically equal to the potential of the electrode nearest to the screen provided that the accelerating voltage is switched on. When the beam impinges the screen, the conductance of the latter turns to be insufficient for quickly balancing the potentials of the screen and conductive coating. The true potential of the screen is set up in accordance with the value of the secondary emission coefficient of the screen and the energy of the electrons of the beam as shown above.

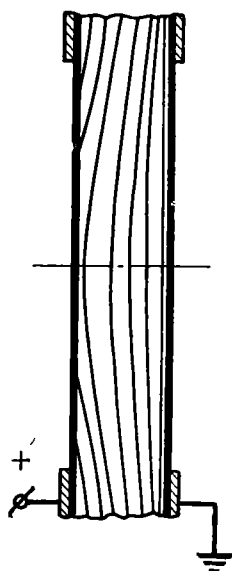


Fig. 6.13. Electric field in bulb face solid

and the energy of the electrons of the beam as shown above. In some cases perceptible screen conductivity may cause distortion to the electric field near the screen surface and defocusing of the beam or may disturb the beam deflection, while a considerable potential drop in the centre portion of the screen may reduce the luminance. Let us discuss the potential distribution in the glass face of the bulb having volumetric electrical conductivity (Fig. 6.13). Assume that the cathode-ray tube is operated with the gun cathode earthed and the conductive coating highly positive with respect to the earth applied to the bulb face. The bulb is mounted in a supporting ring having the earth potential. Since the surface conduction of the external surface of the bulb face is usually much higher than the internal conduction due to the presence of a film of moisture, the potential of the external surface of the bulb face can be considered with sufficient accuracy equal to the potential of the supporting

ring. The electric field lines are shown in Fig. 6.13. The field is distorted near the screen surface, and the potential drop in the centre portion of the screen may reduce the luminance. The true potential of the screen is set up in accordance with the value of the secondary emission coefficient of the screen and the energy of the electrons of the beam as shown above.

ring, i.e. to the gun cathode potential. Then, the equipotential surfaces in the glass solid will have the shape shown in Fig. 6.13. From this figure it is clear that the potential of the medium portion of the screen (with no electron bombardment) is much lower than the potential of the peripheral portions located near the conductive coating and having a potential close to the gun accelerating voltage.

The electric field caused by such nonuniform potential distribution over the screen surface leads to the above-said undesirable phenomena. The fall of the electron beam on the screen surface at $\sigma > 1$ increases the screen potential. When a sufficiently intensive beam is swept over the entire surface of the screen, the potential of this screen rapidly acquires an equilibrium value approximately equal to the potential of the gun anode. Consequently, the stray electric field caused by the conductance of the bulb glass should be taken into account only when the tube operates at very small currents and when the beam strikes only small portions of the screen. When the current is small, the charge carried by the secondary electrons from the screen may be insufficient for bringing the screen potential to the equilibrium value. When the electrons strike only a small portion of the screen, the potential of this portion will have an equilibrium value, while the presence of adjacent sections at a lower potential will result in nonuniform local fields. The local fields may defocus the beam and interfere with the beam deflection. Therefore, where it is necessary to use small separate areas of the screen, it is expedient to regularly irradiate the entire screen by an intensive electron beam for bringing the potential of the screen to an equilibrium value corresponding to the potential of the conductive coating.

Thus, when operating electron-beam devices, one must select accelerating voltages lying in the range of $U_{cr1} \leq U_{ac} \leq U_{cr2}$. Where it is necessary to have $U_{ac} > U_{cr2}$ (e.g. to obtain very high luminance), the screen potential should be kept equal to the accelerating voltage of the gun. This, for example, can be attained by coating the screen with a thin conductive film electrically connected to the gun anode (see Section 6.6).

6.6. Types of Screens

Electron-beam devices can be provided with screens of different types depending on their application. The properties of a screen are determined by the composition of the phosphor, the thickness of its layer, the grain size, the method of application of a luminophor and the presence of additional coatings. To meet the standing requirements standard types of screens have been designed which possess certain parameters such as colour and persistence, light output (or luminance), and ultimate (second critical) potential. Some types of

screens meet additional requirements associated with the operation of the device. The screens of cathode-ray tubes made in the USSR have the typical designation in the form of one of the letters of the Russian alphabet. Foreign-made tubes are also standardized. For example, screens of tubes made in the USA have a typical designation in the form of the letter P and a serial number.

As has been said above, the main properties of the screens are determined by their phosphors. Of a great variety of substances capable of glowing under the effect of electron bombardment the most common are zinc *sulphide*, cadmium *sulphide*, zinc *silicate* and calcium *tungstenate*.

Sulphide phosphors have remarkable properties such as very high luminous efficiency (up to 15 c/W), high luminance, and respectively the possibility of varying the colour of luminescence within a wide range (practically over the whole visible spectrum) depending on the composition and type of the activators used. In the similar manner, depending on the type and composition of the activator, sulphide phosphors have different persistence: from 10^{-3} s (silver-activated zinc sulphide) to about 10 s (copper-activated zinc-cadmium sulphide). The critical potential of sulphide phosphors is also high—up to 30–35 kV. These properties of screens with sulphide luminophors have stimulated their wide application in electron optics.

However, sulphide luminophors have some disadvantages. First, their physical and chemical stability is low. Intensive electron bombardment and, particularly, impacts by heavy charged particles such as negative ions rapidly destroy the phosphor. The sulphide phosphor parameters such as luminance and colour of luminescence can vary significantly in the presence of contaminating components even in very small concentrations. Another disadvantage of these luminophors consists in considerable reduction of luminous efficiency with reduction in the particle size: the phosphor consisting of 2–3- μm grains has a luminous efficiency much smaller than that of the phosphor with a grain size exceeding 10 μm .

The spectral characteristics of some sulphide phosphors were given in Figs. 6.6 and 6.8. In order to obtain the required colour of luminescence, one can use mixtures of several sulphide phosphors. For example, widely used in coatings of kinescope screens is so-called white mixture—a mechanical compound of zinc sulphide and zinc-cadmium sulphide activated with silver. The spectral characteristic of such a screen is given in Fig. 6.7.

Of the silicate group the widest application enjoys zinc orthosilicate activated with manganese ($\text{Zn}_2\text{SiO}_4 \cdot \text{Mn}$) commonly referred to as willemite by the name of the mineral containing zinc orthosilicate.

Synthetic willemite is a good luminophor having adequately high luminous efficiency (up to 3.5 c/W), average persistence and a

green colour of luminescence. The spectral characteristic of willemite is given in Fig. 6.5.

Willemite has very high physical and chemical stability. Intensive electron bombardment causes comparatively small damage to a willemite screen. Admixtures in small concentrations practically have no effect on the parameters of this phosphor. Willemite does not change its properties after heating to 500-600°C.

Calcium tungstate is used in more rare occasions, mainly when it is necessary to obtain very short persistence. No activators are used with calcium tungstate. The role of an activator is apparently played by stoichiometric excess of calcium, i.e. the composition of tungstate phosphor should be written as $\text{CaWO}_4 \cdot \text{Ca}$.

Calcium tungstate has a blue-violet colour of luminescence and very short persistence, in the order of 10 microseconds. The physical and chemical stability of calcium tungstate is satisfactory.

The disadvantage of tungstate screens is a small luminous efficiency—about 0.2 cd/W. The spectral characteristic of calcium tungstate is shown in Fig. 6-14.

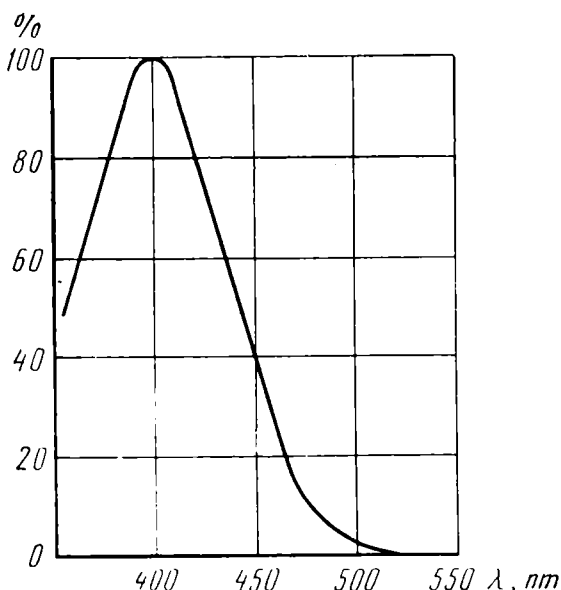


Fig. 6.14. Spectral characteristic of calcium tungstate

Fluorides, phosphates, selenides, oxides and photoluminophores based on rare-earth elements have recently found some application for making screens of special cathode-ray tubes.

The rare-earth luminophors are adequately efficient and have higher physical and chemical stability than the sulphide luminophores. A distinctive feature of the rare-earth luminophors is a very narrow radiation band on the spectral characteristic which provides high purity of the colour on the screen.

Red phosphor consisting of yttrium orthovanadate and activated with europium ($\text{YVO}_4 \cdot \text{Eu}$) has found practical application in screens for colour kinescopes. This phosphor has a narrow radiation band with its maximum near $\lambda = 619 \text{ nm}$. Good results have also been obtained with yttrium and gadolinium oxides activated with europium ($\text{Y}_2\text{O}_3 \cdot \text{Eu}$ and $\text{Gd}_2\text{O}_3 \cdot \text{Eu}$). These phosphors also have maximum of radiation in the long-wave portion of the visible spectrum and their light output exceeds that of yttrium orthovanadate. The

rare-earth phosphors are rather promising; however, high cost of rare-earth compounds prevents their wide application.

The basic parameters of some commercial phosphors are given in Table 6.1.

The screen properties greatly depend on the thickness of the layer of phosphor and the size of its grains. As has been mentioned in Section 6.2, the optimum thickness of the phosphor layer depends on the accelerating voltage: the greater the energy of the electrons flying to the screen, the thicker must be the phosphor layer to eliminate penetration of electrons through the luminophor to the substrate.

Table 6.1

Composition and Properties of Some Phosphors

Name and composition	Colour of luminescence	Luminous efficiency, cd/W	Persistence, s
Silver-activated zinc sulphide $\text{ZnS} \cdot \text{Ag}$	Blue	0.5-2	0.001
Silver-activated zinc-cadmium sulphide $[\text{Zn}, \text{Cd}] \text{S} \cdot \text{Ag}$	Yellow	5.0-8.0	0.001
Silver-activated zinc sulphide and zinc-cadmium sulphide (white mixture) $\text{ZnS} \cdot \text{Ag} + [\text{Zn}, \text{Cd}] \text{S} \cdot \text{Ag}$	White	7.0-8.0	0.001
Copper-activated zinc-cadmium sulphide (photoluminophor) $[\text{Zn}, \text{Cd}] \text{S} \cdot \text{Cu}$	Yellow	1.5-2.0	3.0-8.0
Willemite $\text{Zn}_2\text{SiO}_4 \cdot \text{Mn}$	Green	2.0-3.5	0.05
Calcium tungstate CaWO_4	Blue	0.18-0.2	0.00001

Since the phosphor layer applied to the substrate is far from solid (is loose), the thickness of the coating cannot be measured directly. However, owing to the fact that in the voids between the grains of phosphor the electrons fly without energy loss, we may calculate the optimum weight load, i.e. the amount of phosphor in grams per sq. cm of the screen surface. To this end, it will do to find the required phosphor thickness by formula (6.3) and then calculate the weight load by the specific gravity of the phosphor. Of course, the apparent thickness of an actual loose coating will be much greater than the calculated one but the conditions of absorption of electrons will approximately be the same in both cases. Consequently, the thickness of the phosphor layer is practically estimated by the value of its weight load. The screen thickness (weight load) varies from tenths of milligram per sq. cm to 5-8 mg/cm², depending on the parameters of the tube and its field of application.

The luminous efficiency (or the brightness) of the screen depends on the size of the phosphor grains. As a rule, screens with coarse-grained phosphors have a higher luminous efficiency. However, in

many cases the use of coarse-grained phosphor layers is inexpedient. The grain size limits the resolving power of the screen, as the luminous spot on the screen cannot be smaller than the crystal of phosphor glowing under the action of the electron beam. The decrease in the luminous efficiency is especially noticeable with use of finely ground sulphide phosphors. Therefore, the screens of sulphide phosphors usually have comparatively large grains, up to 5-8 μm . Silicates and tungstenates are less sensitive to disintegration so that willemite and calcium tungstate with a grain size less than 1 μm can be used for making high-resolution screens.

It is interesting to note that the grain size has some effect on the optimal (providing maximum luminous efficiency) weight load of the screen under otherwise equal conditions. This is due to the fact that the electron beam has a greater probability of passing through the phosphor layer without interacting with the coarse-grained phosphor. As an example, Fig. 6.15 illustrates a curve of dependence of the optimal thickness of a layer of sulphide phosphor on the grain size and at the energy of bombarding electrons of 14 keV.

The requirement for high resolving power made it necessary to develop structureless screens in the form of a thin uniform film. It is clear that such a screen has many advantages. However, the attempts to develop a structureless screen using the known commercially available phosphors have failed. Satisfactory results have been obtained by using some new luminophors which can be applied to the screen through vacuum evaporation (sublimation). In this case the phosphor is deposited on the substrate in the form of a thin uniform film. However, such phosphors have much lower luminous efficiency than ordinary grained phosphor and so far have found limited application.

The only structureless screen which has found practical application is a screen with recording by a black trace. The active layer of this screen does not glow under the effect of electron bombardment but becomes darker and acquires a violet-brown colour where the electron beam strikes. A change of colour under the action of electron bombardment is inherent in crystals of alkali-halide salts. In particular, colourless (transparent) crystals of potassium chloride become coloured violet-brown under the action of an electron beam.

The change in the colour of potassium chloride is due to the centres of light absorption which occur in the crystals subjected to electron bombardment. The crystal lattice of potassium chloride may have vacant places in the points normally occupied by potassium and chloride atoms. When an electron is brought to the lattice point normally occupied by a chloride atom this point becomes a centre of light absorption. The electron localized in the lattice site can oscillate at a certain (resonant) frequency. When the crystal with "defects" in the crystal lattice (presence of electrons in the places of

chloride atoms) is illuminated, the light waves corresponding to the resonance frequency of the electrons localized in the lattice sites are intensively absorbed. The resonance frequency of potassium chloride corresponds to the yellow-green portion of the spectrum. Therefore, when a crystal of KCl is subjected to electron bombardment, it looks as being coloured additionally in violet colour.

When the crystals of potassium chloride are bombarded by electrons, the primary electron beam excites secondary emission in the crystal and the secondary, slower electrons can occupy vacant places

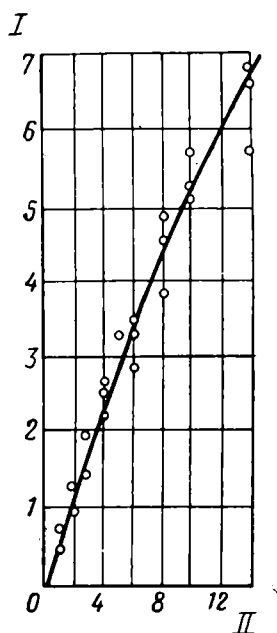


Fig. 6.15. Optimum ratio of layer thickness and size of phosphor grain

I —luminophor layer thickness, mg/cm^2 ; II —grain size, μm

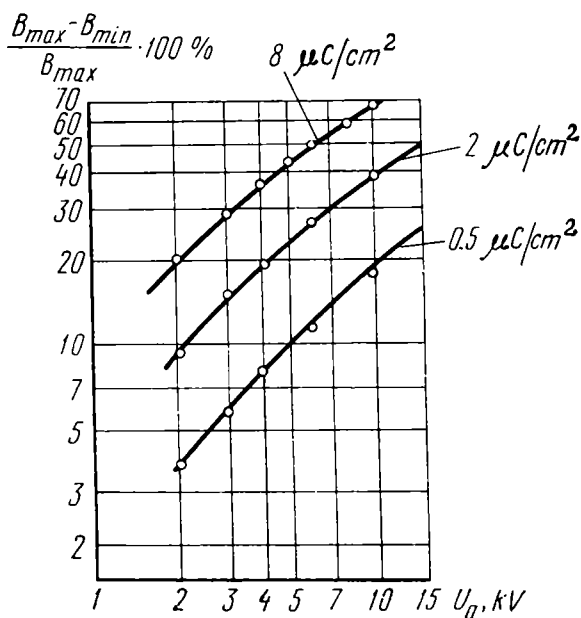


Fig. 6.16. Skiatron blackening versus accelerating voltage

in the lattice sites, thus forming centres of light absorption. The absorption centres are very stable and the colour of the screen may persist for several hours after the electron bombardment has ceased. Removal of the trace of the electron beam (screen discoloration) is effected by heating the screen or by exposing it to a source of intensive light. When the screen is heated, the thermal vibration of the crystal lattice causes redistribution of electrons, the centres of absorption disappear and the crystals of KCl become transparent again.

The intensity of blackening of the screen with recording by a dark trace depends on the energy of the electrons (accelerating voltage) following approximately the square law (Fig. 6.16).

In contrast to the phosphors, the intensity of blackening of the screen depends not on the current density but on the density of the summary electric charge brought by the electron beam on 1 cm^2 of the screen surface (Fig. 6.17)

The optimum thickness of the layer of KCl is determined by the requirement for complete absorption of the beam electrons in the thickness of the layer. For example, at accelerating voltages of the order of 10 kV the thickness of the layer of KCl must be at least $10 \text{ }\mu\text{m}$. The layer of potassium chloride is applied by vacuum evaporation to a substrate (special glass plate or bulb face). Thus obtained

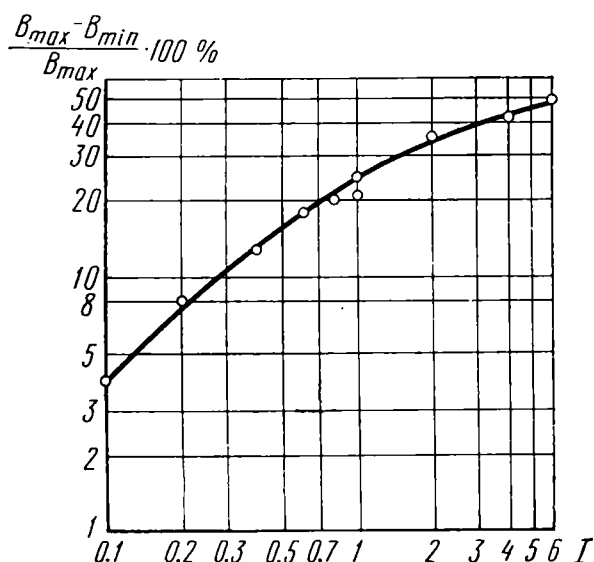


Fig. 6.17. Skiatron blackening versus charge density
 I —charge density, $\mu\text{C}/\text{cm}^2$

is a structureless film having an optical contact with the glass. The optical contact is necessary for observing the trace of the electron beam through the substrate, since in the absence of such a contact much of light incident to the screen would be reflected from the internal surface of the glass without passing through the layer of potassium chloride and without being absorbed therein.

In addition to high resolution, the screens with recording by a dark trace have other advantages. They have adequate resistance to the electron bombardment and withstand heating to comparatively high temperatures (the maximum admissible temperature is limited by the sublimation point of KCl). The persistence may be up to several hours. When necessary, the dark trace can quickly be destroyed by heating or intensive illumination of the screen.

A two-layer screen with cascade excitation of luminescence is one of long-persistence screens widely used in electron optics. The deve-

lopment of two-layer screens was caused by the absence of commercial cathodoluminophores with persistence of up to a few seconds. At the same time, it was known that some phosphors, for example, copper-activated zinc-cadmium sulphide, when illuminated by short-wave light, are capable of glowing for a long period of time after the illumination has been ceased. Thus, alongside with cathode luminescence, zinc-cadmium sulphide features photoluminescence and phosphorescence, the persistence of the latter exceeding the beam-excited luminescence by a few orders of magnitude. The long persistence during the excitation by light is due to the fact that the centres of luminescence of the phosphor crystals capable of being in an excited state for a long period of time are most probably excited not by electrons but by electromagnetic radiation with a wavelength corresponding to the short-wave edge of the visible spectrum close to the ultraviolet region.

Thus, for long persistence the photoluminophor is preferably excited by illuminating with short-wave light rather than the electron bombardment. The theory of luminescence shows that luminescence (fluorescence and phosphorescence) always is of longer waves as compared with the radiation causing this luminescence. Hence, in order to match the prolonged phosphorescence of a photoluminophor with the medium region of the visible spectrum, having the highest apparent brightness, the glowing must be excited by short-wave radiation.

The photoluminophor, applied directly to a transparent substrate (usually the bulb face) of practically employed two-layer screens, consists of zinc-cadmium sulphide activated with copper and having a long-time yellow luminescence when excited with blue or violet light. The photoluminophor is excited by the radiation of the cathodoluminophor excited by an electron beam. The colour of luminescence of the cathodoluminophor must be close to blue, the persistence of the cathodoluminophor having no importance. The cathodoluminophor is usually represented by silver-activated zinc sulphide which, when excited by an electron beam, produces bright azure-blue glowing. The spectral characteristics of a two-layer screen are given in Fig. 6.18.

Since the colour of luminescence of the phosphor (ZnS , $\text{CdS}\cdot\text{Cu}$) differs from the colour of luminescence of the cathodoluminophor ($\text{ZnS}\cdot\text{Ag}$), when the screen is excited by an electron beam, there is seen a bright blue flash. Then, after cutting off the electron beam, the yellow luminescence of the photoluminophor is observed on the screen for a long period of time. With raster excitation of a two-layer screen the colour of luminescence also differs considerably from the colour of luminescence of both phosphors, although measurements show that the cathodoluminophor luminance does not exceed 5-7% of the total brilliance of the screen. It is clear that the best

conditions of excitation of photoluminophor would be at complete absorption of the cathodoluminophor radiation by the photoluminophor layer, but for that case very thick layers of photoluminophor would have to be used. The use of thick photoluminophor layers is inexpedient, since much of the photoluminophor radiation itself would be scattered in these layers and this would cause a decrease in the brilliance and resolving capacity of the screen (a larger luminous spot). Hence the thickness of photoluminophor layer is in practice selected so that it ensures sufficient absorption of the exciting radiation of the cathodoluminophor and, at the same time, it absorbs, but little the light of the photoluminophor itself. Of course, that part of the cathodoluminophor radiation which is not

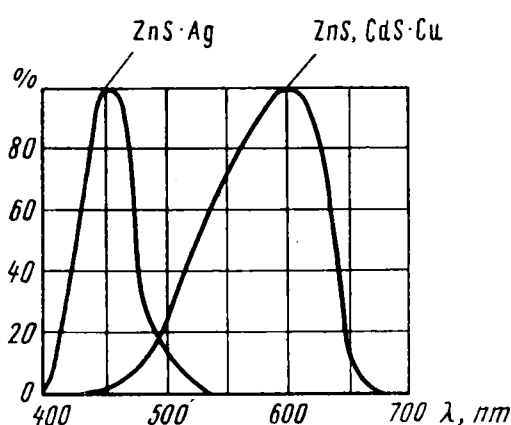


Fig. 6.18. Spectral characteristics of two-layer screen

absorbed by the photoluminophor is seen by the observer. A pronounced flash of luminescence of the cathodoluminophor is sometimes undesirable. Therefore, the two-layer long-persistence yellow screens are often covered from the observer's side with a yellow-orange light filter transparent for the photoluminophor radiation but absorbing the short-wave radiation of the cathodoluminophor.

Extensive use has recently been made of the so-called *aluminized* screens having a metal coating in the form of a thin aluminium film over the phosphor layer from the electron gun side. The screens with a metal coating have many advantages. First, a metal opaque coating eliminates parasitic illumination of the screen by the light emitted by the phosphor into the bulb and incident to the screen either directly (due to the concave surface of the screen) or after reflection from the bulb walls. The absence of internal illumination of the screen considerably increases the image contrast (see Section 9.1). Second, the light radiated by the phosphor towards the metal film is reflected by the latter towards the observer. Therefore, the screen luminous efficiency is increased. Consequently, aluminized screens have higher light output. Third, in the presence of a metal

layer electrically connected to the gun anode the screen potential is independent of the secondary-emission characteristics of the phosphor and the energy of the electrons bombarding the screen. This potential is uniquely determined by the accelerating voltage of the gun. Hence, the tubes with aluminized screens can operate at accelerating potentials exceeding the second critical potential of the cathodoluminophor. Finally, a metal film prevents bombardment of the phosphor layer by heavy charged particles, negative ions, which destroy the phosphor. Thus the screens with a metal coating have a higher service life, particularly in the case of using high accelerating voltages. The tubes with aluminized screens require no ion traps (see Section 9.3).

The metal film for coating the phosphor must satisfy several requirements. It must be transparent for electrons and opaque for light and heavy charged particles. The coating must have high reflective capacity and does not enter chemical reaction with the phosphor. It must allow heating to high temperatures and be well degassed during heating. Finally, the process of application of this film to the phosphor layer should not be difficult. An aluminized coating fairly meets all these requirements. Thin aluminum layers ($0.1\text{--}0.5\text{ }\mu\text{m}$) are transparent for high-speed electrons (with energy higher than $6\text{--}8\text{ keV}$) and opaque for light. A thin aluminum layer traps practically all heavy charged particles (negative ions). A smooth surface of aluminum efficiently reflects light. The aluminum does not react chemically with phosphors involved and is well degassed during heating. A comparatively low melting point of aluminum (660°C) allows the films of this metal to be applied by means of vacuum evaporation.

As mentioned above, the thickness of an aluminum film is equal to a few tenths of one micron. To obtain thinner solid coating is an engineering challenge. Furthermore, an aluminum layer thinner than $0.1\text{ }\mu\text{m}$ is transparent for light. At the same time, such films are easily shot through by electrons having sufficiently high energy. Low-velocity electrons (with energy lower than $5\text{--}6\text{ keV}$) are absorbed in an aluminum layer and their path is changed. They are scattered in aluminum. The absorption of electrons reduces the efficiency of utilization of the electron beam, the screen brilliance drops and the scattering results in a decrease in the screen resolution. Hence, an aluminized screen features a lower luminous efficiency at low accelerating voltages as compared to a non-aluminized screen. With accelerating voltages of $6\text{--}8\text{ kV}$ the luminous efficiency of both aluminized and non-aluminized screens is approximately the same, while with voltages higher than 10 kV the luminous efficiency of an aluminized screen is by $50\text{--}60\%$ higher than that of a non-aluminized one. Therefore, aluminized screens are advantageous only in high-voltage tubes. The approximate dependence of the luminous effi-

ciency on the accelerating voltage for an aluminized and non-aluminized screen with a sulphide phosphor is given in Fig. 6.19.

Some special cathode-ray tubes are equipped with screens coated with several different phosphors. For example, colour television kinescopes have a screen coating consisting of either separate points grouped in triplets and glowing by blue, green and red colour or of alternating strips glowing by the same colours. There are also screens whose coating consists of alternating strips with orange and blue colours of luminescence. Table 6.2 presents the basic parameters of typical screens used in Soviet-made electron-beam devices.

To obtain a high-quality screen, certain conditions should be observed. The substrate (bulb face) and phosphor must be of high

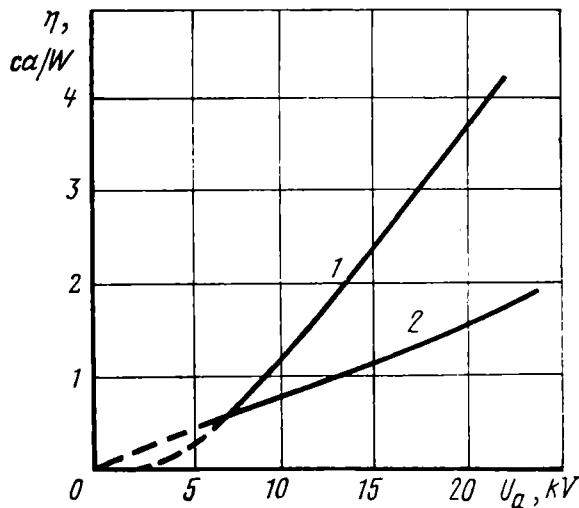


Fig. 6.19. Luminous efficiency of aluminized (1) and non-aluminized (2) screens versus accelerating voltage

purity since even a tiny amount of impurities can have a deleterious effect on the screen parameters such as luminance and colour of luminescence, and cause points and spots which are either dark or have a colour different from the colour of the screen. The materials and solvents used for applying the phosphor on the substrate must be clean.

Moreover, the phosphor must be applied as a uniform layer strongly adhering to the substrate. Finally, the engineering operations, performed after applying the phosphor (such as application of an electrically conductive coating, bulb sealing, activation of the gun cathode, etc.), must not change the screen properties.

The process of screen manufacture requires that the vacuum hygiene is strictly observed. The room where phosphors are applied must have certain constant temperature and air humidity, contain no dust and fumes having chemical activity. Before applying the

Table 6.2

Parameters of Typical Screens

Symbol	Colour of luminescence	Colour of afterglow	Persistence	Applications
A	Blue	Blue	Short	Oscillography, photography of oscillograms
B	White	White	Medium	Television
V	Blue-white	Yellow	Long	Radar
G	Dark	trace	Very long	Radar
D	Blue	Green	Long	Radar
Zh	Blue-green	Blue-green	Very short	Transmission of image by a travelling beam
I	Green	Green	Medium	Oscillography, radar
K	Pink	Orange	Long	Radar
M	Cyan	Cyan	Short	Oscillography, photography of oscillograms
S	Orange	Orange	Long	Radar
U	Green	Green	Medium	Oscillography, radar

phosphor the bulbs involved must be carefully washed with degreasing solutions and a weak solution of fluoric acid to remove inorganic contaminants. Final washing of the bulbs must be effected with demineralized water.

Of the many methods of applying the phosphors two are most common at the present time. These are the method of deposition and the method of spraying. The earlier method of dry deposition of luminophors is rarely employed because this technique cannot provide for a uniform phosphor coating.

The deposition method provides the most uniform coating. This method is used in making kinescope screens, two-layer (cascade) screens of radar tubes and screens of many special devices. Since sulphide phosphors have relatively large grains, the basic method of applying such luminophors is deposition of a phosphor material from a liquid medium. This medium must serve as a binder of phosphor particles with each other and with the substrate (bulb face). The medium binding the phosphor is usually a solution of potassium silicate. When potassium silicate is dissolved in mineralized water, besides the true solution of K_2SiO_3 , there is also formed a colloid solution of silicon dioxide (SiO_2) having the property of polymerization. The colloid particles of SiO_2 drag around the phosphor grains and reliably secure them to the substrate. Since the polymerization under usual conditions is a very slow process (a few dozens of hours would be necessary for securing the phosphor), the so-called

coagulator is added to the solution of the potassium silicate to accelerate the process. Strontium nitrate is usually used as the coagulator. In the presence of a coagulator reliable binding of phosphor is effected within 30-40 minutes.

A bulb prepared for applying a screen coating is mounted with its neck up and the solution of potassium silicated with an addition of a coagular is poured into the bulb. A suspension of phosphor is prepared separately by stirring powdery phosphor in a solution of potassium silicate. The suspension is poured into the bulb through a silk

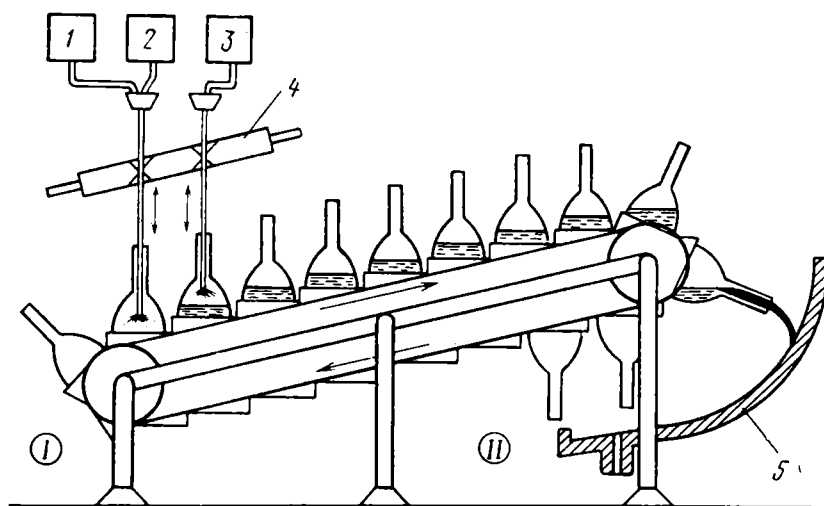


Fig. 6.20. Semiautomatic machine for applying phosphor by the precipitation method

I—bulb loading; II—bulb unloading

sieve and a special spraying funnel. After approximately 1 hour the liquid is removed from the bulb by gradually draining it off by means of a siphon. The deposition method allows high-quality screens to be produced, however, it is of low efficiency. The efficiency is increased considerably when the screens are coated by means of a conveyer-type semiautomatic machines in which the supply of solutions and suspensions into the bulb, as well as maintenance and removal of the liquid are carried out automatically. A schematic diagram of such a machine is shown in Fig. 6.20.

The bulbs are placed on a belt conveyor. Tanks 1, 2, 3 containing the required solutions are mounted above the belt. The solutions are batched into the bulbs by means of a special carriage and a batcher 4.

The belt speed is such that during the motion of the bulbs from the charging station to the discharger 5 the phosphor is reliably secured to the substrate.

The two-layer screens are also coated by the deposition method. The deposition is usually carried out with the use of potassium silicate solution preliminarily poured into the bulb. First suspension of photoluminophor is admitted into the potassium silicate solution and 15-20 minutes later suspension of cathodoluminophor is added to the same solution. The solution is removed after the cathodoluminophor has been deposited completely.

Phosphors capable of being disintegrated to a grain size of lower than 5 μm can be applied by the spraying method. This method is

widely used in manufacture of commercial oscillograph tubes with willemite screens. The spraying methods are also used for applying calcium tungstate. A schematic view of a luminophor spraying machine is shown in Fig. 6.21.

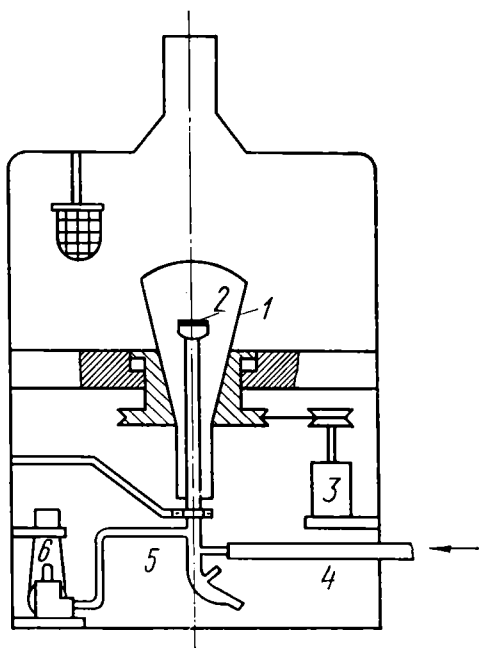


Fig. 6.21. Applying phosphor by the pulverization method

Bulb 1 with its neck down is secured in a holder rotated by a motor 3 at a speed of 50-60 rpm. Placed into the bulb is a sprayer 2 capable of oscillating in a vertical plane along the screen diameter. The sprayer is fed through pipes 4 and 5 with suspension of luminophor atomized by a stream of compressed air from a pump 6. The willemite screen suspension is prepared in alcohol and before the use is diluted with acetone. The tungstate suspension is prepared in demineralized water. The quality of a sprayed screen largely

depends on the time of drying the luminophor composition, therefore, using an alcohol-acetone suspension, the bulb is heated to 50-60°C, while in the case of a water suspension the bulb is heated to 100-120°C. The heating of the bulb provides adequately fast drying of the screen, thus assisting in production of phosphor layers which have a uniform structure and strongly adhere to the glass. The efficiency of this method is satisfactory, however, the quality of the sprayed screens is somewhat inferior of that of the deposited screens.

The screens having coatings applied by the spraying or deposition techniques, although meet the basic requirements for the screens of most of electron-beam devices, have some disadvantages. The phosphor deposited or sprayed on the substrate forms a comparatively loose layer so that the apparent thickness of the coating is rather large. Hence, the light radiated by the phosphor grains undergoes

noticeable scattering in the layer thickness with resultant apparent increase in the light spot diameter on the screen and, as a result, reduction of the resolving power of the tube and drop of the image contrast (see Sections 7.1 and 9.1). With coarse-grained phosphors the limitation of the resolving power may be associated with the grain size, since it is clear that the diameter of the luminous spot on the screen cannot be smaller than the phosphor grain glowing under the action of an electron beam.

Free of these disadvantages are the so-called sublimate screens of thin structureless films having an optical contact with the substrate. The sublimate screens can be obtained by means of sublimation and deposition of a phosphor on a substrate by spraying the phosphor on a refractory substrate (for example, quartz) with subsequent annealing. However, in spite of good uniformity, high resolving power and contrast, the sublimate screens have not yet gained general recognition mainly due to insufficient luminous efficiency of the luminophor and sophisticated production process.

Oscillograph Tubes

7.1. General

An oscillograph tube is an electron-beam device serving as an indicator in electronic oscillographs—instruments for detection and study (quantitative and qualitative) of rapid phenomena. Since a cathode-ray tube is the basic component of an oscillograph, its other units are designed in conformity with the tube. The parameters of the oscillograph tube must be matched with the parameters of the electronic circuit of the oscillograph.

The requirements placed on an oscillograph to a great extent are dictated by the requirements placed on the oscillograph tube.

A modern electronic oscillograph must provide for study of signals having a frequency of at least several dozens of megahertz without perceptible distortions. In this case the phase and frequency values of the signals under study must be measured with sufficient accuracy. These requirements determine the construction of amplifiers for the vertical and horizontal deflection of the beam (wide band and linearity) but they are also significant for the oscillograph tube. The requirement for no amplitude and phase distortions during the deflection of the beam across the entire screen determines such requirements for an oscillograph tube as linearity of both deflection systems and a high value of their input impedance. The latter requirement dictates fairly small input capacitances and inductances of the deflection systems. Moreover, study of the radio-frequency processes dictates that the transit time of the electrons passing through the deflection systems be very short.

One of the basic requirements placed on oscillograph tubes is high resolving power. This parameter determines the amount of information displayed on the tube screen; the higher its resolution the more complete and detailed data can be obtained about the process being studied with the aid of the oscillograph. The resolving power of an oscillograph tube can be estimated by a number of separately distinguishable luminous spots per unit of the screen surface area. In practice the resolving power is estimated by an amount of separately distinguishable lines per centimetre of the screen height or the

entire working surface of the screen. The simplest method of determining the resolving power is by compression of the raster. Both deflection systems are fed with saw-tooth voltage with multiple frequencies to form a raster having a known number of horizontal lines on the screen. Then the raster is gradually compressed in height by reducing the amplitude of the vertical deflecting voltage until separate lines are no more distinguishable. If the raster has n lines and these lines are no longer distinguishable at a raster height h , the resolving power is determined by the number N of lines laid through the entire working height L of the screen

$$N = \frac{Ln}{h} \quad (7.1)$$

It should be noted that the current distribution through the cross section of the beam (see Section 3.2) determining the distribution of

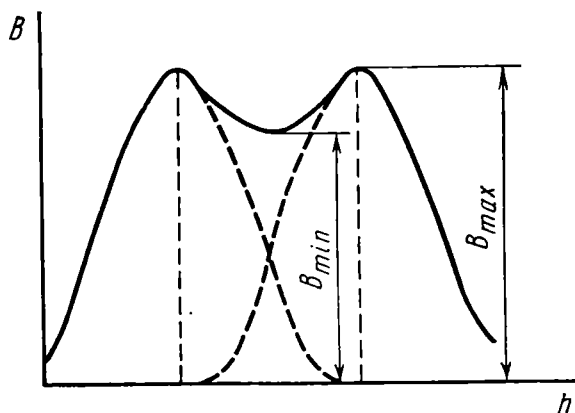


Fig. 7.1. Brilliance distribution when bringing adjacent spots (lines) close to each other

brightness over the spot plays an important role. When the raster is compressed, i.e. the adjacent spots (lines) approach one another, the distinguishability of each line is determined by the modulation brilliance factor (Fig. 7.1).

$$M = \frac{B_{\max} - B_{\min}}{B_{\max}} \times 100\% \quad (7.2)$$

where the values $B_{\max}$ and $B_{\min}$ depend on the curve of brilliance distribution (beam current) and mutual disposition of two adjacent spots (lines).

A human eye can distinguish two luminous lines at a brilliance modulation factor of at least 10-15 %. Assuming that reliable resolution may be obtained at $M \geq 20\%$, the calculation of the resolving power can be made by taking the width of the curve $B = B(h)$ at the level $0.4 B_{\max}$ as the conventional diameter of the spot.

In this case the total brilliance at the places of superposition of the peripheral zones of adjacent spots will not exceed $0.8 B_{\max}$, i.e. the value M will be equal to 20 % and the adjacent spots will be easily distinguishable.

The requirement of high resolving power limits the cross section of the beam in the screen plane. In any case a gun of a modern oscilatron must provide a diameter of the beam in the screen plane (at a level 0.4) not greater than several tenths of a millimetre. Of course, aberration of the lenses, mainly spherical aberration of the projection lens, as well as the influence of initial velocities and the Coulomb repulsive force, results in that the resolving power is substantially less than that calculated by means of the conventional diameter of the crossover and the magnification of the projection lens. Furthermore, the resolving power at the edges of the screen, particularly at high deflection angles, is much less than at the centre of the screen due to the influence of the deflection systems on the focusing (see Section 5.4.)

The resolving power depends on the beam current and the accelerating voltage. The lower the beam current and the higher the accelerating voltage, the higher is the resolution. The resolving power is affected by the screen properties. When the phosphor is coarse-grained, the resolving power is limited by the scattering of light radiated by separate phosphor particles at the grain boundaries. Formation of halos due to complete inner reflection in the glass of the bulb face can also affect the resolving power (see Section 9.1).

An oscillograph tube must provide a sufficiently high recording rate. When studying rapid processes, the speed of travel of the beam on the screen required for detailed consideration of the time variation of the voltage fed to the deflection system may prove to be very high.

On oscillographing high-frequency processes at a frequency f , the recording rate is

$$v_r = lf \quad (7.3)$$

where l is the length of the sweep line (diameter of the working portion of the screen).

It is considered that one period must be "swept" over the entire screen for detailed study of the process.

For example, at a frequency $f = 10$ MHz and a length of the sweep line of 10 cm the recording velocity is equal to 10^6 m/s = 1000 km/s. At such a velocity of movement of the beam over the screen surface the energy delivered by the beam to each element of the screen (phosphor grain) might turn to be insufficient for exciting luminescence of the screen. The screen brilliance at high recording rates can be increased by raising the accelerating voltage. In this case an increase in the beam current is inexpedient, since it is associated with defocu-

sing, greater effect of the Coulomb repulsive forces and, finally, reduction of the resolving power. The recording velocity can be increased by using more efficient phosphors having a higher luminous efficiency.

An electronic oscillograph must provide for a possibility of analyzing signals with very low amplitudes (a few millivolts), i.e. it must have very high sensitivity. Of course, a deflection system having such a high sensitivity is practically unrealizable and the oscillograph is usually equipped with an input amplifier having a high amplification factor. However, the higher the sensitivity of the deflection system, the lower should be the amplification factor of the amplifier to provide the same sensitivity of the oscillograph. Therefore, the deflection sensitivity of the oscillograph tube must be as high as practicable to ensure the required sensitivity of the oscillograph at a moderate amplification factor of the amplifier.

The second deflection system of oscillographs (X) is usually used to provide the time base of the signal being analyzed. In this case the electron beam travelling at a constant velocity along the horizontal axis of the screen from left to right quickly returns to the initial point. Although the retrace of the beam is performed within a very short period of time, the trace of the returning beam can be noticeable and can cause an error in observation or image photography on the highly efficient screen and with adequate power of the beam. To avoid this, the beam is cut off during its retrace by feeding a negative voltage pulse to the tube modulator during this stroke. This blanking pulse must provide complete cutoff of the electron gun but its amplitude should be not too high in order to avoid employing an involved blanking pulse generator. To this end, oscillograph tubes must have comparatively low cutoff voltage, not higher than some tens of volts. Finally, the oscillograph must be simple in manufacture and economic in service.

7.2. Construction of Oscillograph Tube

The general requirements given in Section 7.1 determine the main design features of oscillograph tubes. Modern tubes generally have a gun with electrostatic focusing and electrostatic deflection systems.

An electrostatically focused gun with a relatively small beam current and an accelerating voltage higher than 1 kV produces a luminous spot with a diameter of 0.2-0.3 mm (at a level 0.4) on the screen, which usually provides the required resolution. At the same time, an electrostatic gun is very economic in service. Furthermore, it preserves focusing of the beam on the screen in case of the supply voltage variations (when all the electrodes of the gun are connected to a single source through a voltage divider).

Electrostatic deflection also has a number of advantages. It practically has no persistence up to very high frequencies. Electrostatic deflection systems are featured by small values of capacitances and inductances which is very important for study of high-frequency signals. Moreover, electrostatic deflection systems practically do not consume power, i.e. they do not load the circuit being studied and can be connected to low-power signal sources without disturbing their normal operation.

The screens of oscillograph tubes are usually of a circular shape and comparatively small in diameter (3, 5, 7, 10, 13 cm). Only some special tubes have screens of 18, 23 and 31 cm in dia and are of a rectangular or square shape.

In order to provide good focusing within the entire screen, the deflection angles of the electron beam are selected to be small ($12-18^\circ$). In this connection, oscillograph tubes are relatively long (up to 30-40 cm).

To provide high recording rates, (high brilliance of the screen) without considerable reduction of the sensitivity, many tubes have an after-accelerating electrode for acceleration of the electrons after the deflection. To obtain high deflection sensitivity and convenient operation, oscillograph tubes are usually designed for relatively low accelerating voltages (1.5 to 3 kV). Therefore, to provide convenient insertion of the tube into an electric circuit, all its electrodes are fed through a common base having 12-14 pins. The terminal of the additional accelerating electrode is located on the side of the bulb.

The above considerations are not necessary for all types of oscillograph tubes. In some cases oscillographs may be equipped with magnetically focused tubes or have an accelerating voltage up to 25 kV. They may have the terminals of deflecting plates on the side of the bulb, etc.

7.3. Envelope (Bulb) of Oscillograph Tube

The construction of an oscillograph tube envelope is determined by its overall dimensions. The envelope must have sufficient strength, i.e. withstand the atmospheric pressure. Moreover, the shape of the envelope must be such as to prevent reflection of the light radiated by the phosphor into the envelope and back to the screen. The "back" illumination of the screen by reflected light produces a general light background and hinders with observation or photography of the oscillograms, particularly at high recording rates. Consequently modern oscillograph cathode-ray tubes have their wide portion similar to a truncated paraboloid (Fig. 7.2).

With such a shape of the envelope the light beams incident to the walls of the bulb are reflected inside and absorbed by the inner black

coating. Moreover, a paraboloid bulb has good mechanical strength due to its convex external surface.

The wide part of the envelope serves as a substrate for the screen phosphor. To provide for the deflection linearity, the screen is preferably made flat (see Section 5.4). At the same time, to provide good

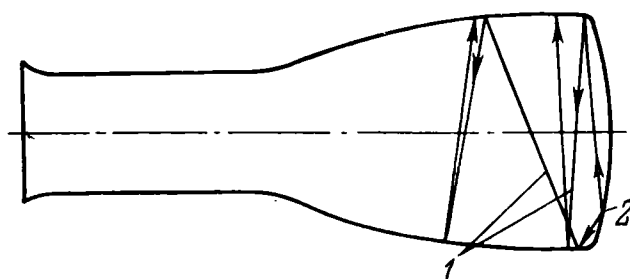


Fig. 7.2. Envelope of oscillograph tube
1—reflected light rays; 2—spot

focusing during the deflection of the beam to any point of the screen, the surface of the latter is preferably given a spherical shape with a centre of curvature coinciding with the centre of deflection of the

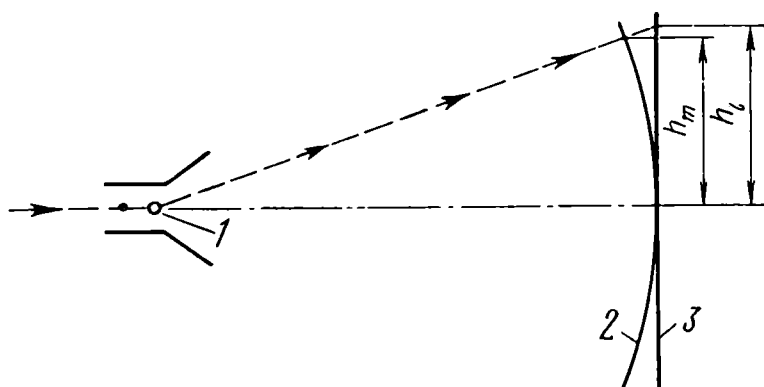


Fig. 7.3. Influence of screen shape on beam deflection
1 — deflection centre; 2—spherical screen; 3—flat screen

beam. For a spherical screen (Fig. 7.3) the value of deflection of the beam measured by the displacement of the spot on the screen will be smaller than that for a linear deflection system. The error Δh is determined as a difference between the linear deflection and the deflection measured on the screen and related to the linear deflection

$$\Delta h = \frac{h_l - h_{meas}}{h_l} = 1 - \cos \alpha \quad (7.4)$$

where α is the beam deflection angle.

With $\alpha = 15^\circ$ the value Δh is equal to 3.5%, i.e. the linearity is significantly disturbed. Consequently, a screen of commercial oscil-

lograph tubes is slightly convex with a radius of curvature 1.5-2 times the distance from the screen to the centre of deflection of the beam. With such a compromise solution the linearity error is comparatively small and the focusing during the deflection of the beam to the screen edge changes, but little. In order to provide high linearity, some tubes are equipped with a flat screen manufactured separately from the envelope and welded to the wide part of it made by blowing in a mould. The convex screens are usually made integral with the envelope wide portion. The thickness of the envelope walls and convex screens is equal to 2-2.5 mm; the thickness of the flat welded faces is somewhat higher to provide greater strength, since the atmospheric pressure on the screen may be as high as a few hundreds of kilograms.

The wide portion of the bulb transforms into a narrow cylindrical portion, a neck. The diameter and length of the neck are determined by the external dimensions of the gun and deflection system. For oscillograph tubes with electrostatic focusing and deflection the neck diameter is selected in the range of 50 to 55 mm and the length of 120 to 150 mm for convenient mounting of the gun and deflection system.

The walls of the envelope are covered from the inside with an electrically conductive coating, a layer of finely dispersed graphite to trap the secondary electrons from the screen, the beam electrons (at too high deflection angles) and scattered electrons which can in a small amount be radiated by the gun. Thus the electrically conductive coating prevents accumulation of electric charge on the glass, which otherwise could considerably distort the electrostatic field in the tube and cause defocusing and deflection errors. The conductive coating also protects the electron beam from external electrostatic fields which could induce charges on the walls of the tube. Finally the black graphite coating absorbs the light radiated by the screen into the bulb.

7.4. Electron Gun

Most of oscillograph tubes employ an electron gun with electrostatic focusing built around a two-lens system. The first lens (immersion objective) is formed by the cathode, modulator and the lower (nearest to the cathode) edge of the accelerating electrode. The first lens forms a crossover shown on the screen by means of the second lens of the gun. The second (single) lens is formed by the upper edge of the accelerating electrode, the first (focusing) anode and the second anode.

In order to obtain adequate brilliance of the screen at moderate accelerating voltages, the tube gun is designed for a beam current of up to 100 μA . In this case it is assumed that with electrostatic fo-

causing a part of the electron beam (up to 80 %) is cut off by the diaphragms of the accelerating electrode and (to a lower extent) of the second anode.

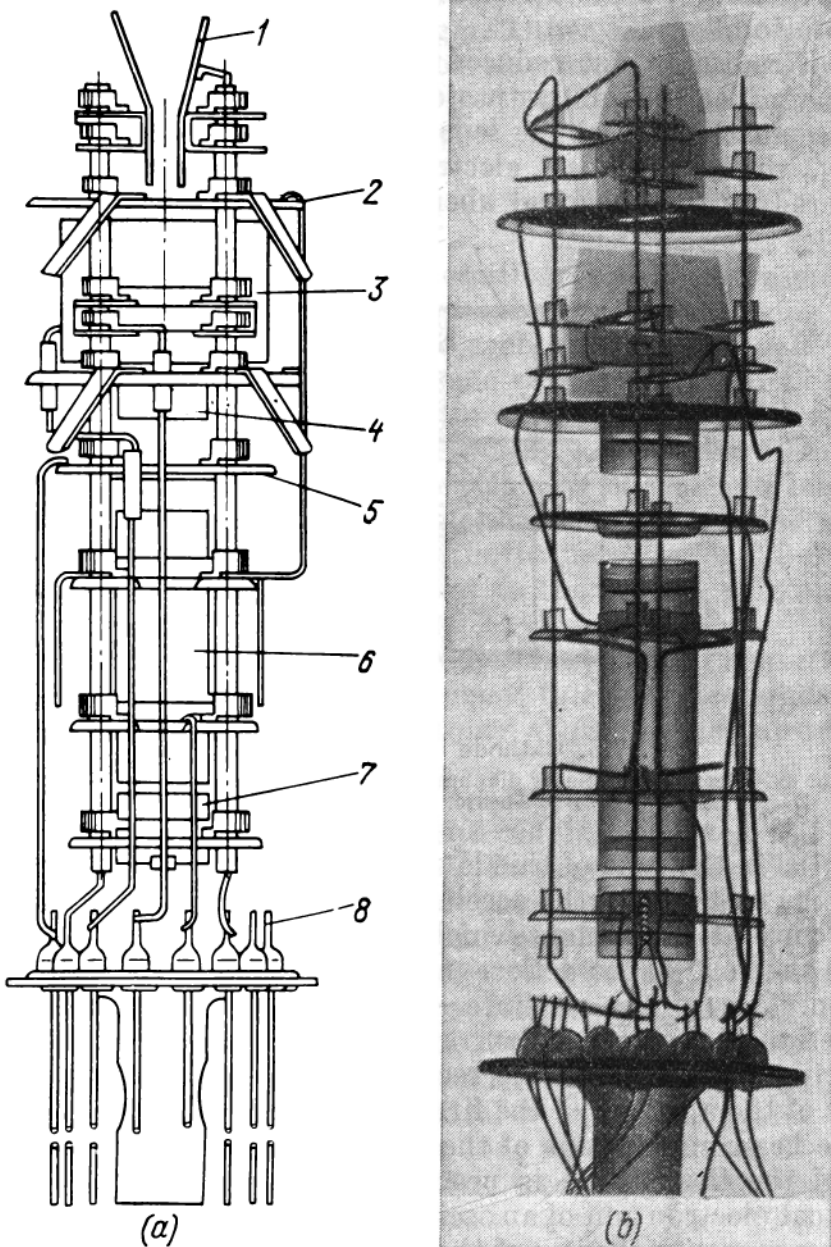


Fig. 7.4. Gun and deflection system of oscillograph tube

(a) schematic view; (b) X-ray photograph; 1—upper plates; 2—screen; 3—lower plates; 4—second anode; 5—first anode; 6—accelerating electrode; 7—modulator; 8—terminals

cond anode. The gun cathode must provide an emission current not less than a few milliamperes in order that the necessary beam current be obtained without using the total working surface of the cathode. The emission only from the central part of the cathode causes

a decrease in the beam diameter in the immersion objective and, due to reduction of the aberrations, makes it possible to obtain a cross-over and, therefore, a lower-radius spot on the screen thus providing high resolution.

The spot radius is also reduced due to a decrease in the value of optical magnification of the projection lens obtained by removing the centre-plane of the single lens from the object (crossover). Consequently, the accelerating electrode is usually made sufficiently long. To reduce the spherical aberration of the projection lens, the

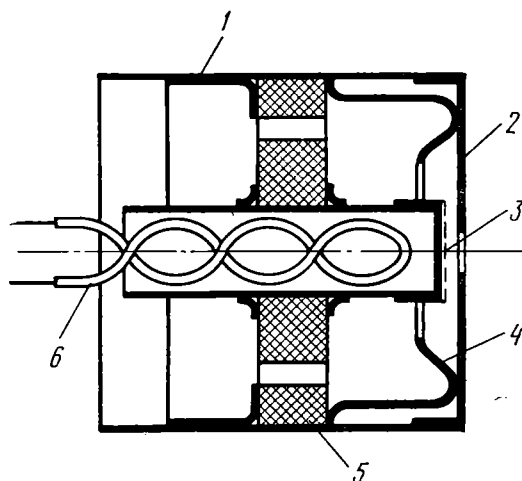


Fig. 7.5. Cathode unit of electron gun

1—modulator cylinder; 2—modulator diaphragm; 3—cathode; 4—spacer; 5—ceramic plate;
6—heater

beam radius in the zone of this lens is limited by a diaphragm mounted near the exit side of the accelerating electrode. The second anode also has an exit diaphragm which to some extent limits the beam radius in the zone of the deflecting plates and this reduces deflection distortion. Due to the presence of limiting diaphragms a considerable portion of the cathode current is branched to the circuit of the accelerating electrode and the second anode. At the same time, the diameter of the aperture of the first anode is several times the diameter of the beam in the zone of the second lens and the current in the circuit of the first anode is practically equal to zero.

A typical electron gun of an oscillograph tube is shown in Fig. 7.4, while a cross-sectional view of the cathode unit is shown in Fig. 7.5.

Since the accelerating electrode is at a high positive potential (equal to that of the second anode) with respect to the gun cathode, a small value of the cutoff voltage is obtained by decreasing the diameter of the modulator aperture and the distance between the modulator and cathode. The cathode-modulator distance in many types of oscillograph tubes is only of 0.12 to 0.16 mm. Since this distance has a substantial effect on the cutoff voltage of the tube,

it must be ensured with adequate accuracy. This distance is adjusted by means of a special-shape spacer resting with its lower edge on the ceramic plate, in which the cathode is pressed, and with its upper edge, on the modulator diaphragm. Other methods of setting the cathode-modulator distance can be also used, for example, by illuminating the modulator aperture with two oblique light rays and observing the edges of the light spots on the cathode surface.

7.5. Deflection Systems

As has been mentioned above, oscillograph tubes are mostly provided with electrostatic deflection systems. The beam is deflected in two directions at right angles to each other by two pair of plates which are usually curved. The geometric sizes of the deflecting plates are selected to obtain deflection sensitivity sufficient for the given type of the tube. In order to obtain a low value of input capacitance and inductance, the plates should be not too large. With practically employed deflecting plates the sensitivity of the lower (nearest to the gun) pair of plates is in the order of 0.2-0.6 mm/V, the sensitivity of the upper (nearest to the screen) pair of plates is smaller by 10-15 %.

The input capacitance of the deflecting plates (measured at the terminals of the common base) is equal to a few picofarads and this makes it possible to employ the tubes at frequencies up to 10 MHz without noticeable phase errors.

To reduce mutual influence of the fields of both pairs of deflecting plates, they are provided with shields mounted therebetween and connected to the second anode. The same shields are preferably mounted between the second anode and the first pair of plates, as well as after the second (upper) pair of plates.

Since it is very important to eliminate deflection errors, mainly trapezoidal distortion, during the precise quantitative investigations by means of an oscillograph, the cathode-ray tube must provide a possibility of symmetric connection of the deflecting plates and controlling the average potential between the plates with respect to the second anode. To this end, all four plates of the deflection system must have independent terminals on the base (or on the side of the bulb).

7.6. Screens of Oscillograph Tubes

The image on the screen of an oscillograph tube is either observed directly or is photographed. Therefore, the colour of luminescence is selected green (or yellow-green) for direct observation or blue (light-blue) for photography. The luminous efficiency (screen brilliance) must be sufficiently high to provide a clear trace of the electron

beam at high recording rates. When analyzing periodic processes, the persistence time is of no practical importance and may be in the range of 0.01-0.1 s. When single processes are analyzed, high-persistent screens with afterglow time of up to several dozens of seconds may be necessary.

Screens of oscillograph tubes must be fine-grained to provide a required resolving power. When oscillograms are photographed from small-diameter tubes (5-8 cm), the picture may require photographic enlargement for more accurate measurements. In this case the grain structure of the screen must not be noticeable.

Until recently these requirements were met by silicates (willemite) and tungstanates (silicate—for direct observation, tungstanate—for photography) having the necessary spectral composition. These materials can be finely dispersed to obtain high resolution of the screen and can be sprayed on a substrate.

However, the luminous efficiency of silicates and tungstanates is insufficient for required high brilliance of the screen at high velocities of sweeping the beam. Highly effective sulphides have recently been developed. These can be disintegrated without marked reduction of their light output. Such sulphides with blue and green luminescence gradually render silicates and tungstanates obsolete. Yet silicates are still widely used in manufacture of oscillograph tubes due to their high physical and chemical resistance.

Spraying is the most efficient method of applying phosphors to screens of oscillograph tubes. Since high uniformity of coating for these tubes is not necessary, the screens coated by the method of spraying have adequate light and mechanical properties. High-persistence oscillograph tubes employ two-layer (cascade) screens similar to those in radar tubes (see Section 8.2).

7.7. Types of Oscillograph Tubes

Oscillograph tubes with after-acceleration (additional acceleration) of electrons, tubes for studying processes in the terms of polar coordinates and double-beam tubes have gained general recognition.

The oscillograph tubes with after-acceleration of the beam electrons after the deflection have been developed in connection with the necessity to increase the resolving power and screen brilliance to provide for high recording rates without perceptible reduction of the deflection sensitivity.

After-acceleration of the deflected beam materially increases the screen brilliance, since it rises up approximately in proportion to the square of the accelerating voltage for most of phosphors. Such additional acceleration can be obtained very simply by mounting

before the screen a circular electrode at a higher potential than that of the last electrode of the gun.

The after-acceleration electrode is made in the form of an electrically conductive coating separated from the tube neck coating. The additional circular layer has a separate terminal which is fed at a voltage 2-2.5 times the voltage applied to the second anode. The after-acceleration electrode is usually referred to as a third anode. Should the electron beam intersect the equipotential surfaces at a right angle in the after-acceleration zone, the deflection sensitivity is kept constant. Under real conditions, however, the field between

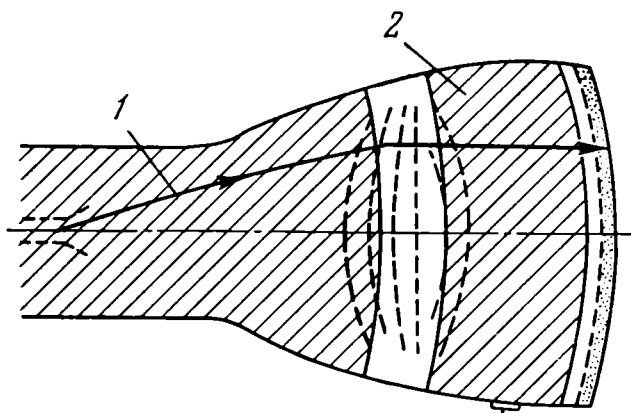


Fig. 7.6. Influence of third anode field on electron beam
1—electron beam; 2—third anode

the conductive coating connected to the second anode and the third anode acts as an immersion lens so that the electron beam is somewhat displaced to the tube axis (Fig. 7.6).

Thus the after-acceleration lens reduces the tube sensitivity. The greater the curvature of the equipotential surfaces in the additional acceleration zone, the lower the sensitivity. Moreover, when the beam intersects strongly curved equipotential surfaces there may appear distortions of focusing and disturbance to the deflection linearity.

These adverse effects can be compensated for by reducing the optical power of the lens and increasing its diameter. Hence, the after-acceleration electrode (separate portion of the conductive coating) is arranged near the screen, in the widest part of the tube. However, the width of the after-acceleration layer (extension along the tube axis) must be sufficiently large, otherwise the potential near the centre of the screen may be lower than the potential of the third anode, i.e. the effect of after-acceleration will not be utilized completely. Furthermore, this is associated with defocusing of the beam.

When using after-acceleration in tubes with envelopes of an ordinary shape, satisfactory results (relatively low drop of the tube sensitivity, as well as the absence of heavy distortion and disturbance

to the deflection linearity) can be obtained only under condition that the third anode potential exceeds the potential of the second anode of the gun by a factor not higher than 2 to 2.5. It should be noted that the after-acceleration makes it possible to improve somewhat the focusing, i.e. to obtain a spot of a smaller diameter. This is accounted for by the fact that the scatter of initial velocities of the electrons and the Coulomb repulsive forces have less influence at high accelerating voltages. Therefore, in spite of some drop of the real sensitivity, the relative sensitivity (i.e. the ratio of the sensitivity to the spot diameter) is practically the same. When necessary, the screen image can be enlarged by an optical (or photographic) technique without reducing the sharpness.

Sometimes, in order to obtain very high luminance or high recording rate, the after-acceleration voltage must be brought up to

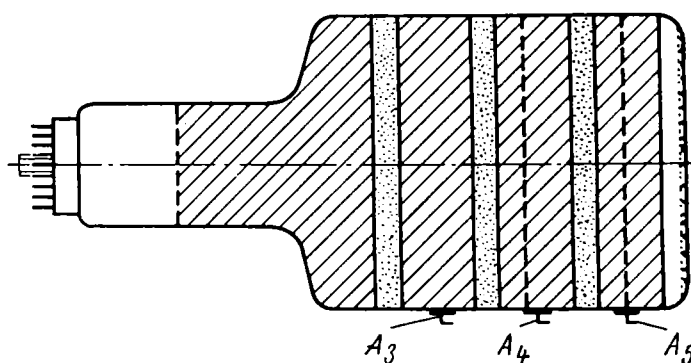


Fig. 7.7. Tube with multiple after-acceleration of beam electrons

20-25 kV. In such cases the use of a single after-acceleration electrode would result in forming a very strong lens in the after-acceleration zone with considerable decrease in the sensitivity and disturbance to the focusing and deflection linearity. Consequently, the after-acceleration zone is subdivided into several sections (3-5) with gradually increasing potentials. Instead of a single lens there are formed a number of comparatively weak lenses of less effect on the sensitivity and focusing. In order that the after-acceleration lenses have a greater diameter, the envelope of a tube with multiple after-acceleration is given a cylindrical shape (Fig. 7.7).

The multiple acceleration tubes with the last electrode at 25 kV make it possible to obtain a well distinguishable image at the spot movement velocity up to 50,000 km/s. When using high after-acceleration voltages, it is necessary that the maximum potential of the phosphor on the screen be not lower than the voltage applied to the last accelerating electrode.

The potential difference between the separate rings of the electrically conductive coating serving as the after-acceleration electrodes

can be of a few kilovolts, and an electric breakdown may occur between the adjacent rings. To eliminate this, the intervals between the rings of the conductive coating are filled with a semiconductor material of high resistivity. Such a semiconductor coating levels the potential distribution along the tube wall. The material of this coating usually consists of chromium oxide or iron oxide applied to the glass in the intervals between the rings of the graphite coating.

But even if an oscillograph tube is provided with several stages of additional acceleration, i.e. several rings of conductive coating at a gradually increasing potential, the immersion lenses formed in

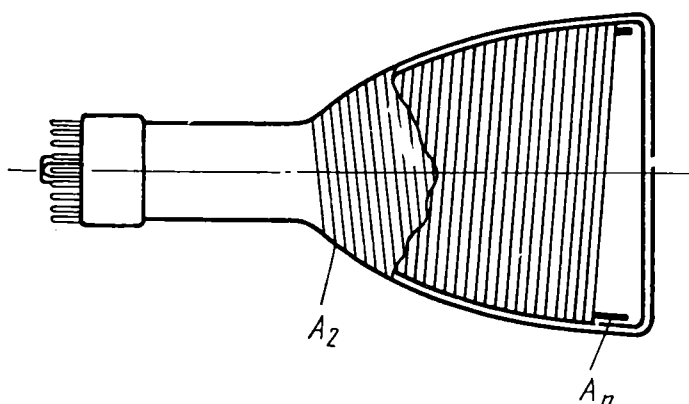


Fig. 7.8. Tube with spiral after-acceleration system

the intervals between the rings considerably deflect the beam to the tube axis. Consequently, we always have some drop of the deflection sensitivity. Since the optical power of electron lenses is determined by the second derivative of the potential along the axis (see Section 1.8), the after-acceleration without additional focusing is possible only when $U_0''(z) = 0$. This condition leads to a necessity of producing such a potential distribution that $U_0'(z) = \text{const}$ in the after-acceleration zone. This requirement can approximately be satisfied if the internal surface of the envelope between the deflection system and the screen is provided with a narrow strip of conductive coating in the form of a spiral with a pitch of a few millimetres. One end of the spiral (the closest to the gun) is connected to the second anode of the gun, while the other end (near the screen) has a terminal fed with a positive voltage several times the value of U_{a2} . In this case the potential in the after-acceleration zone rises practically linearly. Therefore, $U_0'(z) \approx \text{const}$ and the optical power of the immersion lenses is close to zero.

A tube with a spiral after-acceleration system is shown in Fig. 7.8.

To provide for a small current flow in the after-acceleration spiral, the spiral strip is made of a high-resistivity semiconductor mate-

rial. In this case the designer proceeds from the fact that the spiral resistance must be not less than 100 megohms. The tubes with a spiral system of after-acceleration make it possible to obtain an undistorted oscillogram with a ratio $U_{aa}/U_{a2} \leq 10$. In this case the gain in luminance and, therefore, in attainable recording rate, is significant while the loss in the sensitivity is low, at any rate, much lower than when using separate rings for after-acceleration (Fig. 7.9).

The disadvantage of the tubes with after-acceleration consists in excitation of parasitic glowing of the screen due to the secondary electrons knocked out from the deflecting plates (at great deflection angles). The secondary electrons accelerated by the field of the after-acceleration anodes reach the screen with energies sufficient for

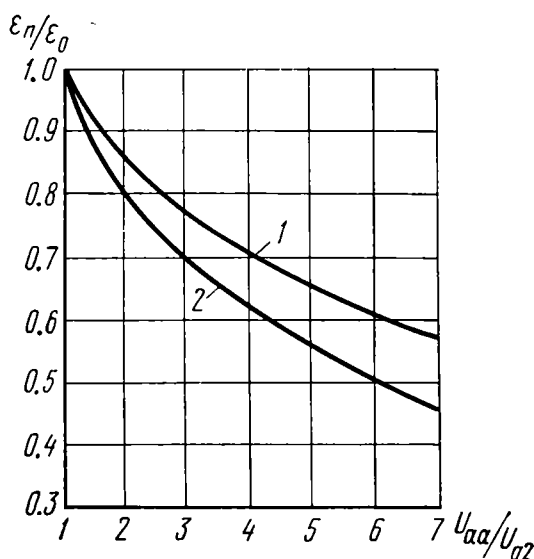


Fig. 7.9. Decrease in sensitivity due to after-acceleration

1—tube with spiral system; 2—tube with three rings of Aquadag coating

exciting luminescence of the screen. As a result the tubes with after-acceleration used for analyzing signals with a high amplitude sometimes have rather a bright background hindering the observation of the oscillograms. Hence, when using tubes with after-acceleration, the beam should not be allowed to fall on the deflecting plates or the after-acceleration anodes, since it inevitably causes parasitic illumination of the screen at the expense of the secondary electrons.

Attempts of increasing the deflection sensitivity have resulted in development of oscillograph tubes with diverging lenses placed between the

deflection systems and the screen. In fact, a diverging electron lens arranged behind the deflection system will cause additional deflection of the beam from the tube axis, i.e. the displacement of the spot on the screen will be greater at the same value of the deflecting voltage and this is equal to an increase in the sensitivity. The simplest method of obtaining the effect of additional deflection of the beam from the tube axis near the screen is to cut off the converging region of the immersion lens formed between the rings of the conductive coating in the zone of after-acceleration of the beam. Any immersion lens has a converging and diverging regions (see Section 1.8), the converging region being always near the electrode at a lower potential.

In the tubes with after-acceleration the converging region of the immersion lens is formed to the left from the centre-plane of the lens (see Fig. 7.6), i.e. near the edge of the conductive coating connected to the second anode of the gun; the diverging region lies to the right from the centre-plane—nearer to the screen. If a flat (or slightly convex towards the screen) fine-structure grid at the potential of the second anode is placed near the edge of the conductive coating connected to the second anode, it is clear that the converging region of the lens will disappear, since the space between the second anode and the grid will be equipotential. At the same time, the action of the diverging region will be preserved completely, since the equipotentials between the grid and the screen are curved to the screen side. Thus we shall obtain additional deflection of the beam from the centre of the screen, i.e. a gain in sensitivity. If the diverging lens has low aberrations, the deflection linearity is not disturbed.

However, this apparently simple and effective method of increasing the sensitivity is practically useless. An increase in the sensitivity is useful only when the resolving power remains unchanged.

The diverging lens causing additional deflection of the beam, i.e. increasing the deflection sensitivity, at the same time changes the slope of the paths of the beam electrons thus increasing the spot diameter. In this case the specific sensitivity (see Section 5.1) determining the scope of information reproduced on the tube screen, according to V. Miller, does not increase. On the contrary, it even drops down to some extent. Moreover, the presence of a grid reduces the contrast of the image due to illumination of the screen by secondary electrons knocked out from the grid and the luminance of the oscillogram is reduced due to the fact that some of the electrons are intercepted by the grid wires.

In some cases it is necessary to analyze a phenomena in the terms of polar coordinates. For this purpose a circular sweep of the beam is employed. In order that the spot on the screen describe a circle, required are two pairs of deflecting plates fed with sinusoidal voltages shifted in phase through 90° . The deflection of the beam in the radial direction can be effected by varying the voltage simultaneously on both pairs of plates. This method, however, has not gained general recognition due to the difficulties associated with the supply of the deflecting voltage and calculation of the magnitude of the deflecting signal by the oscillogram being observed. More frequently the investigations in the terms of polar coordinates are carried out with the aid of tubes which, besides an ordinary deflection system for shifting the beam in two directions at right angles to each other, have a deflection unit for displacing the beam in the radial direction. The circular motion of the beam is effected by two pairs of deflecting

plates, while the signal in question is fed to the radial deflection system regardless of the sinusoidal sweep voltages.

Two types of radial deflection systems have found practical applications. These are a system of two truncated cones and a system with a central rod (Fig. 7.10).

In the former case two truncated cones (R) with different radii and with the generating lines at different angles to the tube axis are arranged on this axis behind the ordinary deflection systems (X , Y). The plates X , Y shift the beam along the conical surface between the

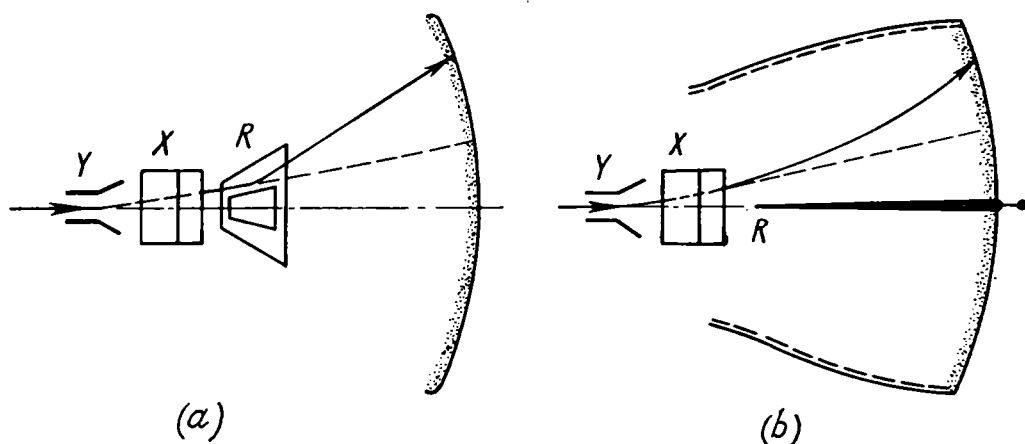


Fig. 7.10. Radial deflection systems
(a) shaped as two truncated cones; (b) with central rod

radial deflection cones, while the signal to be analyzed is fed to the cones deflecting the beam in the radial direction. In the latter case the field deflecting the beam in the radial direction is created between the electrically conductive coating of the wide portion of the envelope and a rod arranged along the tube axis. The rod is brazed in the centre of the screen and a signal to be analyzed is fed thereto. The circular sweep, as in the first case, is performed by the plates X , Y .

Both system are disadvantageous in that the field deflecting the beam in the radial direction is nonuniform, i.e. the system is nonlinear. Moreover, the system with a central rod has very low sensitivity due to a great distance between the rod and the electrically conductive coating. Consequently, the tubes with a conical system of radial deflection have gained a wider acceptance.

Two simultaneously occurring processes are conveniently studied by means of a double-trace oscillograph on the screen of which two oscillograms can be observed at a time. These oscillographs are equipped with special double-beam tubes. Such a tube usually has two independent electron-optical systems comprising an electron gun and deflecting plates. Both guns are mounted so that their axes intersect in the centre of the screen. To eliminate mutual influence

of the two systems, a metal shield is mounted between them. By feeding a d-c voltage to the deflecting plates, the pictures on the screen produced by both of the beams can be brought in coincidence or, on the contrary, be spaced one from another.

The double-beam tubes are presently designed so that the electrodes of the optical systems have separate terminals and the voltages on the modulators and focusing anodes of both guns can be controlled independently. In the same manner all four pair of deflecting plates have separate terminals. Thus a double-beam tube includes essentially two independent systems enclosed into a common envelope and having a single screen.

Previously employed double-beam tubes with a single gun and a special plate for splitting the beam have come out of use mainly because they did not allow for separate control of each beam and also because the controls of both beams are mutually dependent.

Simultaneous investigation of several processes is accomplished by means of multi-beam tubes whose envelope contains four, five or six independent guns and deflection systems.

It has been mentioned above that ordinary oscillograph tubes make it possible to study periodic processes at frequencies up to a few megahertz. Widening of the frequency range is obtained by reducing the electron transit time in the deflection systems due to an increase in the accelerating voltage and a decrease in the length of the deflecting plates. Furthermore, in order to reduce the input capacitances and inductances, the terminals of the deflecting plates are made not in a common base but directly in the tube neck in the form of short straight lengths of wire welded into the glass of the neck opposite to every deflecting plate.

The increase of the accelerating voltage and decrease of the length of the plate result in a drop of the deflection sensitivity [see (5.5)]. However, if the diameter of the spot on the screen is reduced simultaneously (the resolving power is increased), the specific sensitivity determining the amount of useful information will remain the same. In this case we can obtain on the screen a microoscillogram having a high sharpness of the image (due to the small diameter of the spot). This microoscillogram can then be enlarged by optical devices. Under conditions of good focusing and high resolution of the screen it is possible to obtain 100-fold magnification. The enlargement of an oscillogram by optical devices is equivalent to an increase of the sensitivity of the oscillograph. Thus it is possible to obtain a sensitivity sufficient for noticeable shift of the spot (after the enlargement) even at a small amplitude of the deflecting voltage.

The disadvantage of the microoscillograph is low brilliance of the screen due to a low beam current and a high recording rate. Microoscillograph tubes usually have electrostatic focusing, since a well designed tetrode gun provides for a spot of a required diameter,

while the tube is in this case more economic. The deflection system must generally be electrostatic.

The deflecting plates have small dimensions. Their length is equal to 3-5 mm, the width is the same or somewhat larger. The deflection system is enclosed into a shield to prevent mutual influence of the fields of both pairs of plates. The terminals of the plates are made by short lengths of coaxial conductors directly through the sides of the envelope neck. The capacitance of the plates can be less than one picofarad.

A microoscillograph tube has a screen of a small diameter (7 to 10 cm), since the beam deflection usually does not exceed a few

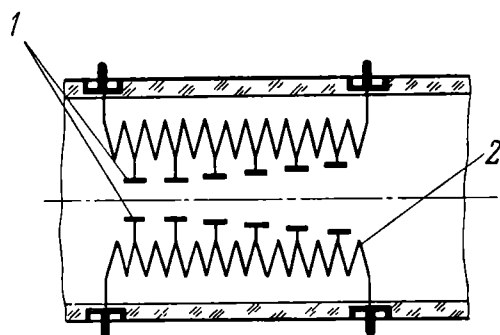


Fig. 7.11. Travelling-wave deflection system

1—deflecting plates; 2—spiral

millimetres and only the medium part of the screen is employed during the operation. To reduce distortion, the screen is made of a flat glass and then is welded to the conical portion of the envelope. The terminal of the last accelerating electrode (high-voltage electrode) is run through the internal conductive coating to the side of the conical portion of the bulb.

Since in this case a high resolving power is required, the phosphor applied to the screen must be fine-grained (the grain size of 2-3 μm) and have high luminous efficiency to provide required luminance at high recording speeds.

The microoscillograph tubes are employed for photography of processes at a frequency of 5,000 to 6,000 MHz. If some decrease in the sensitivity and some distortion of the shape of the curve are tolerable, the frequency range can be raised up to 8,000-10,000 MHz. In this case the recording rate is equal to 8,000-10,000 km/s. The luminance is sufficient for visual observation. The trace of the electron beam can be photographed and enlarged by using a highly sensitive photographic film and high-speed optical lenses.

Microwaves processes are presently oscillographed by means of tubes having a travelling-wave deflection system (see Section 5.5). An example of practical making of such a system is shown in Fig. 7.11.

As is seen from the drawing, the travelling-wave deflection system has a number of short plates mounted on turns of a spiral decelerating system, the distance between the plates being gradually increased with deflection of the beam. The deflection sensitivity of a travelling-wave system may be as high as a few mm/V. The brilliance of the screen is increased by additional acceleration of the beam electrons, for example, by means of a spiral strip on the internal surface of the wide portion of the envelope. The travelling-wave tubes make it possible to study processes at frequencies of a few thousands of megahertz.

Oscillograph tubes with a self-scanning deflection system are not yet in common use.

Cathode-ray tubes employed in indicator radio location systems are sometimes called radar indicators, however, an indicator of a radar station usually includes a cathode-ray tube, its supply sources, sweep generators and other auxiliary units.

Chapter Eight

Radar Tubes

8.1. General

Cathode-ray tubes employed in indicator radio location systems are sometimes called radar indicators, however, an indicator of a radar station usually includes a cathode-ray tube, its supply sources, sweep generators and other auxiliary units.

Cathode-ray tubes have found wide application in radar equipment. None of the other measuring instruments can provide a required speed of response to an incoming signal at adequately high accuracy of measurements. It is very valuable property of cathode-ray tubes that they make it possible to display on their screen not only the magnitude but also the shape of the incoming signal which in many cases allows the operator to judge of the size or type of the object detected.

A modern radar station (ground-type, ship or airborne) makes it possible generally to determine three spherical coordinates of the object being located at a required accuracy. These are the range, azimuth and elevation. However, a screen of a radar indicator makes it possible to observe simultaneously (usually in a conventional scale) only two coordinates—range and azimuth or range and elevation. Therefore, usually at least two cathode-ray tubes are required for determining the three coordinates. One of the coordinates of the object can be determined by using either the movement of the beam on the screen (the signal controls the position of the spot on the screen) or the measurement of the brightness of the spot moving along the screen at a certain speed in a horizontal, vertical or radial direction (the signal controls the spot brightness). In the former case the signal to be analyzed is fed to the deflection system, in the latter case it is applied to the tube modulator, i.e. in the former case we have position modulation and in the latter one, beam intensity modulation.

The indicators with position modulation make it possible to estimate only one coordinate (usually the range). Since the time base scanning must provide for detecting an object over the whole range of the given radar, the scanning frequency must be in the order of 10^4 to 10^2 Hz. At such a scanning frequency satisfactory results can

be obtained by using an indicator built around an ordinary oscillograph tube with good focusing and low persistence. Hence, typical oscillograph tubes can be employed as unidimensional indicators with position modulation.

Cathode-ray tubes employed as two-dimensional indicators in radar stations usually operate with beam intensity modulation. These tubes have some special features and are often used in radar equipment.

Plan-position indicators are widely used in radar stations. Such indicators are characterized by circular scan, the arc described by the spot on the screen determining the azimuth (in natural or transformed, for sector scanning, scale) and the distance between the spot and the centre indicating the range. In this case the circular scan frequency is low, since it corresponds to the frequency of rotation of the radar aerial. Hence, high-persistence screens are required for obtaining a clear image.

Thus, the basic requirement for radar tubes is good focusing providing for high resolution over the entire screen with high luminance which is necessary for observation of the marks on the screen under conditions of external illumination.

Since plan-position indicators are often used for display of a radar map on a screen, the indicator tube must have a high contrast. Screens of radar tubes must as a rule be sufficiently large so that the marks on the screen can be measured with a high accuracy. High linearity of deflection is an important requirement, since the magnitude of displacement of the spot on the screen must accurately reproduce the target coordinates. In some cases radar tubes must meet additional requirements determined by the specific conditions of application of these tubes in radar systems.

8.2. Construction of Radar Tubes

In accordance with some general requirements the units of radar tubes have specific characteristics. Provision of high resolution and high brightness of the screen requires high accelerating voltages (5 to 10 kV and higher). Under these conditions it is expedient to use magnetic focusing of the electron beam. High linearity of deflection, low defocusing by the deflection system in combination with large deflection angles (large screens) dictate the use of magnetic deflection of the beam in most of the radar tubes.

Magnetic deflection and magnetic focusing allow the neck of the bulbs to be sufficiently narrow, 30-32mm in dia. Screens of radar tubes have diameters of 23, 31, 45 cm and more. The bulb face is made separately from the bulb of high-quality uniform glass and then is welded to the wide portion of the bulb. Some tubes have a flat face, but more often it is slightly convex. Since the magnetic

systems allow the beam to be deflected for a large angle without noticeable defocusing and nonlinearity, the bulbs may have a moderate length even at a large diameter of the screen. A typical shape of a bulb of a radar tube is shown in Fig. 8.1.

High anode voltages does not allow the anode terminal to be made in the common base. Therefore, the accelerating voltage is fed to the gun through an internal electrically conductive coating and a terminal at the side of the wide portion of the bulb.

As has been mentioned above, the gun of a radar tube usually has a magnetic projection lens. Consequently, the gun either employs a triode scheme cathode—modulator—anode (immersion objective) + magnetic lens or a tetrode scheme: cathode—modulator—accelerating electrode (immersion objective) + accelerating electrode—anode + magnetic lens. In the tetrode gun between the modulator and the high-voltage anode mounted is an accelerating electrode at a

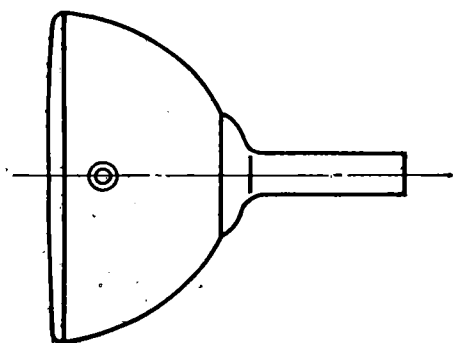


Fig. 8.1. Radar tube bulb

potential of a few hundreds of volts, i.e. much less than the anode voltage. A tetrode gun is shown in Fig. 8.2.

Such a gun has certain advantages. The modulation characteristic at a low voltage of the accelerating electrode may be sufficiently short, while the cutoff voltage may be less than 50 V at adequate permeability of the modulator. The short modulation characteristic, i.e. low percentage modulation is particularly important for tubes

with a brightness mark, since in this case a required value of the signal fed to the modulator (triggering pulse) is determined by the modulation factor. Moreover, in the tetrode gun the cutoff voltage can be controlled to some extent by varying the voltage of the accelerating electrode. Finally, the presence of the second electrostatic lens makes it possible to reduce the beam divergence angle after the crossover, i.e. to confine the electron beam in the zone of the projection (magnetic) lens. In this case the aberrations of the magnetic lens are reduced and, therefore, the focusing is improved. In the tetrode gun the required dielectric strength can be provided more easily, as between the cathode unit and the high-voltage anode there is an intermediate electrode.

A beam confining diaphragm is usually not used in the tetrode gun. The diaphragm is provided only at the lower end of the accelerating electrode. This diaphragm, however, does not cut off the outer electrons of the beam since its diameter is usually 2-2.5 times the diameter of the modulator aperture. The magnetic lens is made in

the form of a short coil with a ferromagnetic envelope put on the tube neck from the outside.

Magnetic deflection systems of radar tubes are made in the form of two (or four) coils with ferromagnetic cores or without them. The use of ferromagnetic cores is quite possible, since scanning frequencies in radar indicators are usually low. Such cores make it possible to reduce the power consumed for the deflection of the beam. However, coreless coils provide for better deflection linearity which in

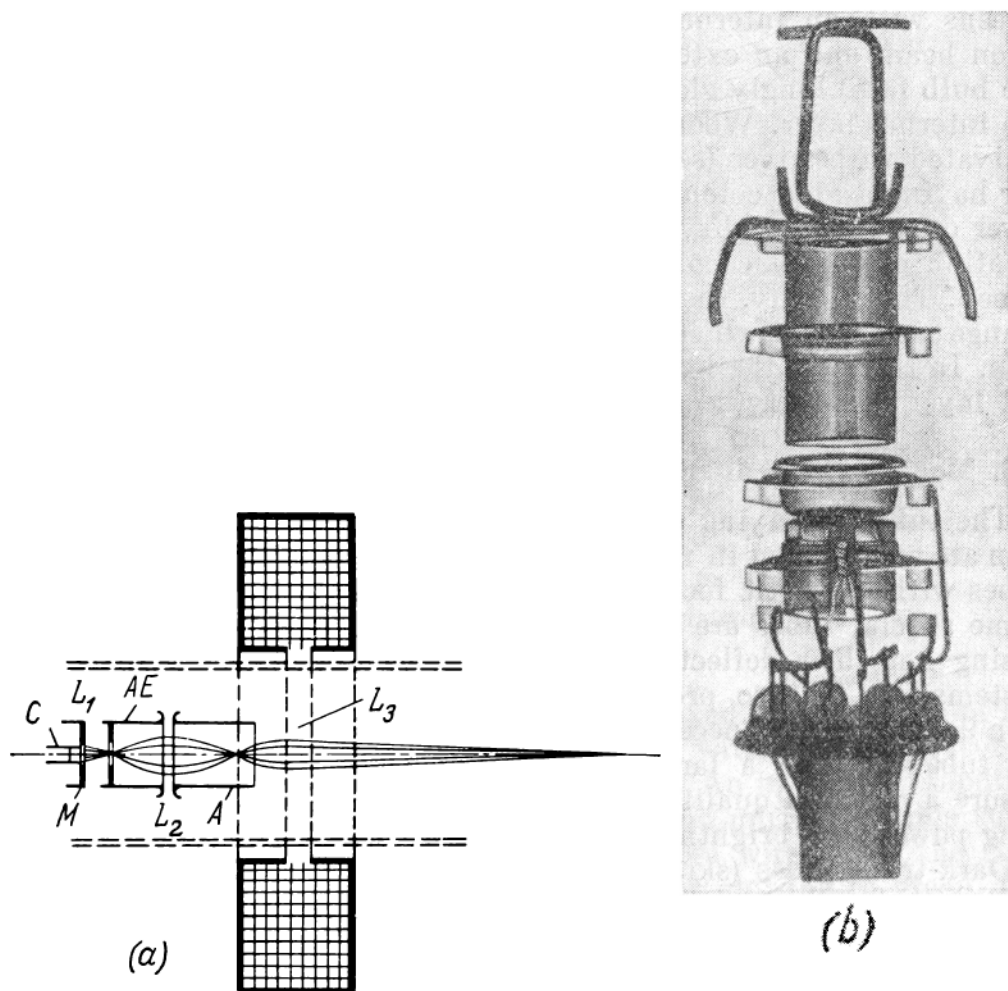


Fig. 8.2. Tetrode gun with magnetic projection lens
(a) schematic diagram; (b) X-ray photograph; L_1 —immersion objective; L_2 —immersion lens;
 L_3 —magnetic lens

some cases is a decisive factor. Screens of radar tubes operating under the conditions of intensity modulation must have: (1) required persistence commensurable with the scanning period (along one of the axes in the Cartesian coordinates or in circular scanning);

(2) high uniformity of glowing over the entire surface of the screen. This is necessary for distinguishing weak signals against the background of a luminous screen in the presence of permanent illumination by the scanning line. The presence of nonuniformities, such as bright luminous spots, with uniform excitation may result in location of non-existing targets; (3) high resolution which allows the positions of the brightness mark to be measured with adequate accuracy.

These requirements are to a great extent satisfied by two-layer screens with an internal shortly glowing layer excited by an electron beam and an external (lying directly on the inner surface of the bulb face) longly glowing layer excited by the light radiation of the internal layer. When the internal layer consists of zinc sulphide activated with silver (see Section 6.5), a bright burst of this phosphor having a blue colour of luminescence is clearly seen through the layer of the longly glowing phosphor and this hinders with accurate location of the place of appearance of the brightness mark on the screen. That is why, the screens of radar tubes are protected with an orange light filter which almost completely absorbs the blue radiation. In this case the long-time yellow-orange glowing of the external layer is clearly seen through the light filter.

8.3. Modifications of Radar Tubes

The tubes satisfying the requirements given in the preceding section are widely used in radar technique. Many types of high-persistent tubes with magnetic focusing and magnetic deflection are available. Some special tubes are equipped with guns having electrostatic focusing but their deflection systems are generally magnetic. Radar systems can be also provided with projection kinescopes (see Section 9.6) where it is necessary to project the image from the indicator tube screen to a large screen. The projection kinescopes can ensure a required quality of a radar picture due to their high resolving power and brightness of the screen.

Dark-trace tubes (skiatrons) have found application in the radar engineering. The skiatrons feature a long time of residence of a dark trace on the screen (see Section 6.6.). The dark trace of the electron beam is clearly seen at intensive external illumination. When necessary, the skiatron picture can be projected to a large screen, in which case the skiatron screen is illuminated by a high-power light source and the picture is transferred to a large screen by means of a light-optical system, i.e. in this case the skiatron is used as a projection kinescope. A general view of a skiatron is shown in Fig. 8.3.

Skiatrons usually employ magnetic focusing and magnetic deflection. The screen of a skiatron is coated with a thin film of potassium chloride applied by vacuum evaporation, i.e. it is structureless and together with good focusing of the beam by the magnetic lens provi-

des high resolution. Since the dark trace on the skiatron screen can be kept for a long time (up to several days and even months), the skiatron must provide for a possibility of erasing the written information. The erasing of the screen is obtained by giving it short-time heating by means of an electric current passed through the transparent layer serving as a substrate for potassium chloride. Such an operation provides for erasure of the trace within a few seconds. The screen can be also erased by heating it with the aid of an external erasing oven but in this case the erasure takes much more time.

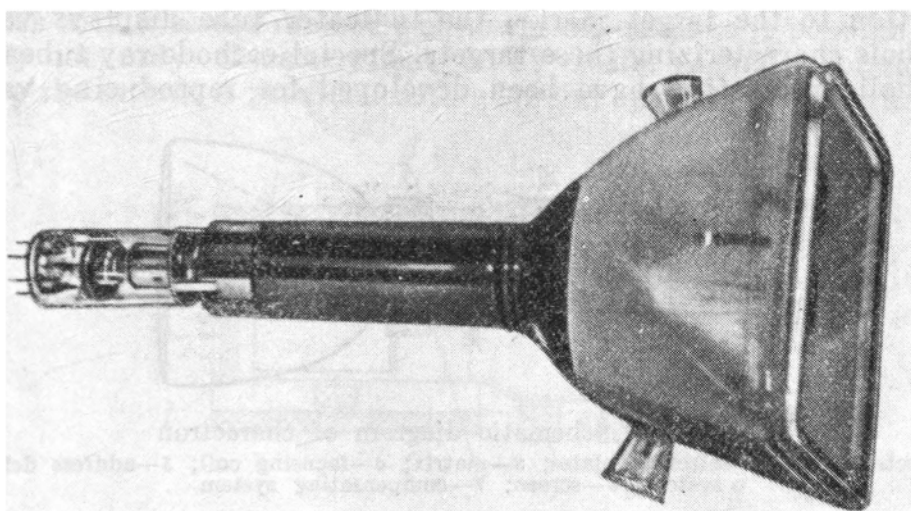


Fig. 8.3. General view of skiatron

To increase the scope of information displayed on the screen of an indicator tube, radar systems are equipped with two-colour tubes similar to colour kinescopes (see Section 9.7). An interesting modification is a two-colour tube with an electronic mirror. In this tube there is mounted a grid near its screen whose wires from the side opposite to the gun are coated with phosphor glowing in green, for instance. The screen is coated with a phosphor glowing in another colour, for example, in red. The screen has a transparent electrically conductive layer provided with a terminal. The grid is also provided with an individual terminal. If the screen potential positive with respect to the gun cathode is higher than the grid potential (which is also positive), the beam electrons passing between the wires of the screen reach the screen and excite its luminescence. The phosphor applied to the grid is not excited, as the electrons do not reach the side opposite to the gun. When the conductive coating of the screen is connected to the gun the screen becomes an electronic mirror reflecting the electrons having passed through the grid back to the latter.

The electrons falling on the grid from the screen side excite the phosphor applied to the grid. Thus, different colours of luminescence can be obtained by varying the screen potential. The glowing of the phosphor applied to the grid is observed through a semitransparent phosphor layer on the bulb face. Since the grid is located close to the screen, the electrons reflected by the screen fall on the grid near the point of passage of the electron beam through the grid. Consequently, the sharpness of the picture obtained in both colours (on the screen and on the grid) is practically the same.

The scope of information can considerably be increased if, in addition to the target marks, the indicator tube displays various symbols characterizing these targets. Special cathode-ray tubes with symbolic indication have been developed for reproducing various

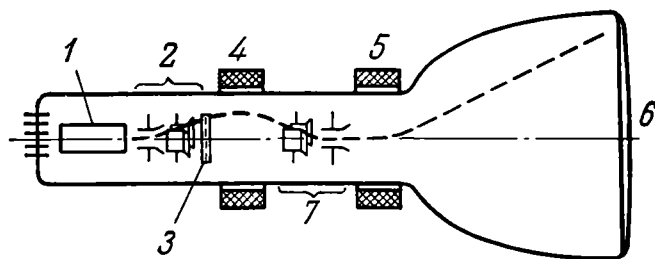


Fig. 8.4. Schematic diagram of charactron

1—electron gun; 2—selecting plates; 3—matrix; 4—focusing coil; 5—address deflection system; 6—screen; 7—compensating system

characters—digits, letters, etc. A characteristic feature of these tubes is the presence of a metal plate (matrix) at the path of the electron beam. The matrix is provided with a number of holes, each having a shape of a letter, digit, or another character. The electron beam passing through the matrix acquires in its section a shape of the corresponding symbol and correspondingly excites the phosphor on the screen. The shape of the luminous spot on the screen will correspond to the shape of the cross section of the electron beam, i.e. the screen reproduces the symbol through which the electron beam has passed in the matrix.

A tube with character indication known as “charactron” is shown in Fig. 8.4.

In this tube the gun with electrostatic focusing shapes a slightly diverging beam with a diameter in the matrix axis of the order of 1 mm. By the electrostatic deflection system consisting of two pair of deflecting plates (called selective) the electron beam is directed to a definite symbol of the matrix and during its passage therethrough acquires a cross section whose shape corresponds to the shape of this symbol. Then the beam is additionally focused by a magnetic coil and is passed through a second electrostatic deflection system

called a compensating system parallel to the tube axis. The third (magnetic) deflection system called an address system directs the beam to that place of the screen where the selected character must be shown. In order to increase the luminance the tube has a spiral system for after-acceleration.

The characters on the matrix have a height of about 0.5 mm. The height of the character on the screen is equal to 0.7-1 mm. With a screen having a diameter of 18 cm, 125×125 characters can be displayed on this screen at a writing rate of up to 20,000 characters per second. Known in the art of radar engineering are character-printing tubes with a screen diameter of 45 cm which make it possible to display not only characters but a TV raster as well. For this purpose the matrix is provided with a special aperture passing the entire

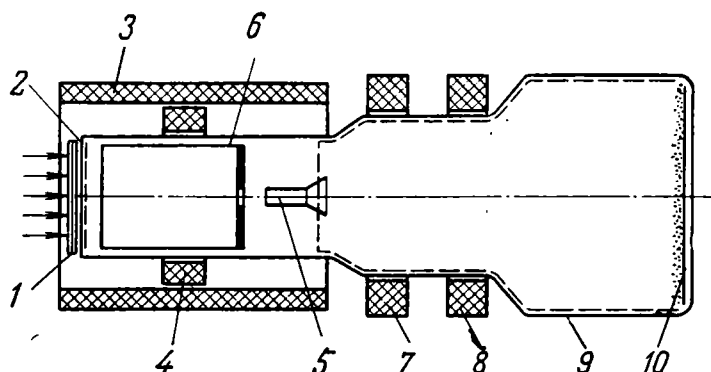


Fig. 8.5. Schematic diagram of compositron

1—matrix; 2—photocathode; 3—focusing solenoid; 4—selecting system; 5—focusing electrode; 6—anode; 7—magnetic lens; 8—address deflection system; 9—conductive coating; 10—screen

electron beam, which is then additionally focused by a magnetic lens and is swept on the screen by a deflection system. The character tubes are employed in radar systems as plan-position indicators on the screens of which, in addition to ordinary brightness marks, is displayed an additional character information.

The tubes with character indication can display on the screen only those characters which are available on the matrix, since replacement of the matrix is impossible without breaking the tube envelope. There are tubes with an external matrix free from the above-mentioned disadvantage. Such a tube commonly referred to as "compositron" is given in Fig. 8.5.

The neck of this tube terminates into a flat face the internal surface of which is coated with a semitransparent photocathode. The matrix is applied to the neck on the outside and is projected to the photocathode by means of an external light source. The neck is placed into an elongated solenoid producing a uniform longitudinal magnetic field. This field transfers the electronic image of the matrix

from the photocathode to the anode having a rectangular aperture whose area is equal to that occupied by one character of the matrix. The magnetic deflection system for character-selection deflects the electronic image of the matrix so that the required character coincides with the anode aperture. The electron beam having a shape of the selected character in its cross section passes through the anode aperture. Next, it is focused and accelerated by an electrostatic immersion lens. After that the beam is additionally focused by a short magnetic lens and is directed to the screen. Mounted between the magnetic lens and the screen there is an address deflection system directing the beam to the required place of the screen. The necessary screen brilliance (at a low photoelectron current) is obtained due to the use of a high accelerating voltage (up to 25-30 kV). The advantage of the tubes with an external matrix is simple replacement of the matrix, their disadvantage consists in the necessity of using a high accelerating voltage and magnetic deflection which to some extent reduces the speed of displaying the characters.

Memory tubes (potentialsopes) with a visible image have found some application in radar engineering. These devices reproduce tone gradations (see Section 11.4). Potentialsopes have some advantages over ordinary tubes, since a sufficiently bright and sharp image on the potentialscope screen is preserved for a long time and if necessary, is easily erased.

Kinescopes

9.1. General

A kinescope is a cathode-ray tube designed for reproducing a television picture. The quality of a television picture depends on the operation of all the components of a television channel starting from the camera tube (see Chapter 12). The kinescope is the final element of television channel and, as the case is with any component of the channel, must satisfy certain requirements with a view to obtaining a high-quality television picture.

First, the image of the kinescope screen must have sufficient brilliance, i.e. be clearly visible in a normally illuminated room. Second, the image must have sufficient contrast, i.e. the ratio of the brightness of the most illuminated places of the image to the brightness of the least illuminated places must be of the order of tens. Third, the kinescope must have resolution allowing a viewer to distinguish the finest details of the TV picture.

The requirement for a definite size of the television picture is important. The optimum size of a television screen can be estimated on the basis of the following considerations. If the distance between the viewer and screen is equal to 2 m, then, taking into account that an object visible at an angle of sight of about 15° is most convenient for observation, the screen diagonal $d = 2 \tan 15^\circ \approx 0.55$ m.

The phosphorescence of ordinary monochromatic kinescopes must be close to standard white colour. The phosphor persistence must not markedly exceed the period of vertical sweep. When the after-glow time is too long, the picture is smeared.

The requirement for a short length of the kinescope is also very important, since the length of the picture tube defines the overall dimensions of the TV receiver, which, naturally, should be not too large.

Wide application of television has led to a situation in which kinescopes have found the most wide application of all the electron-beam devices. As a result, a modern kinescope, besides meeting the above-mentioned requirements, must be economical, simple in manufacture, and inexpensive in mass production, have long service life and provide trouble-free operation.

A tendency of completely satisfying the above requirements has provided development of up-to-date kinescopes having 47, 59 and 65 cm in diagonal, beam deflection angle of up to 110° , white colour of luminescence of the screen, neutral (smoked) light filter and aluminized screen.

Improved technological processes and automatic production lines have made it possible to manufacture high-quality kinescopes practically satisfying all the above-mentioned requirements.

9.2. Kinescope Bulb

The kinescope bulb is a very important element of the tube. Such significant parameters of a kinescope as the screen size, contrast, resolution, as well as overall dimensions of the TV receiver as a whole, depend on the geometric shape and size of the bulb.

Earlier picture tubes started commercially with round bulbs. Nowadays, the tubes are shaped as a frustum of a rectangular pyramid whose base has the same ratio of its sides as a standard television picture, i.e. 3 : 4. The kinescope bulbs are substantially made as all-glass components. Glass-and-metal tubes are gradually becoming obsolete.

Since the picture quality depends essentially on the shape of the bulb face (magnitude of curvature, presence of nonuniformities), as a rule, the bulb face is made of high-quality uniform glass having no blisters or foreign inclusions. The bulb face is obtained by the method of pressing in a hot mould followed by annealing. The most popular sizes of the screen of rectangular kinescopes are 35, 43, 47, 53, 59 and 65 cm in diagonal (round kinescopes had screens of 18, 23, 31 and 40 cm in diameter). Smaller screens are used in special kinescopes, such as TV control monitors, and in projection kinescopes. Small size kinescopes having 10 to 15 cm in diagonal have been developed for transistorized portable TV receivers.

The bulb face is made of comparatively thick glass to provide required mechanical strength, since the atmospheric pressure on the screen of a large kinescope can exceed 1000 kgf. At the same time, it is inexpedient to make the screen convex (with a small curvature radius) in view of the following two reasons: (1) a picture on a convex screen will be distorted; (2) the light radiated by the phosphor of a non-aluminized screen inside the tube will illuminate the entire screen reducing the contrast of the image. Therefore, the face of the kinescope bulb is made only slightly convex in order that the distortion of the picture due to screen curvature be low.

The thickness of the bottom affects the contrast due to formation of halos and scattering of light in the screen solid. The halo is formed due to complete internal reflection of the light emitted by the phosphor from the external surface of the face (Fig. 9.1).

The light from the phosphor comes outwards only at a relatively small angle Θ in the order of 40° . Therefore, more than a half of the light emitted by the phosphor is partially wasted in halos and partially is diffusively scattered in the glass mass. In both cases the contrast is reduced. It is clear that reduction of the contrast is more pronounced in the case of large halos. One can easily see (Fig. 9.1) that the halo radius is

$$r = 2d \tan \Theta \quad (9.1)$$

where d is the thickness of the bulb face.

Since a decrease in the thickness of the bulb face is limited by its mechanical strength, the diameter of the first (usually the brightest) halo is approximately 20 mm, and this of course, can reduce the contrast to a considerable degree. Consequently, the effect of the halo on the image quality can be reduced only at the expense of absorption of a portion of light in the glass mass of the bulb face. Such a method of attenuation of the halo brightness is accompanied by some decrease in the luminance of the entire image. However, taking into account that the path of light in the glass to form a halo is comparatively long (from the screen to the external surface of the glass, from the glass to the phosphor and back), while the light from the screen coming outwards passes through the glass only once, the attenuation of the halos by such a method may be considered as justified.

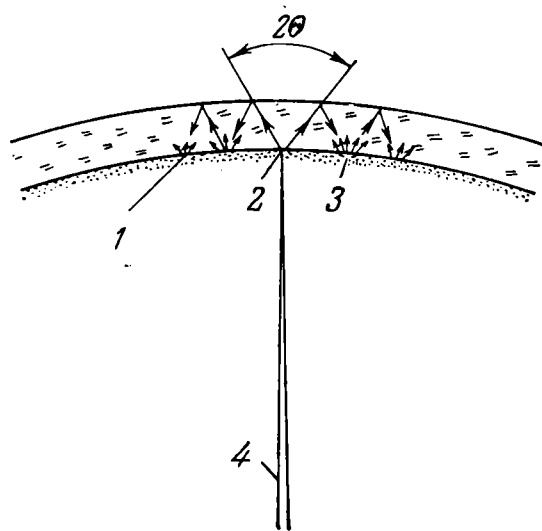


Fig. 9.1. Formation of halos

1—second halo; 2—spot; 3—first halo; 4—electron beam

Illumination of the screen by an external light source has a noticeable effect on the picture contrast. One can easily see that the absorption of a portion of the light in the glass of the bulb face leading to a decrease in the halo brightness can prove to be useful also for reducing the influence of the external illumination on the contrast. In fact, the external light passes through the glass layer two times (from the source to the screen and from the screen to the viewer), while the light radiated by the phosphor—only once. Experience shows that use of smoked glass for making the bulb face considerably increases the picture contrast since such glass serves as a

neutral light filter with a transmission coefficient of 0.6 to 0.7.

A small drop in the luminance of the picture is not very important, since it is usually compensated for by a corresponding increase in the beam current.

One of the most important parameters of kinescopes is their resolution defined by a number of picture elements that can be reproduced on the screen. Since a TV picture features a line structure, the resolution is conveniently estimated by the number of lines clearly seen on the screen. With a television scanning of 625 lines (accepted in the USSR and many other countries) the kinescope resolution must apparently be defined by 625 lines at any place of the screen. It is interesting to note that the standard of 625 lines is well grounded. A human eye serving as a receiver of information reproduced on the screen of a television set with normal vision can distinguish between two elements, if their angular value (the angle at which the element is seen) is not less than $1.5'$. With a "convenient" visual angle of 15° (see Section 9.1) the optimum number of lines providing for perception of the entire scope of useful information is equal to

$$n = \frac{\alpha_{im}}{\alpha_{el}} = \frac{15 \times 60}{1.5} = 600 \quad (9.2)$$

where α_{im} is the angle at which the entire picture is seen; α_{el} is the angular value of a separately distinguishable element.

The resolution is primarily determined by the quality of focusing, i.e. by the diameter of the spot on the screen. If the screen has optimum size (about 0.5 m in diagonal), the raster height is equal to 0.3 m with the accepted ratio of television image sides being 4:3. Thus, during the resolution into 600 lines the optimum diameter of the spot (at the level 0.4) on the screen is equal to 0.5 mm. Such a value of the beam section in the receiver plane is easily ensured by guns with electrostatic focusing, not mentioning those with magnetic focusing.

Since commercial phosphors, even relatively coarse-grained sulphide luminophors, have a grain size of less than 0.1 mm, the discrete structure of the screen has no influence on the resolution of modern large-screen kinescopes. It should be noted that when using single-type guns for kinescopes with different screen sizes, the resolution is first increased with a rise of the screen size and then starts decreasing. This is explained by the fact that the number of elements positioned on the screen increases with the screen size with a constant diameter of the spot, i.e. the resolution rises up. However in order to keep the same luminance of the phosphor on a large-size screen, the beam current must be increased so that the beam widens and, as a consequence, the resolution is reduced. Particularly noticeable decrease in the resolution is observed when the phosphor is

saturated. An increase in the beam current does not lead to an increase in the luminance in the centre of the spot but considerably increases its diameter. Consequently, there is an optimum size of the screen for which the best resolution can be ensured with a gun of the given type.

The widening portion of the bulb of round kinescopes had a shape close to paraboloid. The kinescopes with a rectangular screen have a bulb shaped as a four-face pyramid with convex faces and rounded edges. Modern kinescopes are usually all-glass. The glass-and-metal kinescopes had a widening portion of the bulb made of a ferrochromium sheet. Although glass-and-metal kinescopes have a higher

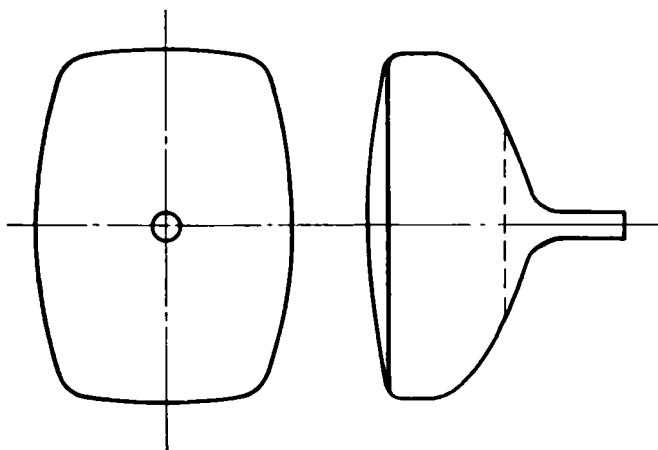


Fig. 9.2. Kinescope bulb

strength and a lower weight than all-glass kinescopes, their application involves some operation difficulties associated with the necessity to have a large metal surface at a high voltage.

The neck of the kinescope bulb has a small diameter (29-36 mm) and a small length, since in the case of magnetic deflection it is expedient to reduce the space between the deflecting coils arranged on the neck, in order to reduce the scattering of the magnetic fluxes.

Moreover, with magnetic deflection it is possible to align the vertical (frame) and horizontal (line) deflecting magnetic fields in a single space and this makes it possible to reduce the length of the neck (as compared to the tubes having electrostatic deflection).

Magnetic deflection makes it possible to use large beam deflection angles without significant disturbance to the focusing and linearity. Hence, modern kinescopes have beam deflection angles of up to $50-55^\circ$ (full span of up to $100-110^\circ$). With such large beam deflection angles the kinescope bulb is comparatively short. A simplified view of a modern kinescope bulb is shown in Fig. 9.2.

The internal surface of the walls of glass bulbs is usually coated with an electrically conductive graphite layer to be used for applying

a high voltage to the tube anode. The external surface of the conical portion of the glass kinescopes operating at an anode voltage higher than 8-10 kV is also covered with a conductive coating which is not electrically connected to the internal layer. The two layers of the graphite coating (internal and external) divided by a dielectric (bulb glass) form a capacitor of the filter of the high-voltage rectifier feeding the kinescope.

Many types of kinescopes have the so-called explosion-proof bulbs. Earlier all-glass bulbs were subject to occasional explosions under mechanical actions or due to insufficient strength of the bulb itself caused by high stresses at the joint of the face with the widening portion. In this case the bulb was destroyed and pieces of glass flew apart in different directions at a high speed. Hence, in order to protect viewers from possible injuries during the explosion of the bulb, the TV receivers were provided with special protective shields made of shatter-proof organic glass mounted in front of the kinescope screen. Modern explosion-proof kinescopes have a special metal frame — holder (Fig. 9.11) encircling the bulb near the screen. The space between the holder and the bulb wall is filled with a hardening mass (for example, gypsum) expanding in the process of hardening. In this case the joint between the bulb and screen and the screen itself are squeezed in a radial direction and mechanical breaking of the bulb results only in cracking of the screen without formation of flying glass fragments. The use of explosion-proof bulbs made it possible to eliminate the protective glass shield, align the kinescope screen with the front surface of the TV receiver, and even to position the convex screen somewhat beyond the front of the receiver thus reducing the depth of the TV receiver.

9.3. Kinescope Gun

A high-quality bright and contrast TV picture under the conditions of a normally illuminated room can be obtained only at high brilliance of the screen in the order of dozens of nits for small-size kinescopes and 200-300 nits for kinescopes with a screen having more than 0.5 m in diagonal. In the case of using sulphide luminophors with a luminous efficiency of several candles per watt the required luminance can be obtained only with a beam having a power output of several watts. The necessary beam power is provided by increasing the accelerating voltage to 10, 16 kV and higher at a beam current of a few hundreds of microamperes. An increase in the beam power is inexpedient, since the higher the beam current, the more difficult is the maintenance of a required resolution due to the action of the Coulomb repulsive forces so that it is necessary to use a greater area of the cathode surface which increases the aberrations of the lenses, etc. Moreover, a high beam current can result in saturation of

the phosphor brightness (see Section 6.2) which also reduces the resolution due to the increase of the apparent (at a level 0.4) diameter of the spot. It should be remembered that the luminance of sulphide luminophors increases with an increase in the accelerating voltage approximately by the square law, i.e. the luminous efficiency of the screen increases.

The above considerations show that the kinescope gun must produce a beam with a current of a few hundreds of microamperes at an acceleration voltage of at least 10 kV. In this case, to provide a required resolution, the spot diameter (at a level 0.4) must be not higher than 0.2-0.3 mm for kinescopes with a screen diagonal of up to 35 cm and may reach 0.5 mm for kinescopes with a screen larger than 50 cm in diagonal. Too small diameter of the spot (several times smaller than the distance between the lines) is even undesirable, since the sharpness of the picture determined by the amount of scanning elements (number of lines) in this case is not increased while the line structure of the picture becomes more noticeable. From the viewpoint of artistic perception, some overlapping of the lines is tolerable.

Since the video signal producing a television picture is fed to the kinescope modulator, the gun must have an adequately steep modulation characteristic. High steepness of the modulation characteristic makes it possible to obtain considerable variations of the beam current with a moderate amplitude of the video signal thus producing a contrast picture. From the point of view of simplicity of the video signal amplifiers it is necessary to have linear dependence of the screen brightness on the modulator voltage. However, since the camera tubes feature not quite linear light characteristic, (see Section 12.4), the characteristic $B_{screen} = f(I_l)$ is also not quite linear. Moreover, development of a gun with a linear modulation characteristic is a difficult problem. Therefore, the shape of the modulation characteristic of a kinescope gun must not satisfy rigorous requirements, except for the requirement for a small modulation factor. The nonlinearity of the elements of a television channel is compensated for by using the so-called gamma correction consisting in a special disturbance to the linearity of the transmission characteristics of the video amplifiers, which is reverse in comparison with the nonlinearity of the characteristics of electron-beam devices.

An electron gun adequately satisfying the above-mentioned requirements can be built around a two- or three-lens optical system with an electrostatic or magnetic projection lens. In earlier widely used round kinescopes having a comparatively small screen (18, 23, 31 cm) magnetic focusing was employed, i.e. the gun was based on the immersion objective—magnetic lens scheme. At the present time most of the kinescopes have a gun with electrostatic focusing, the magnetic projection lens being used only in projection tubes (see

Section 9.6) and some special-purpose kinescopes having small screens and improved resolution.

Kinescopes equipped with guns having electrostatic focusing are fairly economic in service which is a significant characteristic in the case of mass production of television receivers. Furthermore, when the entire electron-optical system of the gun is fed from a single power supply source, variations in the supply voltage do not disturb the focusing, while in the case of magnetic focusing these variations cause defocusing.

The gun with electrostatic focusing usually employs a three-lens (pentode) scheme, i.e. mounted between the accelerating electrode

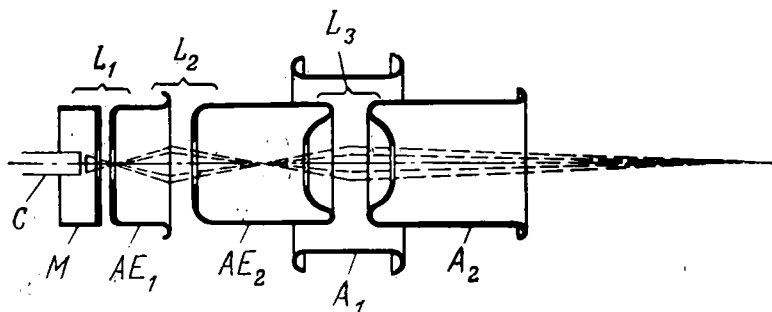


Fig. 9.3. Kinescope gun design

and the modulator is an intermediate electrode which is usually called the accelerating electrode too, but it is at a relatively low potential (from +200 to +600 V) with respect to the cathode. The use of an intermediate electrode is stipulated by the necessity to use a moderate cutoff voltage and a small modulation factor. The immersion lens formed between the first and second accelerating electrodes reduces the beam divergence angle behind the crossover plane, the beam in the projection lens becomes narrower and this reduces the aberrations of this lens, i.e. the focusing is improved. Moreover, at high accelerating voltages (15-20 kV) the accelerating electrode at an intermediate potential facilitates the provision of required dielectric strength. Thus, the gun with electrostatic focusing has the following optical scheme: the cathode, modulator and accelerating electrode form an immersion objective; the accelerating electrode and second accelerating electrode at the potential of the second anode form an immersion lens; the second accelerating electrode, the first anode and the second anode form a single lens. A scheme of such a three-lens gun is shown in Fig. 9.3, its general view is illustrated in Fig. 9.4.

With the operation time of a kinescope using the magnetic deflection of the beam the centre of the screen shows a dark spot considerably reducing the quality of the picture or even making the tube unsuitable for further utilization. The cause of this spot is gradual

destruction of the central portion of the screen by heavy charged particles, negative ions. Hence, the dark spot on the screen is commonly referred to as an ionic spot. A mass-spectrometric analysis of charged particles leaving an electron gun shows that, present among the electrons are some negative ions O^- , Cl^- , CO_3^- and others, in total up to 30 different ions. The resultant current of the ions is

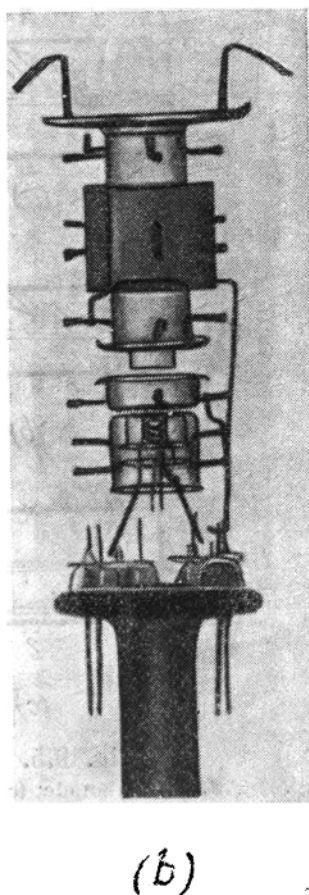
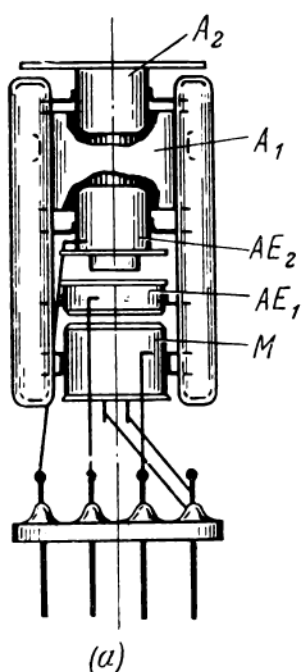


Fig. 9.4. Gun of modern kinescope
(a) schematic; (b) X-ray photograph

small (10^{-10} to 10^{-9} A), but due to high accelerating voltages the energy acquired by the ions is sufficient for gradually destructing the phosphor.

Since the deflection angle of the beam of charged particles caused by the magnetic field is inversely proportional to $\sqrt{m}$ [see (5.2)], the ion beam is deflected but through a small angle when sweeping over the whole screen. For example, with the ions O^- the deflection is about $1/150$ of that for the electrons. The given example shows that

in the case of magnetic deflection the ions destroy only the central part of the screen, i.e. an ion spot is formed.

With electrostatic deflection the deflection angle of the beam of charged particles does not depend on the mass of particles [see (5.1)]. Therefore, negative ions in the tubes with electrostatic deflection

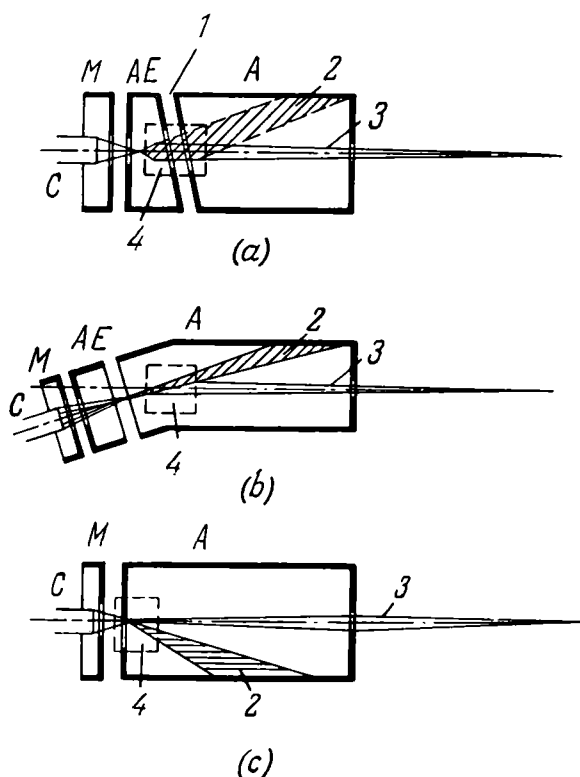


Fig. 9.5. Ion trap

(a) with tilted lens; (b) with curved anode; (c) with displaced diaphragms; 1—oblique lens; 2—ions; 3—electrons; 4—magnet

cause no ion spot in the centre of the screen but merely accelerate "burn-out" of the phosphor on the entire surface of this screen. Since a magnetic field is used in modern kinescopes for deflection of the beam (see Section 9.4), it is necessary that measures be taken for preventing the formation of an ion spot. To this end the penetration of ions to the screen must be prevented. This can be obtained by trapping the ions either in the electron gun (ion traps) or directly at the screen surface (aluminized screens).

The principle of operation of ion is based on the different effect of a magnetic field on the ion and electron beams. In the majority of practically employed ion traps the beam comprising electrons and negative ions (after passing the first lens of the gun) is deflected from the axis tilting the first portion of the gun or by means of an auxiliary electrostatic field. Before entering the second lens, the

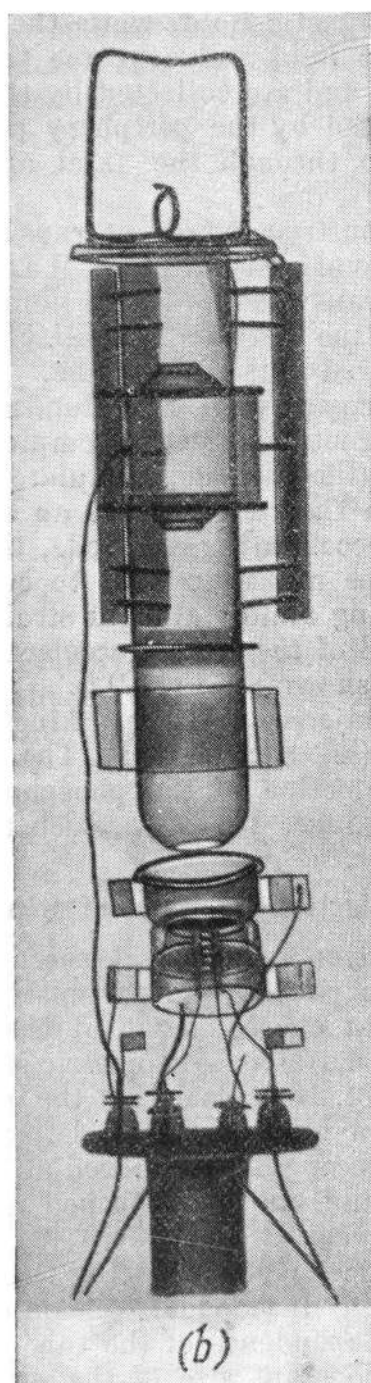
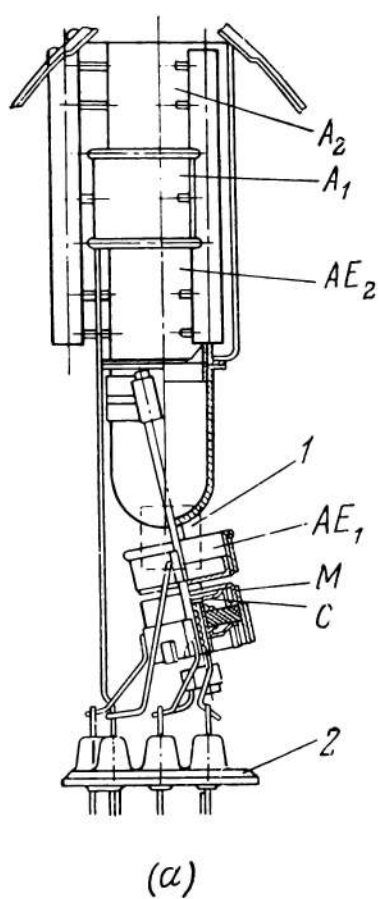


Fig. 9.6. Gun with ion trap

(a) schematic; (b) X-ray photograph; 1—transverse magnetic field (permanent magnet);
2—base

electron beam coincides with the axis under the effect of the transverse magnetic field, while the ions are much less deflected by the magnetic field and continue their movement without approaching the axis and are collected by the metal shield. Such a shield can be constituted by the periphery portion of the diaphragm of the gun electrode through the axial aperture of which the electron beam passes.

The construction of ion traps employing the above system is shown diagrammatically in Fig. 9.5.

The transverse magnetic field necessary for directing the electron beam to the gun axis is usually produced by a small permanent magnet secured outside the tube.

Kinescopes with non-aluminized screens, electrostatic focusing and magnetic deflection employ guns with ion traps whose first portion (the cathode, modulator and first accelerating electrode) is tilted to the tube axis at an angle of $10\text{--}15^\circ$. Before entering the second accelerating electrode, the electron beam is deflected by the transverse magnetic field to coincidence with the axis, while the ions flying almost along a straight-line path are entrapped by the side wall of the second accelerating electrode. The gun with an ion trap is shown in Fig. 9.6.

The majority of modern kinescopes are equipped with aluminized screens (see Section 9.5). The aluminium layer provides for reliable protection of the phosphor against the bombardment by the negative ions. Therefore, such tubes need no ion traps.

9.4. Deflection Systems of Kinescopes

Deflection systems of kinescopes must meet rigorous requirements since the picture quality mostly depends on the deflection system. The main characteristic of the picture quality depending on the deflection system is geometric similarity of the image to the transmitted object. It is clear that the geometric similarity can be obtained only at a high degree of deflection linearity. The high quality of the image must be preserved at all points of the screen, i.e. the focusing should not be disturbed when the beam is deflected over the entire screen.

The necessity of reducing the overall dimensions of the kinescope (its length) dictates large beam deflection angles. Figure 9.7 illustrates the dependence of the tube length on the beam deflection angle with a constant size of the screen.

Kinescopes with a beam deflection angle of 70° have a relatively great length. An increase in this angle to 110° reduces the tube length more than 1.5 times the length of a tube using a beam deflection angle of 70° . Finally, the deflection system must be economic, i.e. should not consume considerable power for deflection of the beam

over the entire surface of the screen. The last requirement is well satisfied by electrostatic deflection systems. The systems, however, do not satisfy the other requirements. Therefore, kinescopes with electrostatic deflection have not found wide application.

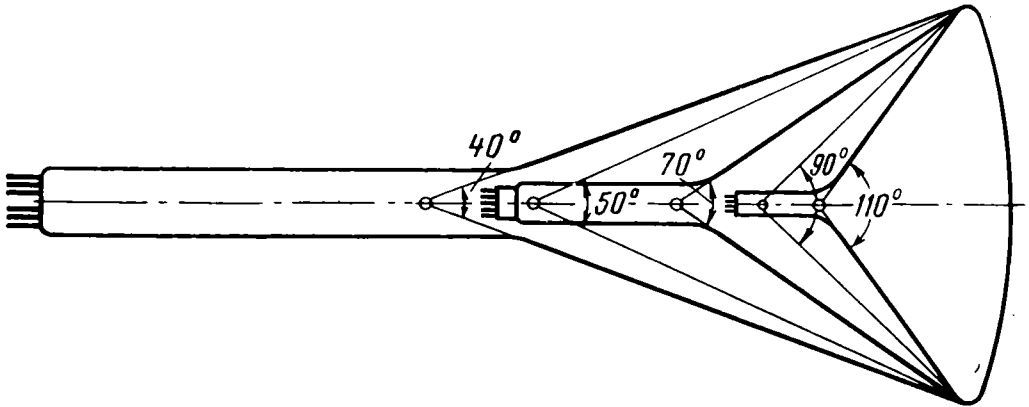


Fig. 9.7. Kinescope length *versus* beam deflection angle

A typical deflection system of a modern kinescope is a magnetic system consisting of four coils without ferromagnetic cores and with series summed magnetic fluxes. The systems for horizontal (line) and vertical (frame) deflection are combined in space to reduce the general length of the deflection system. The frame and line sweep coils are similar in construction but differ in the number of turns and gauge of wire. A magnetic deflection system of a kinescope is shown in Fig. 9.8.

Kinescopes with 110° beam deflection angle (shorted kinescopes) have received wide acceptance. It is evident that at such large angles of deflection it is only a magnetic deflection system that can be used. In order to increase the efficiency of the deflection system, which is very important for shorted kinescopes, it is expedient to reduce the gap between the deflecting coils. The reduction of the distance between the coils makes it possible to obtain a higher magnetic induction in the gap, using the same number of ampere turns, due to a decrease in the stray fields. In order to reduce the distance between the coils, the diameter of the

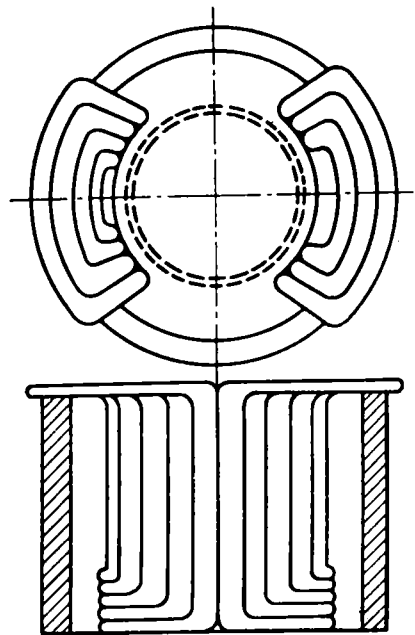


Fig. 9.8. Magnetic deflection system

bulb neck of kinescopes is reduced to 29-30 mm, as mentioned above. The system of close-lying coils makes an efficient (economical) 110° deflection system.

9.5. Kinescope Screens

The quality of a television picture depends on the properties of the screen: colour, luminance and persistence, contrast and resolution. A type of a screen is selected in accordance with these requirements.

The phosphor for coating screens of ordinary (monochromatic) kinescopes must have a white colour of luminescence, comparatively

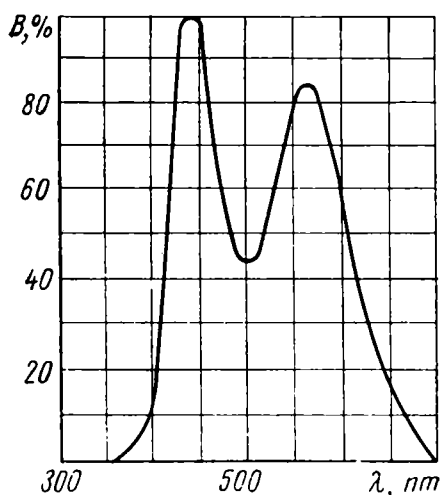


Fig. 9.9. Spectral characteristic of kinescope screen

high luminous efficiency (at least 2-3 cs/W) and persistence not higher than the period of the vertical sweep, high critical potential (not less than the gun accelerating voltage) and adequate resistance to the intensive electron bombardment. Since the requirements for the resolution of kinescope screen are not too rigorous, the phosphor may have fairly a coarse-grained structure.

None of the standard commercial phosphors has a white colour of luminescence. Consequently, the coating of the kinescope screen uses a mixture of two phosphors featuring complementary colours, usually blue and yellow. An earlier yellow component consisted of zinc-

berillium silicate was activated with copper $[(8\text{ZnO} \cdot \text{BeO} \cdot 5\text{SiO}_2 \cdot \text{Cu})]$. However, silicates have insufficiently high luminous efficiency (see Section 6.6). Hence, at the present time silver-activated zinc-cadmium sulphide is most frequently used as the yellow component. Varying the percentage of cadmium and zinc sulphides it is possible to obtain a required colour of the yellow component of the white phosphor. The blue component is usually composed of silver-activated zinc sulphide. The colour of a screen coated with a phosphor consisting of a mechanical mixture of two luminophors is determined by the ratio of the quantities of both components. To obtain a white colour of luminescence, the quantity of blue luminophor must be 4 to 4.5 times (by weight) that of yellow luminophor. It should be noted that this proportion is varied depending on the spectral characteristic of the yellow luminophor which depends on the percentage ratio of the ZnS and CdS and the type of activator. Figure 9.9

illustrates a summary spectral characteristic of the white phosphor composed of a mixture of $\text{ZnS} \cdot \text{Ag}$ and ZnS (48 %) $\cdot \text{CdS}$ (52 %) $\cdot \text{Ag}$.

Although the curve has a gap in the medium portion of the spectrum, the presence of the blue and yellow maxima makes the colour of luminescence of this phosphor close to the standard white one.

With the same proportion of the phosphor components the colour of luminescence varies, depending on the method of coating the screen. When the screen is coated with a mechanical mixture of yellow and blue luminophors it appears more yellow, since a portion of the phosphorescence of the blue luminophor is absorbed by the crystals of the yellow luminophor, while the blue luminophor practically does not absorb yellow colour. If blue luminophor is applied directly to the glass of the bulb face and then, after this layer has dried, yellow luminophor is applied thereto, there will be no loss of the blue radiation, the screen will appear more blue and the total luminous efficiency of the screen will be somewhat higher. However, layer-to-layer application of blue and yellow luminophors complicates the process and makes the kinescope more expensive while the gain in the luminous efficiency is comparatively small. Therefore, in mass production of kinescopes their screens are usually coated with a mechanical mixture of both phosphors.

As mentioned above, the kinescope resolution is limited by the spot diameter (a few tenths of a millimetre), therefore, the phosphor grain may be coarse (4-8 microns). The use of coarse-grained phosphors is expedient, since grinding of sulphide phosphors, as a rule, reduces their luminous efficiency. The thickness of the coating is selected, depending on the value of accelerating voltage, in order to provide complete transfer of the electron energy in the phosphor layer. It has been found experimentally that the maximum luminous efficiency for a white sulphide phosphor is obtained at a thickness of the coating of 3 to 5 mg/cm^2 at an accelerating voltage of 12 to 16 kV.

The method of deposition is usually employed for obtaining uniform coatings of television picture tube screens. A mechanical mixture of both phosphors in the required proportion is suspended in a distilled water and is poured into the kinescope bulbs. The phosphor is secured with the aid of a binder such as liquid glass in a very low concentration. Since the kinescopes usually operate in stationary apparatus, the use of a binder is not strictly necessary, particularly taking into account that the binder to some extent reduces the luminous efficiency.

The use of two-component phosphors essentially solved the problem of the white colour of kinescope screen luminescence, however, such screens are not free of some disadvantages. First, the colour of luminescence strongly depends on the composition percentage of both components and their spectral characteristics which, in their turn,

are not quite stable due to negligible amounts of admixtures or difference in the technological process (for example, different conditions of heat treatment.) The second disadvantage is a change in the screen colour in the process of operation due to different resistance of both components to the electron bombardment. The yellow phosphor "burns out" somewhat earlier and the screen colour becomes more blue. Finally, the colour of luminescence of two-component screens depends on the value of accelerating voltage because of the difference in the luminance characteristics of both components.

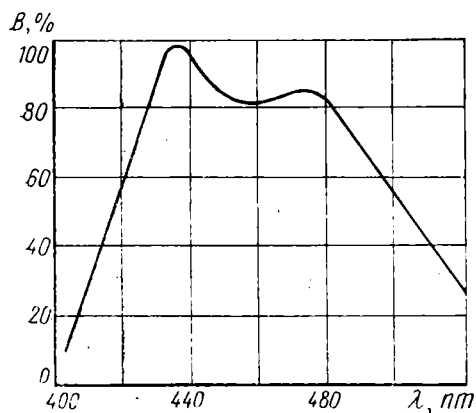


Fig. 9.10. Spectral characteristic of single-component white luminophor

These disadvantages can be eliminated only by using a single-component (simple) phosphor with a corresponding spectral characteristic. The investigations conducted during the recent years have made it possible to produce several types of single-component white phosphors. Such phosphors are based on zinc sulphide with the use of several activators Ag, Au, P, As, etc. A satisfactory white luminophor was obtained by activation of zinc sulphide with silver and phosphor. The spectral characteri-

stic of the luminophor $\text{ZnS} \cdot \text{Ag, P}$ is given in Fig. 9.10.

As is shown in this drawing, the radiation involves a considerable portion of the visible spectrum with a maximum in the blue section. Industrial development of single-component white phosphors will make it possible to produce kinescope screens which are more uniform in colour and more stable in operation.

As mentioned above, the majority of modern kinescopes have aluminized screens. The aluminization of the screens is quite justified at accelerating voltages of more than 10 kV (see Section 6.6). In addition to the reliable protection of the phosphor against the electron bombardment, the aluminization increases the visible brilliance of the screen and increases the contrast by eliminating the internal illumination of the screen by the light radiated by the phosphor. Finally, the potential of the aluminized screen does not depend on the secondary-emission properties of the phosphor.

9.6. Modifications of Kinescopes

A typical modern kinescope is presented by a cathode-ray tube with an electrostatic gun using a three-lens scheme. Such tubes have an accelerating voltage of 15 to 20 kV, a beam current of 300-500

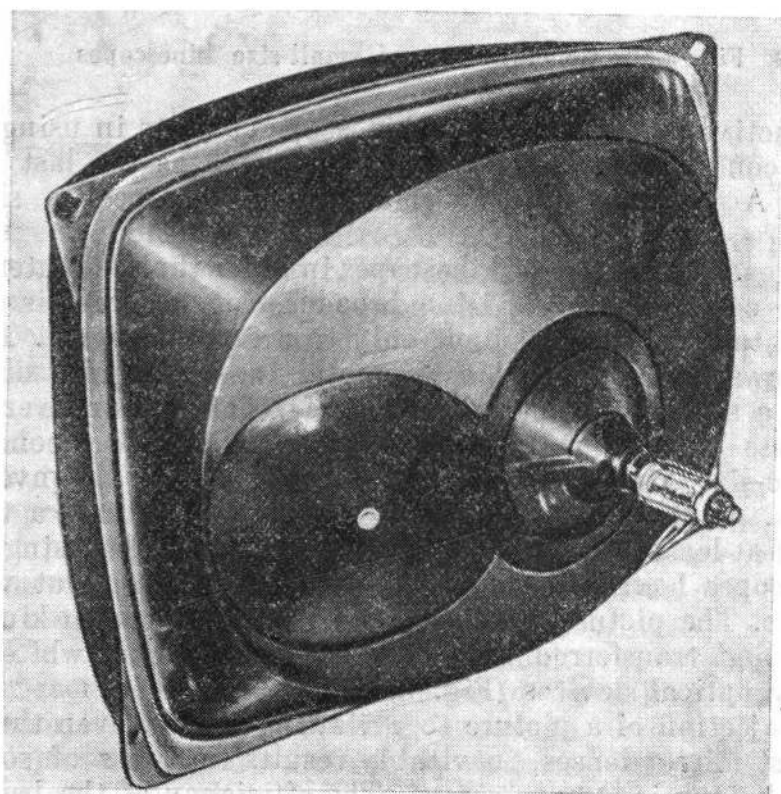
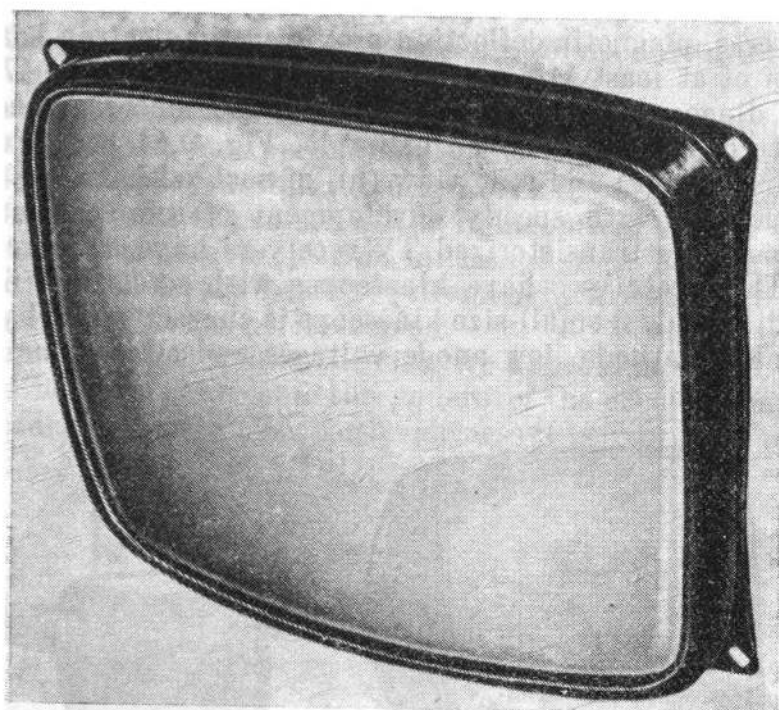


Fig. 9.11. General view of explosion-proof kinescope

microamperes, magnetic deflection providing an electron beam deflection span of at least 110° , an aluminized screen having 47, 59 and 65 cm in diagonal, a white colour of luminescence, and an explosion-proof construction. As an example, Fig. 9.11 shows a general view [front view (a) and rear view (b)] of such a kinescope.

In connection with speedy development of semiconductor engineering, portable transistorized TV receivers have appeared on the market. These receivers have kinescopes with screens of 6, 11 and 16 cm in diagonal. A small-size kinescope is characterized by a highly economical cathode, low anode voltages and a low beam current.

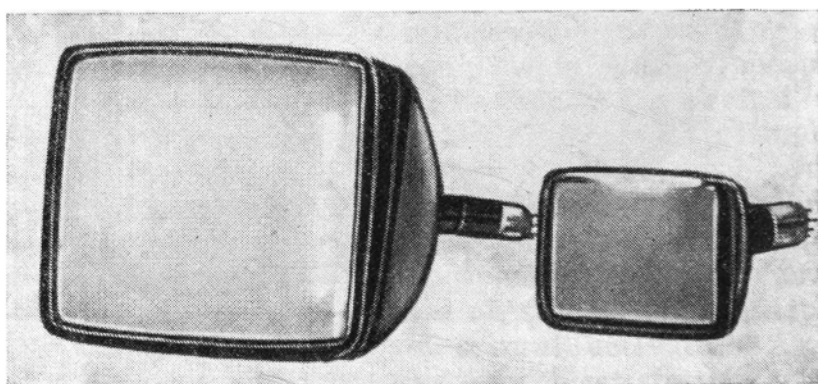


Fig. 9.12. General view of small-size kinescopes

A constructive feature of such a kinescope consists in using the electrically conductive coating on the bulb neck as the last anode of the gun. A general view of small-size kinescopes is shown in Fig. 9.12.

An increase in the size of kinescopes in order that a greater number of people could see a television broadcasting program is economically and technically expedient only to a certain degree. The experience shows that glass and even metal-and-glass bulbs of kinescopes with a screen diagonal in excess of 75 cm become very heavy, the process of their manufacture being considerably complicated. The large-size kinescopes become expensive and inconvenient in operation. At the same time a large audience requires a television picture of at least 3 to 4 m². This problem is solved by using projection kinescopes having a comparatively small screen but very high brightness. The picture from the screen of a projection kinescope is enlarged and transferred to a large light-scattering white viewing screen by optical devices (Fig. 9.13).

The projection of a picture to a viewing screen, even through the use of fast mirror lenses, inevitably results in a loss of some light radiated by the kinescope screen. The efficiency of the best optical systems with spherical mirrors having a diameter of 50-70 cm is

equal to 25-30 %, i.e. the viewing screen receives only $1/3$ the energy radiated by the phosphor.

In order to obtain an adequately good image on a viewing screen, its brightness must be approximately the same as during the projection of an ordinary motion picture, i.e. in the order of $25\text{--}30\text{cd/m}^2$. Taking into account the losses in the light-optical system the brilliance of the screen of a projection kinescope must be of a few thousands of cd/m^2 . Thus, the basic requirement for a projection kinescope is very high brilliance of its screen.

Since the phosphor luminance rises up approximately in proportion to the beam current and the square of the accelerating voltage, then, in order to obtain a higher brightness it is expedient to increase

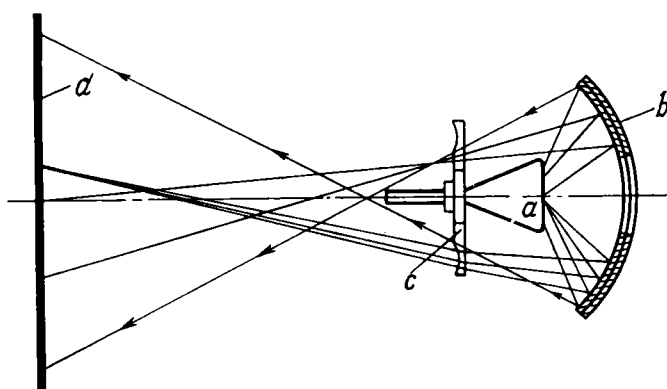


Fig. 9.13. Projection arrangement
a—kinescope; b—mirror; c—correction lens; d—viewing screen

this voltage. A considerable increase in the beam current is inexpedient, since in this case the resolution is reduced (see Section 9.2) and the process of destruction of the phosphor is accelerated. Therefore, projection kinescopes use high accelerating voltages, up to tens of kilovolts. In this case the power brought by the electron beam to the screen may be as high as one kilowatt and higher per mm^2 of the screen surface. Since a considerable amount of the energy of the beam electrons stopped by the screen is converted into heat, the scanning units in a projection kinescope assembly cannot be switched off even for a short period of time. A stationary beam of such power immediately “burns through” the screen. To cool the screen of a projection kinescope use must be made of positive air cooling.

The bulbs of projection kinescopes usually have small screens circular or rectangular, with a diameter (diagonal) of 6-18 cm. To reduce distortions, when projecting a picture onto a large screen, the kinescope screen is made of highly uniform optical glass and has a plane surface. The phosphor consists of a white sulphide mixture or special highly efficient phosphorescent compounds. The brilli-

ance may be as high as 25-30 thousand nits. Since the accelerating voltage exceeds the second critical potential of the phosphor, the screen of a projection kinescope must be provided with an electrically conductive aluminium coating connected to the gun anode in order to maintain the potential at a level of the anode voltage. Moreover, the aluminization of screens, as has been mentioned above, increases the luminance and contrast of the picture and protects the phosphor against destruction by the negative ions.

In most of projection kinescopes the second lens of the gun is made in the form of a short magnetic coil, i.e. the gun is employing a system: bipotential objective—magnetic lens. The magnetic lens makes it possible to dispense with the limiting diaphragms in the anode cylinder. This is expedient, since in this case it is possible to obtain a higher beam current (at the same load of the cathode) and a steeper modulation characteristic (a smaller modulation factor). At the same time, the magnetic focusing makes it possible to obtain a spot diameter lower than 0.1 mm which is necessary for providing resolution of at least 600 lines at a small size of the picture. Since the accelerating voltages are sufficiently high, the terminal of the gun anode is led through the internal conductive coating to the side

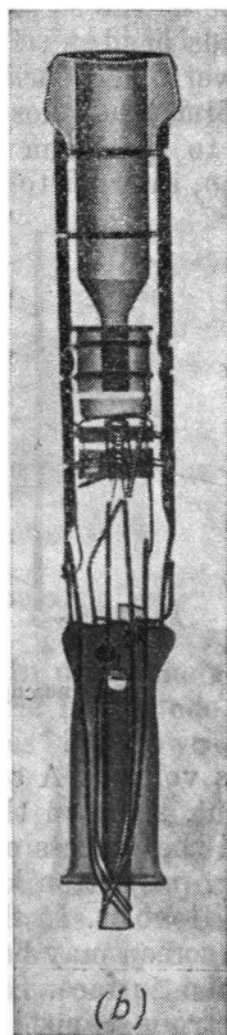
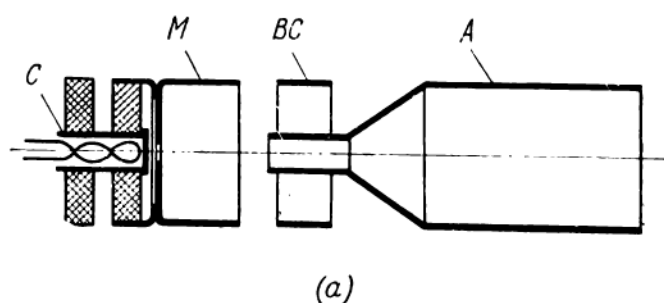


Fig. 9.14. Projection kinescope gun
(a) schematic; (b) X-ray photograph

of the widening portion of the bulb. To avoid electrical breakdown over the external surface of the bulb, the anode terminal is protected by a glass cylinder welded to the bulb wall so that the terminal is "sunk" in a deep (35-50 mm in length) cylinder.

The electrode system of the gun of a projection kinescope is shown in Fig. 9.14.

As shown in the drawing, the anode cylinder has a narrow nose from the modulator side. Arranged around this nose is a short wide cylinder, the so-called spark arrester. The presence of an earthed spark arrester reduces the possibility of an electric breakdown between the anode and the modulator (cathode). General view of a projection kinescope having a screen 6 cm in diameter is shown in Fig. 9.15.

Despite of the high brilliance of screens of projection kinescopes, an adequate quality of a picture with luminance of tens of nits can



Fig. 9.15. General view of projection kinescope

be obtained only on screens whose size is not greater than 3×4 m. At the same time a large audience must be served by a screen of tens of square metres. The problem of obtaining a bright image on a large screen can be solved with the aid of the so-called light-valve devices. In these devices the light from a high-power source is modulated by an electric video signal. The light-valve devices with a Kerr cell or with modulation of a light beam by means of ultrasonic waves propagating through a transparent medium have not accepted practical application due to some inherent disadvantages. The only light-valve device brought to adequate technical perfection and having gained some application is a television projector with an electron optical light modulator known in engineering literature as "Eidophor".

In the "Eidophor" projector (Fig. 9.16) the light modulator is presented by a thin film of a special viscous oil (Eidophor liquid) whose surface is deformed by the electrostatic forces produced by an electron beam brought to the surface of the liquid. The light from a high-power (2 kW) gas-discharge xenon lamp 2 is directed to a mirror slits (diaphragm) 5 by an optical system consisting of a concave reflector and a condensor 1. The mirror slits 5 are formed by a few mirror strips arranged at an angle of 45° . The light beam reflected by the mirror slits falls on a spherical mirror 7. The mirror surface is coated with a light modulating medium, a film of Eidophor liquid about 50 microns thick. The mirror slits are located in the curvature centre of the spherical mirror. If the surface of the light modulating liquid is undeformed, the light from the strips of the mirror

slits set is reflected by the spherical mirror back to the mirror slits (diaphragm) and does not get into the projection lens 4 and onto the screen 3.

Arranged at an angle to the axis of the spherical mirror is an electron gun 6 producing a cathode ray having current of about $10 \mu\text{A}$ at an accelerating voltage of 15-20 kV. The ray diameter at the mirror surface is about 50 microns. A magnetic deflection system scans this beam forming a television raster on the surface of the light modulating liquid. Depending on the value of the signal modulating

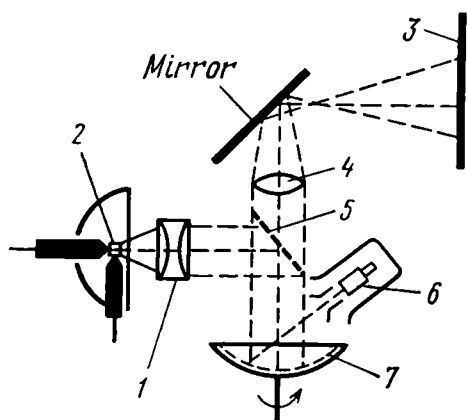


Fig. 9.16. "Eidophor" television projector

the electron beam, a higher or lower electric charge is imposed on the surface of the light modulating film. The charge stored on the film deforms it. The greater the charge brought by the cathode ray, the greater is the deformation.

The deformation of the layer of the light modulating liquid changes the direction of the light beams reflected from the spherical mirror. Some portion of the light flux passes between the strips of the mirror slits and, having passed through the projection lens, produce a television image on a screen. The high-quality reproduction of movable objects is possible only under the conditions which make the deformations remain for the duration of one picture period. To allow the charge and, therefore, the deformations (ripple) to remain on the film for not more than $1/25$ s (one picture period), the viscous liquid must possess a certain electric conductivity providing for leakage of the charge to the metal reflecting surface of the spherical mirror within a time period of not higher than $1/25$ s at a maximum

value of the video signal. To maintain the required thickness of the film, the light modulating liquid is continuously fed through a supply pipe to the mirror surface, while the mirror itself is slowly rotated at a speed of some revolutions per hour. A shaped rule smoothing the layer of light modulating liquid is placed between the supply pipe and the sector of the mirror on which a television image is produced by the cathode ray.

The "Eidophor" system makes it possible to obtain a high-quality television picture on a screen having an area of not more than 50 m² with an average brightness of 15 to 20 nits and a contrast factor of up to 100. The disadvantage of the television projector system with an electron-optical light modulator is the complexity of the entire system and the use of vacuum pumps continuously evacuating the vapours of the light modulating liquid from the vacuum envelope housing, the spherical mirror and the electron-optical system. Due to the presence of oil vapours the electron gun must be provided with a tungsten cathode but even in this event the service life of the cathode does not exceed tens of hours and it must be replaced rather frequently.

9.7. Kinescopes for Colour Television

Colour television allowing a viewer to see the object being televised in natural colours has presently gained wide recognition. A colour image presents many specific features of an object that are unattainable for black-and-white television. A colour picture turns to be more sharp and contrast than a monochrome picture due to different coloration of separate areas which considerably increases the artistic impression from the televised image.

The first experiments in televising a colour image were made in the twenties of our century. Despite technical imperfection, the earlier devices were based on the possibility to obtain any colour by mixing the three fundamental colours (red, blue and green) in definite proportions. The use of these three colours stems directly from the theory of human eye perception according to which the eye has three types of colour-sensitive elements reacting correspondingly only to one certain colour—red, blue or green.

Thus, the complete information about the character of coloration of all sections of a colour picture can be conveyed by a definite quantitative proportion of the three fundamental colours. The principle of a three-colour television transmission is employed in most of the modern television systems.

All colour television systems can be divided into two groups: (1) systems with sequential transmission of the fundamental colours (blue, green and red) and (2) systems with simultaneous transmission of these colours.

In the systems with sequential transmission, as the name implies, a colour image of a frame, line or scanning element is transmitted successively and the eye inertia is used for obtaining a fused colour picture. In systems with simultaneous transmission all the three single-colour images are transmitted concurrently and then are combined into a single colour picture optically or electrically.

Each of these systems has advantages and disadvantages. Of the systems with sequential transmission attention should be given to a system with alternating transmission of single-colour pictures, since

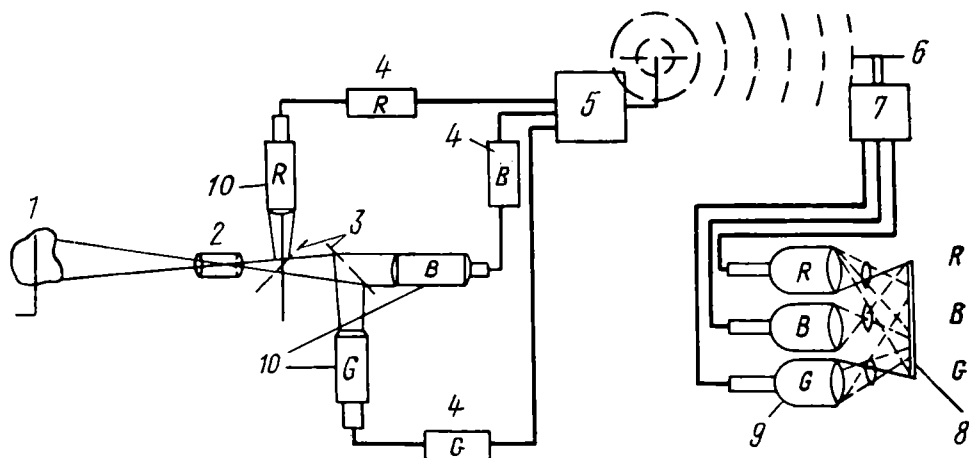


Fig. 9.17. Transmission and reception of colour picture

this system may be equipped with an ordinary (black-and-white) kinescope whose screen is viewed through a rotating three-colour light filter.

The systems with simultaneous transmission are more common, since, despite their complexity caused by the necessity of combining three pictures into one, they provide higher quality of the image being televised.

A schematic diagram of a modern colour television system may be presented as follows (Fig. 9.17).

Element of scene 1 to be reproduced is projected by lens 2 to a system of mirrors 3 providing selective colour reflection to three camera tubes (*R*, *B*, *G*)¹⁰. Because of the use of light filters the tube *R* receives only red colour of the image, while the tubes *B* and *G* receive the blue and green colours, respectively. After amplifying by amplifiers 4, the red, blue and green video signals are fed to a television transmitter 5 and then into a television aerial. The entire colour information can be transmitted through a single television channel using an adequately wide frequency band.

The colour signal received by an aerial 6 of a TV receiver 7 is divided into three video signals and after amplification is fed to three kinescopes 9 whose screens feature red, blue and green colours of

luminescence, respectively. The three single-colour (red, blue and green) images are combined into a single colour picture by means of lenses or mirrors.

The above simplified diagram shows that the separation of the colours at the transmitting side presents no substantial difficulties. Moreover, the camera tubes do not differ from ordinary tubes used in black-and-white television provided that the photocathode of each of the three tubes has high sensitivity to the red, blue and green colours, respectively. On the other hand, at the receiving side the matching of the three pictures is much more complex. Hence, the systems with three separate kinescopes have not accepted wide recognition.

Kinescopes providing for a colour picture directly on their screens have been found to be more promising. Many distinct kinescopes allowing a colour picture to be obtained on a single screen have been proposed. Some types have immediately proved to be unacceptable either because of poor quality of the picture, complex manufacture or operational difficulties. Satisfactory results have been obtained only with kinescopes having a three-colour screen. Such a screen contains three phosphors, each having blue, red and green colour of luminescence, respectively. The luminescence can be excited either by a single electron beam successively exciting each colour of the screen or by three electron beams each of which exciting only one phosphor.

The simplest colour tube is represented by a kinescope with a single gun and a magnetic deflection system forming an ordinary television raster on the screen. A peculiar feature of the tube is a special line filter and an additional electrostatic deflection system. The line filter is a row of strips of phosphor, each strip, when excited by the electron beam, glowing with a red, blue or green colour. To direct the beam to the phosphor strip necessary at a given moment, use was made of an additional electrostatic system shifting the beam, scanned on the screen by the main deflection system, vertically. Such a tube can be used in the systems with sequential transmission of the image by frames or lines. However, in spite of its simplicity, this system operates unsatisfactorily. First, in order that the beam excites only the required colour, the spot diameter must be smaller than the width of the phosphor strip of the given colour and this calls for good focusing over the entire screen. In the second place it is necessary to have a high deflection accuracy, since even a negligible shift of the beam in a vertical direction results in distortion of the colour.

An improvement of the above-described tube is a three-beam kinescope with a line screen. This kinescope has three electron guns whose beams, when scanned over the screen, run over definite phosphor strips. Thus, the beam of the first gun excites only the

blue strips, the beam of the second gun—the red strips and the beam of the third gun—the green strips of the screen. Such a system is suitable for simultaneous reproduction of the three colours. Therefore, the brightness of the picture is increased correspondingly. Nevertheless, the difficulties associated with good focusing over the entire screen and with high deflection accuracy are inherent in this kinescope either.

Widely used in present-day colour television receivers is a kinescope with a shadow mask which to a considerable degree is free from the disadvantages intrinsic in the other types of colour TV tubes. In this kinescope the cross-sectional area of the beam is

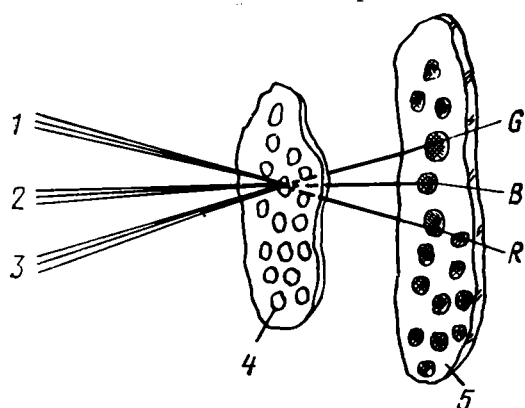


Fig. 9.18. Screen unit of shadow-mask kinescope

1—beam from red gun; 2—beam from blue gun; 3—beam from green gun; 4—mask; 5—screen

limited in the direct vicinity to the screen and penetration of the beam to a “strange” phosphor is practically eliminated.

The principle of operation of a kinescope with a shadow mask is as follows (Fig. 9.18).

The electron beams from three independent guns 1, 2, 3 deflected at any angle are directed into one of many apertures of a metal plate—so-called shadow mask arranged in front of the screen.

Having passed through the same hole in the mask, the electron beams impinge upon three differently coloured dots on the screen spaced at some distance from each other at the apices of a triangle. It is clear that the position of the traces of each beam on the screen is determined by the angle of incidence of the beam and the distance between the mask and the screen. Applied on the screen are separate dots of red, blue and green phosphors forming triangular groups. The arrangement of the phosphor points is selected so that the beam of each gun, having passed through the hole of the shadow mask, can fall on the phosphor dot glowing with a “proper” colour. At any deflection angle the beam of the blue gun excites only the blue phosphor, the beam of the red gun—only the red phosphor, and the beam of the green gun—only the green phosphor.

All three beams are deflected by a common deflection system. Therefore, if the beams are brought into one hole of the mask at any point of the screen, bringing the beams into any other hole of the mask at any deflection angle can be made with sufficient accuracy provided that the physical relations are selected correctly. The location of the phosphor dots on the screen prevents the possibility of exciting a colour that does not correspond to the given gun.

The three video signals controlling the luminance of each colour are fed to the modulators of the three guns. With a sufficiently large number of holes (apertures) in the mask and a corresponding number of colour phosphor dots (three times the number of the holes) the line or dot structure of the raster is quite imperceptible. Such a kinescope makes it possible to obtain high-quality pictures both for the sequential and simultaneous transmission of the colours.

A kinescope with a shadow mask has some disadvantages. First, it should be noted that the mask and screen are complex in manufacture, since, in order to obtain a good image, it is necessary to have at least 500-600 lines and, correspondingly the number of holes in the mask is equal to 300-400 thousand, while the number of phosphor dots is greater than 10^6 . The electron-optical system

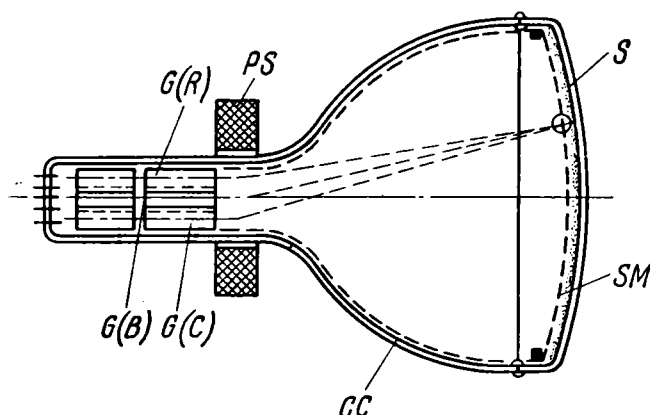


Fig. 9.19. Shadow-mask kinescope

is also very complex and requires additional "converging" elements. Absorption of electrons by the mask is a principal disadvantage of the kinescope with a shadow mask.

Some methods of reducing the loss of electrons in the mask were proposed. In particular a proposal was made to provide additional focusing grids. However, these methods have found no practical application.

A kinescope with a shadow mask is shown in Fig. 9.19.

The kinescope bulb, as usual, consists of three parts: a cylindrical part (neck), a conical part and a face (screen). Since the neck contains three guns, its diameter is greater than that of ordinary single-gun kinescopes. Connected to a cylindrical part is another part of the bulb widening to the size of the screen. This part is conical (in circular kinescopes) or has a shape of a truncated four-face pyramid with convex faces and rounded ribs (in kinescopes with rectangular screens).

The bulb screen has a shape of a shallow cup and is made separately. The phosphors are applied to the slightly convex face of

the bulb, while the shadow mask is carried by special supports in the sides. The mask is made of a thin (0.2 to 0.25 mm) metal sheet and also has a slightly convex shape. In the earlier circular kinescopes the mask was flat while the phosphors were applied to a separate glass plate and the entire screen set consisting of a flat screen and a mask was mounted in the bulb.

A kinescope with a shadow mask has three similar guns producing three electron beams with a current of a few dozens of microamperes at an accelerating voltage of up to 25 kV. Since the axes of the guns do not coincide with the axis of the tube, the magnetic focusing by means of a coil put on the tube neck cannot be used. Hence, kinescopes with three guns employ electrostatic focusing.

Normal operation of a colour kinescope requires that all the three guns have sufficiently close modulation characteristics, since only in this case it is possible to obtain correct reproduction of colours at varying general brightness of the picture. This calls for good workmanship of the gun. The axes of the guns are tilted to the tube axis so that axes of the guns intersect at the centre of the mask. But even in the use of tilted axes of the guns "converging" electrostatic or magnetic fields are necessary to ensure accurate convergence of all the three beams into a single point. Therefore, the tilted position of the axes of the guns is of no importance.

When using a colour kinescope with three electron guns, it is necessary that all the three beams are deflected identically. To this end use should be made of a deflection system common for all the beams. The simplest deflection system is a magnetic coil producing a transverse field sufficiently uniform in the region of deflection of all the three beams. A large region of uniform field can be provided by means of great coils without cores with bent edges arranged on the tube neck.

The three beams are converged into a single hole of the mask either by means of an electrostatic lens formed between the last accelerating electrode and the electrically conductive coating or (what is more often) by using three deflecting magnetic fields produced by three coils on the tube neck. The coils are provided with ferromagnetic cores, while located inside the tube are internal magnetic circuits producing a uniform magnetic field shifting the beam radially (Fig. 9.20).

The converging electrostatic lens or converging magnetic fields provide for passage of all the three beams through one hole of the mask in the centre of the screen. However, even in case of an ideal deflection system, the passage of the three beams through one hole at any area of the mask is disturbed at used deflection angles due to variations in the distance from the deflection centre to the mask surface at different angles. Therefore, additional measures should be taken in colour three-beam kinescopes to provide for the so-called

dynamic convergence, i.e. convergence of the three beams into one hole of the mask at any beam of deflection angles within the operating zone of the screen. When an electrostatic converging lens is used, the dynamic convergence is obtained due to the fact that the converging electrode is fed with a voltage modulated by the line and frame scan voltages. A change in the optical strength of the converging lens causes a change in the angles between the axes of the beams and with correct selection of the amplitudes of the modulating voltages provides for converging all the three beams

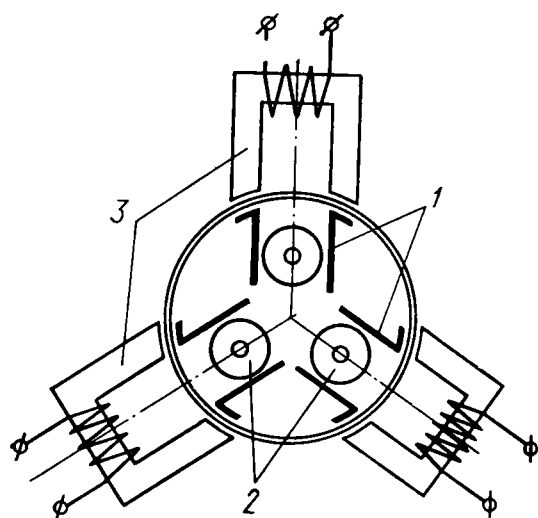


Fig. 9.20. Converging system of colour kinescope

1—pole pieces; 2—second anodes of guns;
3—converging magnets

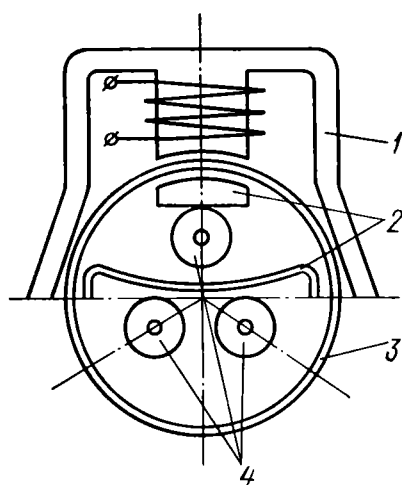


Fig. 9.21. Blue beam correction system

1—external magnetic circuits; 2—internal magnetic circuits; 3—tube neck; 4—second anodes of guns

into one hole of the mask. When a magnetic field is used for converging the beams, the dynamic convergence is obtained by varying the current in the converging coils synchronously with the line and frame scans. However, shift of the beams in a radial direction is insufficient for provision of the convergence of all the three beams in one hole of the mask. To this end at least one beam must be moved at right angles to the radial direction. This is performed by an additional coil with external and internal magnetic circuits which produces a radial magnetic field (Fig. 9.21), and the custom is to utilize this upon the blue beam.

The shadow mask is made so as, on the one hand, to prevent completely the electron beam of each gun from hitting dots of a "strange" colour and, on the other hand, to pass the greatest possible amount of electrons. The latter requirement is accounted for by the necessity to have a sufficient brilliance of the screen which generally is proportional to the energy of the electrons passed through the

mask. At the same time, considerable attenuation of the electrons by the mask is undesirable due to a high load of the mask leading to its overheating and deformation because of thermal expansion.

The amount of electrons passed by the mask is estimated by the mask efficiency defined as the ratio of the screen current I_s to the beam current I_b

$$\eta = \frac{I_s}{I_b}, \quad I_b = I_s + I_m \quad (9.3)$$

It is evident that the greater part of the mask surface occupied by the holes, the higher the efficiency is; however, the hole diameter cannot be greater than $d_{\max}$ equal to

$$d_{\max} = \frac{a}{3} \left(\sqrt{3} - \frac{d_b}{\varepsilon} \right) \quad (9.4)$$

where a is the distance between the centres of the holes; d_b is the beam diameter in the deflection plane; ε is the eccentricity, i.e.

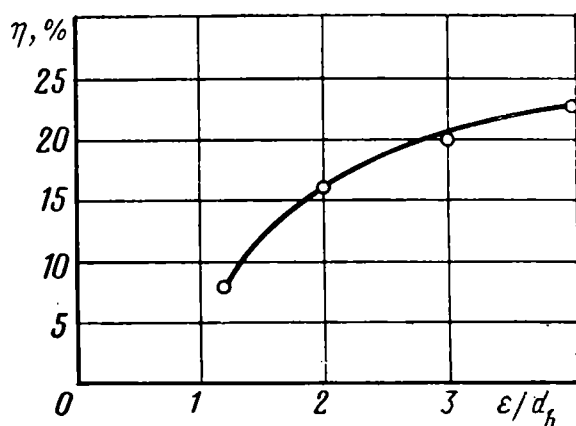


Fig. 9.22. Mask efficiency versus the ratio of eccentricity to beam diameter

the distance from the beam in the deflection plane to the tube axis.

If the hole diameter is greater than the value of $d_{\max}$, the spots on the screen can overlap each other.

The number of holes in the mask is determined by the television resolution standard. Thus, knowing the screen diameter and the number of resolution elements, one can find the distance between the centres of the holes. Then the maximum possible diameter of the mask holes will depend only on the d_b/ε ratio. In this case the mask efficiency is described by

$$\eta = \frac{I_s}{I_b} = \frac{\pi}{8\sqrt{3}} \left(\sqrt{3} - \frac{d_b}{\varepsilon} \right)^2 \quad (9.5)$$

The dependence of the efficiency of the mask on the ratio of the eccentricity to the beam diameter in the deflection plane is given in Fig. 9.22.

It is seen from the diagram that as the ratio ε/d_b rises, the efficiency increases rather fast, but at $\varepsilon/d_b \geq 3$ the change in the efficiency becomes less pronounced.

The mask efficiency is the higher, the greater the eccentricity and the lower the beam diameter in the deflection plane. However, an increase in the ratio ε/d_b meets some difficulties. The greater the distance between the axes of the guns and the axis of the tube, i.e. the greater the eccentricity, the more difficult is the provision of the dynamic convergence under otherwise equal conditions. Furthermore, with a great distance between the beams the region of a uniform magnetic field produced by the deflection system must be very large and this is difficult to obtain.

Reduction of the beam diameter in the deflection plane complicates the electron gun. With electrostatic focusing a beam with a diameter less than 1 mm cannot be usually obtained directly at the gun exit at very small currents. On the contrary, to obtain the minimum cross-sectional area of the beam in the mask plane, it is good practice to have a beam convergence angle which is not too small (see Section 3.1). Therefore, practically the ratio ε/d_b ranges from 2 to 3 which corresponds to an efficiency of 15-20%.

The screen of a colour kinescope with a shadow mask consists of separate phosphor dots glowing respectively in three fundamental colours: red, blue and green. The number of phosphor dots is three times the number of the holes in the mask. Generally the diameter of phosphor dots must exceed the diameter of the holes in the mask. An increase in the dot diameter is expedient in view of the following two reasons. With a properly focused gun the electron beam has the smallest diameter in the plane of the mask and widens to some extent at either side of the mask. Although the distance between the mask and screen is small the spot has a diameter exceeding that of the mask hole due to expansion. For maximum utilization of the beam energy it is necessary that the phosphor dot be not lower in diameter than the trace of the electron beam, i.e. the spot. Since the screens of colour kinescopes are aluminized, the electrons are slightly scattered in the aluminium film and this also may lead to an increase in the spot diameter.

However, an excessive increase in the diameter of the phosphor dots is also undesirable. Should all the phosphor points be applied without intervals, any inaccuracy in the assembly of the guns and the mask-screen system may result in penetration of the electron beam to a "strange" phosphor thus causing distortion of the colour picture.

The red phosphor used in colour kinescopes consists either of zinc phosphate activated by manganese or zinc selenide activated with copper. The green phosphor consists of silver-activated willemite or zinc sulphide-selenide. The blue phosphor consists of silver-

activated zinc sulphide. Separate dots of phosphor are applied by the photographic method. A solid layer of phosphor of one colour is deposited on the screen. Then a layer of a photosensitive emulsion is applied to the phosphor. The phosphor may be also preliminarily mixed with a photosensitive material, this mixture then being applied to the screen. Such a light-sensitive screen is illuminated by a point light source through the mask in the operating position. The light source is placed in the centre of deflection of the electron beam of the corresponding gun. After the exposure, the photosensitive layer is developed and fixed so that the phosphor is fixed in the photographic emulsion where it is illuminated. The areas of the photographic emulsion, which had not been exposed to light, are easily washed off, including the phosphor. After that a new solid photosensitive layer with the phosphor of another colour is applied to the screen and the exposure is effected through the mask. In this case, however, the point light source is placed in the deflection centre of the other beam and the development and fixing are carried out again. The third phosphor is applied in the same way.

The photographic method of making the screen provides for the passage of electron beams through the holes of the mask only to the points of "proper" phosphor under the condition of precise alignment of the deflection centres of the beams with the position of the point light source. However, even a minute misalignment in the assembly of the guns and the kinescope itself may result in penetration of electron beams to a "strange" phosphor thus distorting colour reproduction and breaking the purity of colour. The electron beam is brought to the position excluding the illumination of the "strange" phosphor by means of the so-called colour purity magnet producing a uniform magnetic field transverse to the tube axis. This magnet consists of a coil with a magnetic circuit encircling the tube neck in the space between the guns and the deflection system. By adjusting the position of the colour purity magnet, one can shift the electron beams in such a way that each beam is directed to the "proper" phosphor.

The colour kinescopes with a shadow mask are rather complex tubes both in making and operation. Consequently, in spite of wide application of such tubes, investigations are continuously carried out with a view to developing more simple and reliable colour kinescopes. In particular, a single-gun tube with a strip-type screen and a focusing grid called the chromatron is a simple and promising device in the field of colour television. The screen of this tube consists of a number of vertical phosphor strips applied in the following succession: red, green, blue, green, red, etc. Mounted in front of the screen is a grid of vertical wires, the latter being stretched in parallel to the screen surface strictly opposite to the strips of red and blue phosphors. The grid wires are interconnected alternately,

i.e. the grid has two terminals, one connected to all the wires located opposite the red phosphor and from the wires coinciding with the blue phosphor (Fig. 9.23).

The grid wires are at a constant positive (with respect to the gun anode) potential of a few kilovolts. The screen is provided with an aluminium coating at a potential, which is 10-12 kV higher than the grid potential. Due to the potential difference between the grid and the screen formed between the grid wires are converging cylindrical lenses additionally focusing the electron beam. The

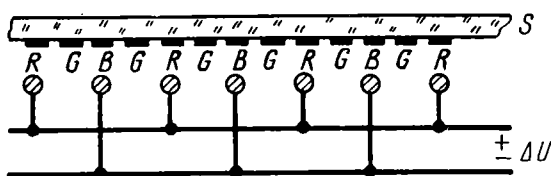


Fig. 9.23. Screen unit of chromatron

physical relations and the potential difference are selected so that the smallest cross-sectional area focused by the cylindrical lenses is aligned with the screen surface. The electrons of the beam converge to the strips of the green phosphor and excite its luminescence due to this additional focusing.

If a potential difference of $\pm \Delta U$ of the order of several hundreds of volts is produced between the adjacent wires of the grid, the beam electrons are subjected to transverse acceleration and strike either red or blue strips of phosphor, depending on the polarity of the deflecting voltage ΔU . Thus, feeding the gun modulator with a video signal comprising an information of the red, green and blue components of the image and simultaneously switching the deflecting voltage one can obtain on the screen alternately the red, green and blue images which merge into one colour picture at a sufficient frequency of the switching due to the persistence of vision.

A chromatron is a kinescope with sequential transmission of colours. Therefore, to obtain the required luminance, one must employ a comparatively high beam current (higher than 100 μA) and accelerating voltages of the order of 20 kV. However, due to the simplicity of this device (a single gun, a simple magnetic deflection system, the absence of additional converging and hue magnets) the development of the chromatron type tubes is effected in many laboratories both in this country and abroad.

The chromatron is used as the basis for development of three-gun television tubes. These kinescopes, like a chromatron, employ a strip-type screen and a grid playing the role of a shadow mask directing the beams of each of the three guns to the strip of the "proper" phosphor.

An interesting modification of a colour kinescope with a strip-type screen is a tube known as a "trinitron".

The gun of this kinescope (Fig. 9.24) has three cathodes C_1 , C_2 and C_3 arranged side by side in a horizontal plane and a single common modulator M with three apertures. Located behind the modulator is a wide short cylinder or a first accelerating electrode AE_1 at a positive potential of several hundreds of volts. The three electron beams shaped by an immersion lens are focused by a pre-focusing lens (immersion lens) formed between the first and second accelerating electrodes AE_1 and AE_2 , respectively. The optical strength of this lens is selected so that the outer beams are deflected to the axis and intersect the medium beam approximately in the

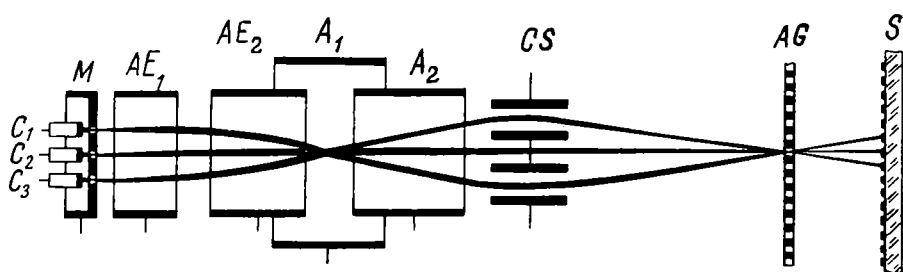


Fig. 9.24. Diagram of trinitron

centre-plane of the main projection lens (single lens) formed by the second accelerating electrode, the first anode A_1 and the second anode A_2 .

Arranged behind the second anode is an electrostatic converging system CS consisting of four parallel plates. The medium plates between which passes the medium beam are electrically interconnected and are at the potential of the second anode of the gun. The outer plates are also interconnected and are negative with respect to the second anode. The transverse field thus formed deflects the outer beams to the axis and they intersect the medium beam in the plane of the aperture grid (mask) AG made in the form of a metal plate with vertical slots. Mounted behind the aperture grid is a screen S coated with vertical strips of phosphors glowing in green, red and blue colours. All the three beams are deflected (scanned over the screen) by a single magnetic deflection system mounted on the tube neck. The video signals are fed to the gun cathodes (cathode modulation), while the modulator is earthed.

In contrast to a single-gun chromatron, the trinitron is a colour kinescope with simultaneous transmission of colours and this makes it possible to obtain high luminance at comparatively small beam currents. As compared to a three-gun kinescope with a shadow mask, the trinitron has some advantages. The aperture grid has much better transparency than the mask and makes it possible to use

the beam currents more efficiently. The location of the three beams in a single plane simplifies the dynamic convergence and hue control. The presence of only one gun makes it possible to reduce the diameter of the tube neck and, therefore, to increase the efficiency of the magnetic deflection system. The trinitron has other advantages. However, this tube is complex in construction and manufacture and so far has gained less application than a three-gun kinescope with a shadow mask.

A colour picture can be obtained on a large screen by means of three projection kinescopes having screens respectively with red, green and blue colours of luminescence. The "red", "green" and "blue" information displayed on the screens is combined on a large screen by optical means. Since all the three colour pictures are projected simultaneously, it is possible to obtain adequate luminance on the screen having the size of 3×4 m. The disadvantage of this system consists in complex optical coincidence of the three pictures and very rigorous requirements to the deflection systems of the projection kinescopes to provide strict identity of the rasters on the screens of the three tubes. A sufficiently bright colour picture on a large screen is obtained by means of the types "Eidophor" television projector provided with a rotary colour light filter. Although in this case the colours are reproduced in succession, high power of the light source allows good pictures to be obtained on a screen having an area of up to 40 m^2 .

Image Converter Tubes and Image Intensifiers

10.1. General Principles of Image Conversion and Intensification

An electron-optical image converter is a device intended for conversion of an optical image from one (usually invisible) into another (visible) region of the spectrum. Image intensifiers are used for intensification of the brightness of an optical image without perceptible varying the spectrum region. The image converters and intensifiers are based on double conversion, a light image into an electronic one and an electronic image into a visible light image. The light image is converted into an electronic image by means of a photocathode which has sufficient sensitivity in the spectrum region used in the given image converter. The electronic image is converted into a light image by means of a screen coated with a phosphor effectively transforming the kinetic energy of the electrons into light radiation.

Using the double conversion one can transfer an image from one spectrum region to another and to obtain considerable gain in brightness. Of course, the photocathode and screen have efficiency lower than unity. However, by accelerating the electrons between the photocathode and the screen by means of an electrostatic field, it is possible not only to completely compensate for the energy loss on the photocathode and screen but also to obtain a material brightness gain.

Simple calculations allow us to find the photocathode illuminance that can cause noticeable glowing of the screen of the image converter. Assume that all the electrons emitted by the photocathode reach the screen. Let us denote the photocathode area S_p , the screen area S_s , the accelerating voltage U_{ac} , the photocathode sensitivity k_p and the luminous efficiency of the screen through η . Let us calculate the dependence of the screen brilliance B on the photocathode illuminance E . The photocathode current is

$$I_p = k_p \Phi = k_p S_p E \quad (10.1)$$

where Φ is the light flux incident on the photocathode.

The power carried by the electrons to the screen at the accelerating voltage U_{ac} is

$$P_s = I_p U_{ac} = k_p S_p E U_{ac} \quad (10.2)$$

The screen brilliance is determined as the light intensity (J) of the screen related to its surface

$$B = \frac{J}{S_s} = \frac{\eta P_s}{S_s} = k_p \eta E U_{ac} \frac{S_p}{S_s} \quad (10.3)$$

The quantities in expression (10.3) have the following dimensionalities: k_p in $\mu\text{A}/\text{lm}$, η in cd/W , E in lx , and U_{ac} in V . In order to express the brilliance in cd/m^2 , it is necessary to use a conversion factor equal to 10^{-6} . Thus, the screen brilliance

$$B = 10^{-6} k_p \eta E U_{ac} \frac{S_p}{S_s} [\text{cd}/\text{m}^2] \quad (10.4)$$

Expression (10.4) makes it possible to estimate the minimum illumination of the photocathode at which the glowing of the screen can be noticed. A well adapted human eye can distinguish an illuminated area with brightness of $10^{-5} \text{ cd}/\text{m}^2$ against an absolutely black background. The sensitivity of photocathodes illuminated by direct light is equal to tens of $\mu\text{A}/\text{lm}$. Let it be taken for $50 \mu\text{A}/\text{lm}$. The maximum luminous efficiency of the phosphor may be equal to $10 \text{ cd}/\text{W}$. In this case, with $U_{ac} = 10 \text{ kV}$ the brilliance of $10^{-5} \text{ cd}/\text{m}^2$ on the screen ($S_s = S_p$) will be obtained at

$$E = \frac{B \times 10^6}{k_p \eta U_{ac}} = \frac{10^{-5} \times 10^6}{50 \times 10 \times 10^4} = 2 \times 10^{-6} \text{ lx}$$

The obtained result gives the minimum value of illumination of the photocathode. Of practical interest is the minimum illuminance of a scene which can be still distinguished by means of an image converter. The illumination of the scene is

$$E = \pi B_0 \sin^2 \frac{\Theta}{2} \quad (10.5)$$

where B_0 is the scene brightness; Θ is the aperture angle of the lens.

If the scene surface is an ideally dispersing plane, the magnitude πB_0 is the illumination of the scene in luxes. The aperture angle Θ for high-transmission lenses is equal to 90° . Then $\sin^2 \frac{\Theta}{2} = 0.5$ and at $E = 2 \times 10^{-6} \text{ lx}$ the minimum illumination of the scene is expressed by

$$\pi B_0 = \frac{E}{\sin^2 \frac{\Theta}{2}} = \frac{2 \times 10^{-6}}{0.5} = 4 \times 10^{-6} \text{ lx}$$

The above-given calculations yield a smaller value of the really distinguishable minimum illumination of the scene. Under real conditions with completely dark photocathode the screen weakly glows due to low thermal and autoelectronic emission of the cathode. Of course, in this case the minimally distinguishable brightness

of the image will be far above 10^{-5} nit and, therefore, the threshold illumination of the scene being observed must be increased correspondingly.

The efficiency of the image conversion is characterized by the radiant flux conversion coefficient G_p determined as the ratio of the light flux Φ_s emitted by the screen of the image converter to the light flux Φ_c incident on the photocathode

$$G_p = \frac{\Phi_s}{\Phi_c} \quad (10.6)$$

Assuming that the spectral composition of the screen radiation is similar to the spectral composition of the flux, received by the photocathode (which is approximately observed in brightness intensifiers), it is possible to calculate the magnitude of the radiant flux conversion coefficient. Assume that a radiant flux Φ_c is incident on a photocathode. With the cathode sensitivity equal to k_p the photocurrent of the cathode is

$$I_p = k_p \Phi_c \quad (10.7)$$

and the power transmitted to the screen by the photoelectrons accelerated in the image converter by a potential difference U is

$$P_s = I_p U = k_p \Phi_c U \quad (10.8)$$

With the energy efficiency (energy output) of the screen equal to η_s (assuming that η_s is independent of U) the flux radiated by the screen is

$$\Phi_s = \eta_s P_s = \eta_s k_p \Phi_c U \quad (10.9)$$

and the radiant flux conversion coefficient is

$$G_p = \frac{\Phi_s}{\Phi_c} = k_p \eta_s U \quad (10.10)$$

As is seen from expression (10.10), the conversion coefficient is the higher, the higher the photocathode sensitivity, the screen efficiency and the accelerating voltage. In modern image converters equipped with multislit photocathodes and sulphide phosphors and operating at accelerating voltages of 10 kV the conversion coefficient is measured in tens, i.e. the luminous flux can be intensified by tens of times.

When estimating the efficiency of brightness intensifiers, it is expedient to introduce an image intensification coefficient G_B determined as a ratio of the brightness B_s of the image on the screen of the converter to the brightness B_0 of the object

$$G_B = \frac{B_s}{B_0} \quad (10.11)$$

Let us assume that an object (scene) r_0 is projected by a lens with an aperture D onto a photocathode PC of a brightness intensifier (Fig. 10.1). The image converter transmits the image r_c of the photocathode to a screen S . It is clear that the size of the image on the screen is equal to

$$r_s = M_s r_c = M_s M_l r_0 \quad (10.12)$$

where M_l and M_s are linear magnifications of the lens and the electron-optical system.

Let us isolate an elementary area Δs_0 on the object surface so that it is located on the axis of the optical system. Assume that

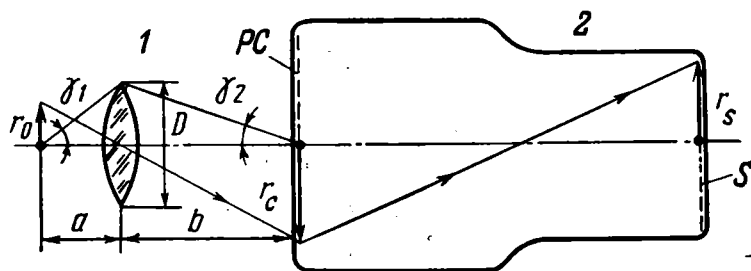


Fig. 10.1. Image conversion
1—lens; 2—image converter tube

the object surface satisfies the Lambert law. In this case the flux incident to the photocathode elementary area Δs_c is equal to

$$\Delta \Phi_c = \pi \tau B_0 \Delta s_0 \sin^2 \gamma_1 \quad (10.13)$$

where τ is the transmission coefficient of the lens; γ_1 is the aperture angle from the scene side.

Using the Lagrange—Helmholtz theorem [see (1.152)], one can write

$$\Delta s_0 \sin^2 \gamma_1 = \Delta s_c \sin^2 \gamma_2 \quad (10.14)$$

where γ_2 is the aperture angle from the photocathode side.

Determining $\sin^2 \gamma_2 = \frac{D^2}{D^2 + 4b^2}$ from the geometric relations (see Fig. 10.1), we obtain

$$\Delta \Phi_c = \pi \tau B_0 \Delta s_c \frac{D^2}{D^2 + 4b^2} \quad (10.15)$$

The image of the area Δs_c of the photocathode is transferred to the screen by an electron-optical system. In this case the flux radiated to the rear semisphere by the screen area Δs_c is equal to

$$\Delta \Phi_s = \Delta \Phi_c G_p = \pi \tau B_0 G_p \Delta s_c \frac{D^2}{D^2 + 4b^2} \quad (10.16)$$

the screen brightness is

$$B_s = \frac{\Delta \Phi_s}{\pi \Delta s_c M_s^2} = \frac{\tau G_p D^2}{(D^2 + 4b^2) M_s^2} B_0 \quad (10.17)$$

and the image intensification coefficient

$$G_B = \frac{B_s}{B_0} = \tau \frac{D^2}{D^2 + 4b^2} \frac{G_p}{M_s^2} \quad (10.18)$$

Using the relations of geometric optics, the value b in the denominator of expression (10.18) can be expressed through linear magnification M_l and the focal length f of the lens

$$b = (1 - M_l) f \quad (10.19)$$

Using (10.19), we shall obtain the following expression for the brightness intensification coefficient

$$G_B = \frac{\tau D^2}{D^2 + 4(1 - M_l)^2 f^2} \frac{G_p}{M_s^2} \quad (10.20)$$

Employing low aperture angles (intensification of images of remote objects), formula (10.20) can be simplified to

$$G_B \approx 0.25\tau \left(\frac{D}{f} \right)^2 \frac{G_p}{M_s^2} \quad (10.21)$$

where D/f is the objective aperture ratio.

With an aperture ratio of 1 : 2 the error in determining G_B by formula (10.21) is equal to approximately 5%. When $(D/f) \leq 1/5$, the error becomes less than 1%.

As is seen from the obtained expressions, the image intensification is determined both by the converter itself (G_p , M_s) and the light-optical system (τ , D/f). Using high-transmission lenses with a transmission coefficient of at least 0.8 and image converters having $G_p = 20$ to 30 and $M_s < 1$, the image brightness can be intensified several times.

One of the decisive parameters of image conversion is resolution. The resolution of image converters is usually estimated by a number of alternating light and dark lines (primes) occupying 1 mm of height of a test pattern (strip mira) which are seen separately (not merged) when the pattern image is transferred to the screen. For separate observation of adjacent lines it is necessary to have contrast of 5% or a contrast transmission coefficient of at least 0.05.

The image converter channel includes light-optical systems such as a lens projecting the image on the photocathode (sometimes a second light-optical system through which the image on the screen is observed) and an image converter itself consisting of a photocathode, an electron-optical system and a screen. The resolution of the entire system depends on each element of the conversion channel.

Every element of the conversion channel can be estimated by its "contribution" to the resolution ability of the whole system, using the so-called frequency-contrast characteristics. The frequency-contrast characteristic is a curve expressing the dependence of the

contrast transmission coefficient $F(N)$ on the spatial frequency of alternation of the dark and bright strips N . With an increase in the number of lines per 1 mm of height of the object the possibility of their separate observation is reduced (differently for various elements of light and electron-optical systems). As an example, Fig. 10.2 gives the frequency-contrast characteristics of some light and electron-optical elements.

As is shown in the drawing, the photocathode reduces the resolution less than the optical and the electron lenses. The frequency-contrast

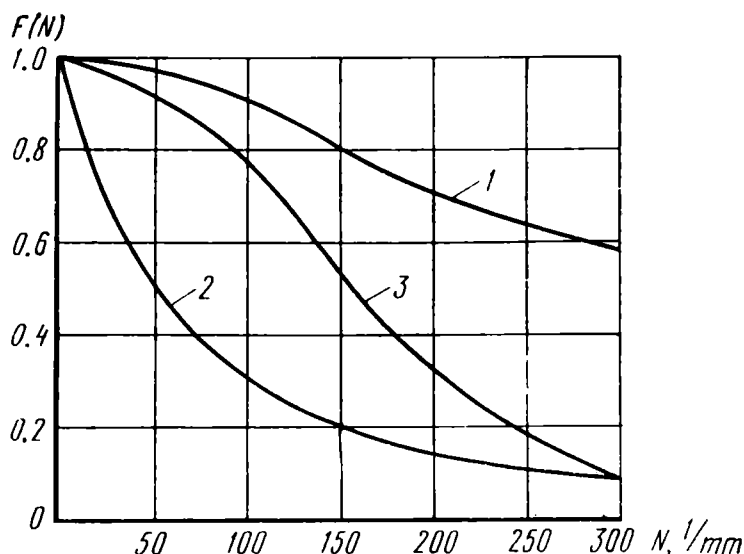


Fig. 10.2. Frequency-contrast characteristics
1—photocathode; 2—lens; 3—electron lens

characteristics can be calculated analytically or found experimentally.

If the frequency-contrast characteristics of separate elements of the conversion channel are known, the points of the common (resultant) frequency-contrast characteristics can be found by multiplying the ordinates of the corresponding points of the entire frequency-contrast characteristic of all the elements. The resultant characteristic of the conversion system can be used for graphical representation of the converter resolution. Assuming that separate lines can be distinguished at contrast of 5% [$F(N) = 0.05$], the resolution of the system is determined by the abscissa of the point of intersection of the straight line drawn in parallel to the abscissae axis at a level of 0.05 with the resultant frequency-contrast characteristic.

The requirements placed on an image converter or an image intensifier vary with their field of application. For example, image converters intended for conversion of invisible (usually infrared) image

into a visible one must provide the required sensitivity of the photocathode in the employed region of the spectrum. One of the most important parameters of image converters and image intensifiers is the threshold sensitivity allowing one to estimate the minimum illumination of an object at which it is possible to obtain a satisfactory image on the screen. Approximate estimation of the threshold sensitivity is discussed in this section. In some cases, in addition to high resolution, high contrast of the image is dictated. Finally, taking into account that electron image converters are often used in mobile stations, it is desirable that the overall dimensions and weight of the devices be low and that they consume moderate power. The last requirement in many cases makes the designer to reject magnetic electron-optical focusing systems.

10.2. Electron-optical Systems of Image Converters

The purpose of the electron-optical system of an image converter is to transfer an electron image from the photocathode to the screen with minimum geometric distortions. During this transfer the electrons emitted by the photocathode must acquire energy sufficient for excitation of bright luminescence of the phosphor. The transfer of the image must be performed either without changing the dimensions or with a slight reduction in its size. The enlargement is inexpedient in view of the following reasons. First, the image brightness is inversely proportional to the square of enlargement ($B \sim \frac{s_p}{s_s} = \frac{1}{M^2}$). Second, an increase in the screen increases the overall dimensions of the image converter and this is undesirable. Third, large images require wider electron beams which result in higher aberrations. Consequently, an electron-optical system of an image converter is designed so as to obtain a reduced (compared with the photocathode) image. If necessary, the image can be enlarged by optical means.

The image can be transferred by various electron-optical systems from the simplest (uniform longitudinal electrostatic and magnetic fields) to sophisticated electrostatic and magnetic lenses with reduced aberrations. Of course, a purely magnetic system is not used, since the electrons must be accelerated in the process of their transfer for which purpose an electrostatic field must be used.

In the simplest image converters the image is transferred by means of uniform longitudinal fields, i.e. the electronic image converter has no electron lenses.

If the cathode and screen are parallel planes, the field between them may be considered as uniform (neglecting the edge distortions). Such a system would be an ideal converter free of aberrations. It is true that the magnification factor of such a system would be equal

to unity. But an ideal image would be obtained under the condition of photoemission of electrons at zero initial velocities. An actual photocathode emits electrons with initial velocities in the range from 0 to $v_{0\max}$. The initial velocities depend on the yield of the photocathode, the value of light quanta and the photocathode temperature. Expressing the initial velocity through the equivalent potential difference u_0 , we may show, that in the case of a uniform electrostatic field, each point of the photocathode is represented on the screen by a scattering circle with a radius

$$r = 2l \sqrt{\frac{u_0}{U_{ac}}} \quad (10.22)$$

where l is the distance between the cathode and the screen; U_{ac} is a potential difference accelerating the electrons.

It is clear that expression (10.22) determines the resolution of the converter with a uniform electrostatic field. One can easily see that with practically realizable distances and accelerating voltages the value $2r$ (the diameter of scattering circle) cannot be lower than 0.1 mm, i.e. only ten pair of lines per millimetre can be seen separately.

Placing the simplest converter into an elongated magnetic coil (uniform longitudinal magnetic field), one can increase the resolution approximately to 50 pair of lines per millimetre, i.e. obtain an electronic image converter suitable for practical purposes.

In spite of simplicity and the possibility to obtain an image with adequate brightness and resolution, the converters with uniform electrostatic and magnetic fields have found limited application due to a series of disadvantages. The resolution of the simplest converters with a uniform electrostatic field is insufficient for practical purposes, while the use of a long solenoid is inexpedient due to high energy consumption, large overall dimensions and weight.

The simplest converters are disadvantageous in the presence of optical feedback. The light emitted by the photocathode into the bulb illuminates the photocathode. The light from the screen absorbed by the cathode excites photoemission, while the photoelectrons transferred by the electrostatic field to the screen increase its brightness which, in its turn, increases the illumination of the cathode. Thus, in the presence of optical feedback the screen will glow even without external illumination of the photocathode, i.e. the converter cannot be used for observation of the objects projected by the lens onto the photocathode. The optical feedback can be reduced considerably by placing a diaphragm with a small aperture between the cathode and the screen. However, such a diaphragm cannot be used in the simplest converters since the electrons from the photocathode move practically along the lines of force of the uniform field and

in the presence of a diaphragm the area of the image cannot be larger than the area of the diaphragm aperture.

In principle, it is possible to reduce the optical feedback by selecting the spectral characteristics of the photocathode and phosphor so that the maximum on the spectral characteristic of the phosphor corresponds to the "gap" of the spectral sensitivity of the photocathode. However, this method is impracticable, since the known infrared photocathodes have rather high sensitivity in the visible portion of the spectrum. At accelerating voltages higher than 10 kV a reliable method of reducing the optical feedback is aluminization of the screen, since a thin film of aluminium freely passing the high-speed electrons is practically opaque for light.

The use of electron lenses for transfer of an image makes it possible to eliminate many disadvantages inherent in the simplest converters. Using lenses, one can increase the resolution up to tens and even hundreds of pairs of lines per millimetre (in the centre of the screen) and this is quite sufficient for obtaining a high-quality image. Furthermore, in the presence of lenses the beam has a crossover, in the region of which it is possible to mount a diaphragm with a small aperture thus considerably reducing the optical feedback. It is safe to say that in the converters with lenses the system of transfer of an image is transparent for the electrons flying from the cathode to the screen and almost opaque for the light rays propagating from the screen to the photocathode.

A typical electron-optical image converter is based on a two-lens system. An immersion objective is used as the first near-cathode lens, since an accelerating field is necessary for effective withdrawal of the photoelectrons near the cathode surface. A diaphragm is provided near the crossover formed by the immersion objective. The second lens transferring the image to the screen is an immersion or single lens. An example of an electron-optical converter system with electrostatic focusing is shown in Fig. 10.3.

In this system an immersion objective field is produced in the cathode—first anode space. The first and second anodes form an immersion lens.

Since the use of wide beams is associated with such significant aberrations as distortion and curvature of the image surface, it is expedient that the screen has a concave surface by coinciding its surface with the surface of the best image. However, observation of a picture on a convex (from the observer's side) screen is inconvenient due to the geometric distortions (see Section 7.3). At the same time, modern optical objectives with corrected aberrations provide for a sharp image on a convex (as viewed from the objective) photocathode. Hence, in many cases it is desirable to make the photocathode concave (as viewed from the electron lens) in order to obtain a high-quality picture on a flat screen.

The influence of the scatter of the initial velocities of photoelectrons can be reduced by means of an approximately uniform magnetic field near the photocathode. Such a field is produced by means of a small solenoid placed into which is the near-cathode part of the

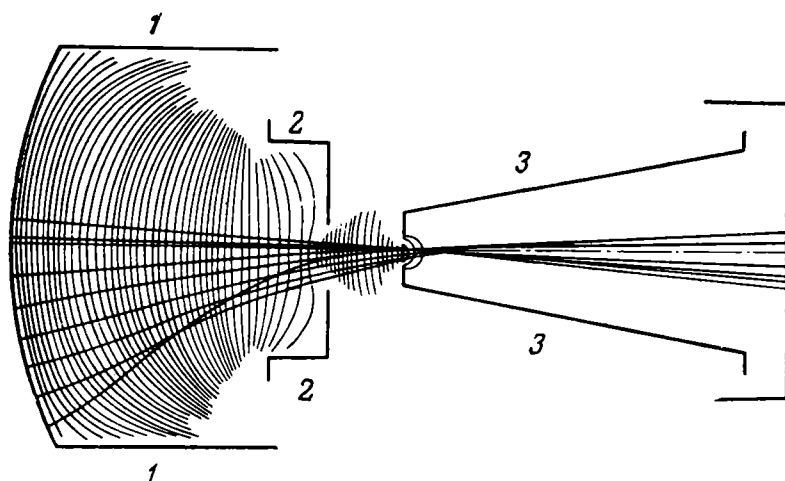


Fig. 10.3. Image converter tube with electrostatic focusing
1—cathode; 2—first anode, 7 kV; 3—second anode, 20 kV

image converter bulb. In this case the electrons emitted by the photocathode are accelerated by the electrostatic field, while the longitudinal magnetic field directs them approximately parallel

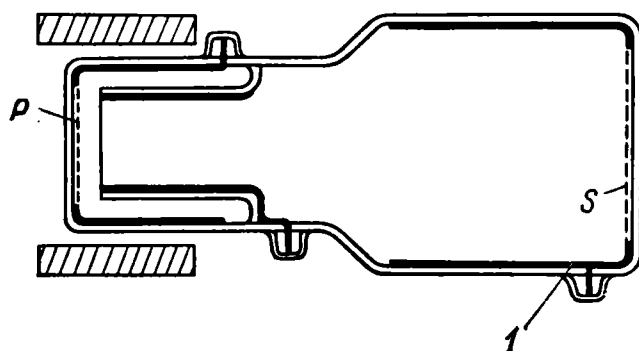


Fig. 10.4. Image converter tube with magnetic focusing
1—metal coating

to the magnetic lines of force. The use of additional focusing by a longitudinal magnetic field in the near-cathode region makes it possible to improve the picture, particularly its edge zone. A schematic diagram of such a converter is shown in Fig. 10.4.

10.3. Types and Constructions of Image Converters and Image Intensifiers

A night vision device, i.e. an electron image converter converting an invisible infrared image into a visible one is a typical example of an image converter. The photocathode of such a converter must have sufficient sensitivity in the long-wave portion of the spectrum. An oxygen-silver-caesium photocathode widely used in such devices features pronounced sensitivity in the infrared region of the spectrum up to wavelengths of 1000-1200 nm. Therefore, many types of image converters intended for night vision are equipped with oxygen-silver-caesium photocathodes. However, insufficient sensitivity of Ag-O-Cs photocathodes forced the specialists to seek new, more

efficient photocathodes. Multislot photocathodes with high sensitivity in the long-wave region of the spectrum having been lately developed, find increasing application in the up-to-date image converters.

In night vision devices the image is usually formed by electrostatic lenses. The use of magnetic focusing is inexpedient due to an increase in the overall dimensions and weight of the device, furthermore, the electric magnets consume considerable power. These factors are very important, since night vision devices are usually used in mobile

equipment. As has been mentioned above, the electron-optical system of the image converter usually consists of two lenses. These are a near-cathode immersion objective and an immersion lens transferring the image to the screen.

Since at a small current in the photocathode the accelerating voltage should be not less than 10-15 kV in order to obtain the required brightness of screen, the screens of image converters can be aluminized. Aluminized screens increase the brightness and contrast and practically eliminate optical feedback. A schematic diagram of an electric converter for a night vision device is shown in Fig. 10.5, and its general view (X-ray photograph) is depicted in Fig. 10.6.

The brightness can be increased by using after-acceleration of the electrons near the screen similarly to the after-acceleration used in oscillograph tubes (see Section 7.7). An electron image converter using after-acceleration is shown in Fig. 10.7.

The sides of the bulb of this converter (near the screen) are provided with a ring of an electrically conductive coating having a sepa-

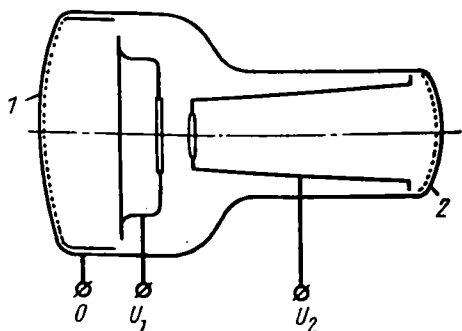


Fig. 10.5. Image converter tube with electrostatic focusing (for night vision)

1—photocathode; 2—screen

rate terminal serving as an electrode for after-acceleration of the electrons. This electrode is at a potential 2 to 2.5 times the potential of the second anode. The use of after-acceleration makes it possible to obtain sufficient brightness of the image at a decreased illumination of the photocathode, i.e. with a sensitivity gain. It should be noted that the immersion lens formed near the screen causes aberrations

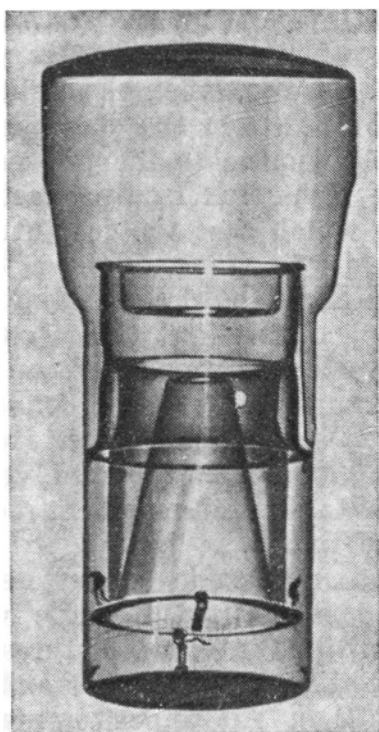


Fig. 10.6. X-ray photograph of image converter tube with electrostatic focusing

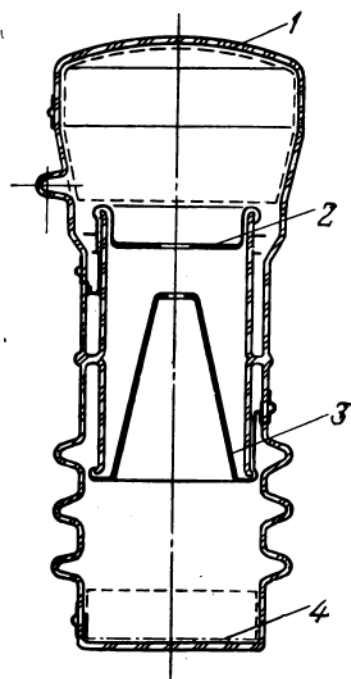


Fig. 10.7. Image converter tube with after-acceleration of electrons
1—cathode; 2—first anode; 3—second anode; 4—screen

tions and somewhat reduces the resolution, particularly near the edges of the screen.

If photocathodes sensitive to infrared rays are used in electron image converters for night vision, the brightness amplifiers employ photocathodes whose maximum sensitivity lies within the visible spectrum. The most important requirement placed on these devices is the provision for obtaining the maximum light intensification factor and resolution.

As has been indicated above, one stage of conversion at the best provides for light intensification several dozens of times which frequently is insufficient (for example, when image intensifiers are used in experimental physics, or astronomy). High light intensification can be obtained by means of two or several stages of image

conversion. Series connection of several converters should theoretically intensify the brightness at least thousands of times. In the simplest case an inadequately bright image from the screen of the first image converter can be optically projected to the photocathode of the second converter, and the brighter image obtained on the screen of this converter is then projected to the screen of the third device, and so on. Such method, however, has not found practical application. First, in optical transfer of the image from the screen of one image converter to the photocathode of another converter some of the light energy is inevitably wasted. Second, when the light passes two times through the glass of the bulbs (from the phosphor layer to the outside and from the optical objective to the photocathode of the second converter) some of the light flux is also wasted and the light is scattered in the glass in which case the resolution and contrast are considerably deteriorated.

The light losses can be reduced by eliminating the optical objective transferring the image from the screen of the first image converter to the photocathode of the second converter. It is clear that an optical objective is not needed if the screen face of the first converter is mounted near the cathode face of the second converter. However, since the bulb walls of vacuum devices cannot be too thin, considerable scatter of light in the glass is inevitable with resultant reduction of the resolution and contrast of the image.

These disadvantages are eliminated in cascade converters where two or three conversion stages are combined in a single vacuum bulb divided by thin transparent partitions into two or three chambers. One side of the partition is coated with a phosphor and serves as the screen of the first conversion stage. The opposite side of the partition is coated with a photosensitive material and serves as the cathode of the second stage. It is clear that the thinner the partition, the lower the scatter of light and the higher the resolution.

Similar to the radiant flux conversion coefficient for a single-chamber image converter [see formulas (10.6) and (10.10)], the radiant flux conversion coefficient for a two-chamber image converter is determined by the formula

$$G = \frac{\Phi_s}{\Phi_c} = \xi \eta_{s1} \eta_{s2} k_{p1} k_{p2} U_1 U_2 \quad (10.23)$$

where η_{s1} and η_{s2} are the energy yield of the screens of the first and second chambers, respectively, k_{p1} and k_{p2} is the sensitivity of the first and second photocathodes; U_1 and U_2 are the accelerating voltages of the first and second chambers.

The coefficient $\xi < 1$ is required for taking into account the discrepancy between the spectral characteristics of the screen of the first chamber and the photocathode of the second chamber.

The construction of the two-chamber converter is shown in Fig. 10.8.

This device consists of two identical electron image converters connected in series. The partition carrying the phosphor and the photocathode has a concave shape to reduce the aberrations. As mentioned above, the necessary condition for effective amplification of the light flux by a cascade image converter is the best possible coincidence of the spectral characteristics of the phosphor and the photocathode sensitivity, i.e. a high value of the coefficient ξ in formula (10.23). Satisfactory results are obtained when using a phosphor in the form of silver-activated zinc sulphide which features maximum radiation in the bluish region of the spectrum and anti-mony-caesium photocathode having its maximum sensitivity appro-

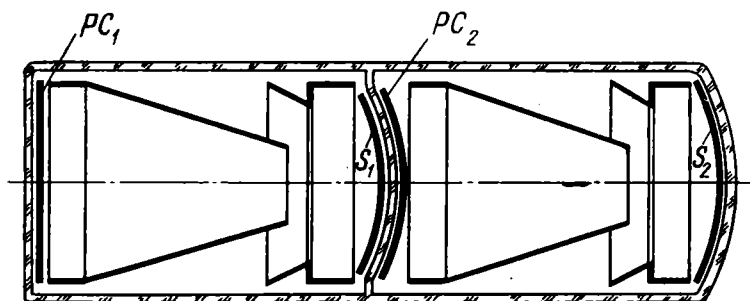


Fig. 10.8. Two-chamber image converter tube

ximately in the same region of the spectrum. For this phosphor-cathode pair the coefficient $\xi \approx 0.9$ which means that the spectral characteristics are in good agreement.

Using two-chamber image converters, it is possible to obtain amplification of the light flux by a factor of 1000 and more. Three-chamber converters allow the amplification to be increased up to 10^4 . However, the resolution of such devices is much lower than that of single-chamber devices. The decrease in the resolution is explained by the presence of aberrations in each stage of conversion and considerable scatter of light in the partitions and phosphor layers. The resolution of the two-chamber converters with magnetic focusing ordinarily does not exceed 15-20 pair of lines per millimetre near the edges of the picture. The resolution of electrostatic electron image converters is 2 to 3 times less. It should be noted that the resolution of image converters with a good electron-optical system is limited not only by the aberrations but also by the grained structure of the phosphor coating. Therefore, during the recent years attempts have been made to develop structureless screens for image converters by depositing the phosphor layer from the vapour phase.

The resolution can be increased materially by using fibrous optics for transferring the image from the screen to the photocathode. A fibre-optic partition between the chambers of an electron image converter consists of many thin glass fibres (light conductors) tightly fitted into a ring limiting the partitions. The light entering each glass fibre spreads only along the light conductor due to multiple complete internal reflection and emerges from the opposite end of the fibre almost without losses. Thus, applying a phosphor layer on one surface of the fibre-glass plate and a photosensitive layer on the other surface, one can transfer an image from the screen to the photocathode with very low light losses. If that is the case the resolution will be limited solely by the dimensions of each fibre, light conductor. At the present time it is possible to obtain fibre-glass plates composed of light conductors 5 to 6 microns in diameter. Such a diameter of fibre glass makes it possible to obtain resolution of 100 pair of lines per millimetre. Internal light losses do not exceed 15% even for comparatively thick fibre-glass plates, the light conductors being 5 to 6 mm long. The fibre optics in combination with structureless screens makes it possible to develop image converters with light intensification coefficients of tens of thousands of times with resolution of several dozens of line pairs per millimetre.

Brightness intensifiers used for X-ray apparatus have some constructive features. Since an X-ray picture of the object being investigated is obtained at a scale of 1 : 1 and the X-ray objects in medicine and flaw detection usually have large overall dimensions, the input photocathode of an X-ray image converter must have a large size, the diameter being of 200 mm and more. Preliminary optical reduction of the X-ray image is inexpedient, since it is associated with light losses. At the same time, it is possible and expedient to apply an X-ray fluorescent screen and a photocathode to different sides of a thin glass plate placed into a vacuum envelope near the inlet face. In this case the X-rays excite the luminescence of the phosphor in close vicinity to the photocathode and using a thin plate separating the phosphor and the photosensitive layer, one can minimize the scatter and loss of light, maintaining good resolution. Since the X-ray screen is placed into the bulb of the device, the inlet face of the bulb is made of glass transparent for X-rays.

The electron-optical system of X-ray converters is designed so as to reduce the image 25 to 200 times (by area) which gives a material brightness gain [see formula (10.4)]. The reduced image on the output screen can be photographed and then enlarged by optical means. The use of an electron-optical intensifier of an X-ray image is particularly expedient in medical diagnostic X-ray apparatus. The image intensification makes it possible to considerably reduce a radiation dose to a patient and to improve the working conditions of the operating personnel. Good results can be obtained by using two-chamber

cascade image converters for intensification of an X-ray picture. Figure 10.9 illustrates the construction of a two-chamber device used to intensify the brightness of an X-ray image.

As is shown in the drawing, the electron-optical systems of both chambers are identical. The focusing fields are built-up between the photocathodes and focusing electrodes made in the form of rings of electrically conductive coating on the cylindrical portions of the bulb and also between focusing electrodes and anodes of a special shape. A glass-fibre plate is used as a separating partition between the chambers. At an accelerating voltage of each chamber equal

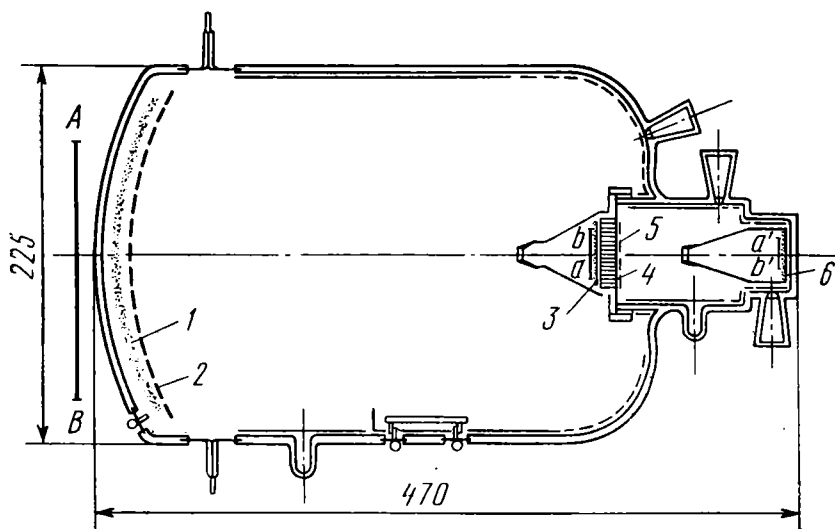


Fig. 10.9. Two-chamber X-ray image intensifier

1—X-ray screen; 2—photocathode of first chamber; 3—screen of first chamber; 4—fibre-glass partition; 5—photocathode of second chamber; 6—screen of second chamber

to 25-30 kV such a converter provides for image intensification by a factor of 200,000 to 300,000 with resolution of up to several tens of line pairs per millimetre.

An interesting modification of image intensifiers is presented by recently developed devices in which the secondary electron emission is used for "shooting through" of an electron picture. The secondary "shooting through" emission is caused by bombardment of thin films of certain materials with high-speed electrons provided that the energy of the primary electrons is sufficient for their penetration to almost the opposite side of the layer being bombarded. In this case the yield of secondary electrons is observed on the film side opposite to that irradiated by the electrons. The number of emitted secondary electrons much exceeds the number of primary electrons, i.e. the electron flow is "multiplied" in the solid of the film. With a small thickness of the film (of the order of $0.1 \mu\text{m}$) there is practically no scatter of electrons within the film, and the secondary

electrons emerge from the elements of the opposite film surface which oppose the elements of the other side irradiated by the primary electrons. The use of the secondary "shooting through" emission allows the image to be transferred from one side of the film (dynode) to the other side with intensification by a factor of 5 to 6.

Let us consider the construction of an image intensifier with secondary-emission dynodes operating as "shooting through" devices (Fig. 10.10).

The bulb of the device is cylindrical in shape. A photocathode is formed on the internal surface of the inlet face, a phosphor is applied to the surface of the opposite face. Partitions (dynodes)

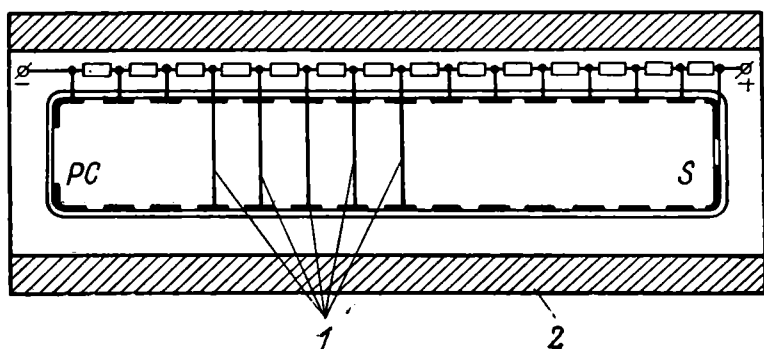


Fig. 10.10. Image intensifier with secondary-emission dynodes
1—dynodes; 2—solenoid

are placed in the space between the photocathode and screen. The dynodes are thin metal supports (aluminum or gold) which carry a thickness less than $0.1 \mu\text{m}$ of a material having a sufficiently high coefficient of secondary "shooting through" emission. Such a material may be, for example, potassium chloride having $\delta \geq 5$ with an energy of primary electrons of about 5 keV. The secondary-emission film can also be supported by a fine-mesh grid. The photocathodes, dynodes and aluminum coating of the screen have leads to supply through all electrodes of the device from an external voltage divider in order to provide a potential difference between adjacent electrodes of the order of 5-6 kV. When the image to be intensified is projected into the photocathode, the emitted electrons are accelerated by an electrostatic field and, penetrating into the dynode film, excite a secondary "shooting through" emission. The secondary electrons emerging from the other side of the first dynode are again accelerated and bombard the second dynode, the secondary electrons emerging from the second dynode bombard the third dynode and so on. The secondary electrons emerging from the last dynode are accelerated by the electrostatic field between the dynode and the screen and excite the luminescence of the phosphor. Using five dynodes and $\sigma \approx 5$, the initial photocathode current is amplified

by a factor of $(3-4) \times 10^3$. Since a simple converter during the transfer of an image from the photocathode to the screen (without dynodes) operating at an accelerating voltage of about 30 kV can intensify the light flux by dozens of times, the total image intensification ensured by a five-dynode intensifier may be of 100,000.

Of course, using solely a longitudinal electrostatic field for transferring an electronic image from the photocathode to the first dynode and between the dynodes, cannot provide high resolution of the device [see formula (10.7)]. Therefore, to obtain a satisfactory picture, the entire intensifier is placed into an adequately strong longitudinal uniform magnetic field produced by an elongated solenoid or permanent magnets. The use of magnetic focusing makes it possible to increase the resolution up to 30-40 line pair per millimetre. A distinctive feature of such intensifiers is the absence of geometric aberrations, since the device has no electron lenses (longitudinal uniform electrostatic and magnetic fields form no lenses), and the image is neat over the entire surface of the screen. Of course, the scatter of initial velocities of the photoelectrons and secondary electrons reduces the resolution and it is correct to say that the image quality is due to the chromatic aberration of the transfer system. The advantage of intensifiers with dynodes is in their simplicity (as compared with the multichamber image converters), since these devices have only one photocathode and one screen. The disadvantage is in the necessity of magnetic focusing which dictates the use of external solenoids or permanent magnets considerably increasing the weight and overall dimensions of the device.

Storage Tubes

11.1. Application and Principle of Operation

By storage tubes are meant electron tubes intended for storage (writing) of certain information and its reproduction at a later time. Hence the storage tubes may also be referred to as electrostatic memory tubes.

The storage tubes have found wide application in radar indicators, special oscillographs with long storage of the signal pattern, in television for conversion of a television picture having one standard of scanning (one number of lines) into a picture with another standard, in computers as memory elements, in automatic and remote control systems, and for program control of various processes.

Most of the storage tubes employ double conversion of information: (1) conversion of a series of input signals containing necessary information into electric charges distributed over the target surface (storage surface); in other words, provision for a certain charge definite potential pattern on the target surface; (2) conversion of the charge pattern on the target into a sequence of output signals reproducing the entered information with sufficient accuracy.

Generally the first conversion is called the information writing. The other conversion is called the information reading. In addition to the writing and reading, some types of storage tubes perform an auxiliary operation of erasing used for removing the stored charge pattern when preparing the storage tube for writing new information.

The information to be stored is entered into the storage tube either in the form of a sequence of electric pulses or by projecting its optical image onto a photosensitive target. The information to be reproduced is read in the form of a sequence of electric pulses. Sometimes it is converted into a visible image displayed on a screen.

The time of storage of the written information may vary within a wide range—from fractions of a second to several hours and even days. In the same manner, the number of reading operations can vary from one to tens and hundreds of thousands. In some types of storage tubes the written information is stored during a long time due to perfect insulation of the target, in other tubes the time

of storage is increased by means of a special electron beam maintaining the charge pattern.

The essentials of a storage tube include a target on the surface of which stored charge pattern is formed, electron guns producing writing and reading beams, and deflection systems. In addition, the storage tubes have a collector to collect electrons (secondary electrons, photoelectrons, and the electrons reflected from the target), grids for producing electrostatic fields of the required configuration and other auxiliary elements. An example is the electrically conductive substrate of the target. The conductive substrate is often called the signal plate, since the signal to be written can be fed to this plate or the stored information is read from it.

11.2. Methods of Writing and Reading

There are several methods used for writing and reading information. In most of the storage tubes these operations are performed by an electron beam scanned over the target surface. A stored charge pattern is usually produced by secondary electron emission.

Generally the target may be regarded as a combination of separate storage elements. In practice the target consists either of a single dielectric layer (more often) or of many conductive particles applied to a dielectric surface ("mosaic"). The secondary-emission properties of a dielectric (or a surface formed by a combination of electrically conductive particles on a dielectric substrate) are graphically described by a curve showing the secondary emission coefficient *versus* the energy of the impinging electrons (see Fig. 6.12). The secondary emission coefficient $\sigma = 1$ at two values of the energy of the incident (primary) electrons is as follows: with $\varepsilon_1 = eU_{cr1}$ (point *a*) and with $\varepsilon_1 = eU_{cr2}$ (point *b*). Moreover, with $\varepsilon_1 \rightarrow 0$ the "apparent" secondary emission coefficient tends to unity.

When the surface of a non-conductive target is bombarded with electrons the potential of its elements may acquire different equilibrium (stable) values depending on the energy of incident (primary) electrons. The energy of the electrons approaching the target but not yet striking its surface is determined by the passed potential difference in the way from the cathode to the last anode of the gun forming the electron beam or to the electrode located near the target (collector or grid). However, the energy of the electrons bombarding the target which determines the secondary emission coefficient depends on the target element potential (with respect to the gun cathode) which in the case of a dielectric surface may materially differ from the potential of the gun anode or collector (see Section 6.5).

If the accelerating voltage (the potential difference passed by the electrons) is less than U_{cr1} , then $\sigma < 1$ and due to the secondary emission the target will loose less amount of electrons than that

brought by the electron beam. In this case a negative charge will accumulate on the target surface and the potential of the target elements will decrease. The accumulation of negative charge and, therefore, reduction of the potential will continue until the target potential becomes equal to the potential of the gun cathode. When the target elements are at the gun cathode potential, a retarding field is built up at the target surface, which prevents electrons from getting to the target. Further accumulation of negative charge is ceased and the target surface assumes a zero (equilibrium) potential with respect to the gun cathode.

When the accelerating voltage is greater than U_{cr1} but less than U_{cr2} and $\sigma > 1$, the amount of secondary electrons emerging from the target becomes greater than that brought to it by the beam, a positive charge accumulates on the target surface and its potential rises up. However, the target potential cannot substantially overpass the potential of the gun anode or collector, since the accelerating field formed near the target surface returns some of the secondary electrons back to the target and the excessive positive charge is partially compensated for. Thus, when $U_{cr1} \leq U_a \leq U_{cr2}$, an equilibrium potential is obtained on the target surface, which is approximately equal to the potential of the last anode of the gun or to the potential of the grid or collector positioned near the target.

With $U_a > U_{cr2}$ and $\sigma < 1$, the amount of secondary electrons lost by the target is less than that brought by the beam and a negative charge is formed on the target surface. However, the potential cannot be reduced to zero (the potential of the gun cathode), as when the target potential becomes equal to U_{cr2} , the secondary emission coefficient becomes equal to unity. Further accumulation of a negative charge is ceased and an equilibrium potential equal to U_{cr2} is set up on the target surface. This equilibrium potential does not depend on the energy of the primary electrons (at $U_a > U_{cr2}$). The dependence of the target potential U_t on the energy of incident (primary) electrons is given in Fig. 11.1.

From the drawing it is seen that the equilibrium potential of the target depends on U_a (approximately equals U_a) only in the region $U_{cr1} \leq U_a \leq U_{cr2}$, while in the regions $U_a < U_{cr1}$ and $U_a > U_{cr2}$ the equilibrium potential of the target is independent of the accelerating voltage being set up at the levels of $U_T = 0$ and $U_T = U_{cr2}$, respectively.

Thus during the preparation of the target for writing the potential of its elements is brought by a scanning electron beam to one of equilibrium values possible for the given energy of the primary electrons. The preparation of the target for writing, i.e. scanning its surface by unmodulated electron beam with no input signals, may be effected as an individual operation.

The following methods of writing are used in the storage tubes: equilibrium, bistable, non-equilibrium and induced-conductivity writing. The redistributed-charge writing employed in television tubes (see Section 12.4) has found but limited application in storage tubes.

When use is made of the *equilibrium writing*, the energy of the electrons of the writing beam is selected to be higher than eU_{cr2} or lower than eU_{cr1} . When scanning the target by an unmodulated beam, an equilibrium potential U_{cr2} or zero potential is set up. If now an input signal is fed to the cathode of the writing gun, a new equilibrium potential will be the same with respect to the

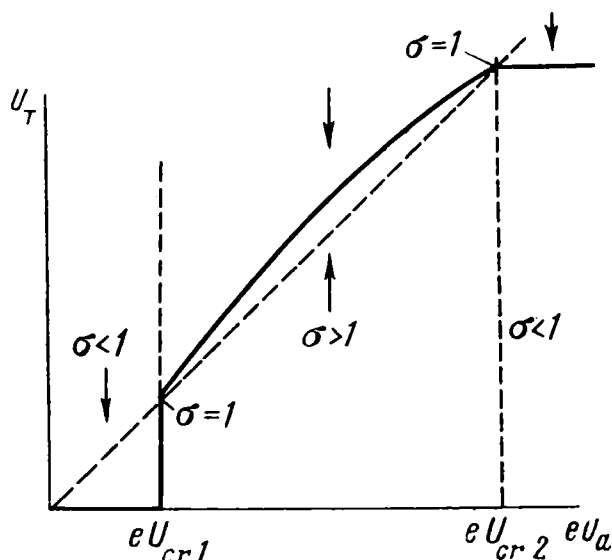


Fig. 11.4. Target potential versus energy of primary electrons

cathode but change with respect to the collector. Thus, formed on the target surface is (with respect to the collector) a stored charge pattern corresponding to the information fed to the cathode.

The equilibrium writing can be accomplished at the energies of primary electrons lying between U_{cr1} to U_{cr2} . In this case the signal to be written is applied to the collector or the signal plate (metal substrate of the target). When scanning the target with an unmodulated beam, the potential of the target elements is brought to an equilibrium value corresponding to the given energy of the primary electrons, i.e. approximately to the potential of the gun anode. However, the charge accumulated by the elements will be different depending on the value of the input signal, i.e. in this case too, a stored charge pattern will be formed on the target surface with respect to the signal plate.

The *bistable* (two-position) writing is used when the information being written can be expressed in binary system, i.e. presented in the form "0-and-1", "yes-and-no", and "black-and-white". In this

case the target potential can have only two greatly differing equilibrium values. For example, one can perform bistable writing by modulating the energy of the primary electrons from $\varepsilon_1 < eU_{cr1}$ to $\varepsilon_1 > eU_{cr1}$ or from $\varepsilon_1 < eU_{cr2}$ to $\varepsilon_1 > eU_{cr2}$. In the first case the first equilibrium potential is equal to zero, the second one is close to the potential of the gun anode. In the second case the first equilibrium potential is close to the anode potential while the second one is equal to U_{cr2} .

The bistable writing permits two modes: "white on black" and "black on white". Before performing the bistable writing in the "white on black" mode, the target potential is brought to a lower equilibrium value (for example, to zero by scanning the target with low-speed electrons, $\sigma < 1$). The writing is carried out by faster electrons with energy exceeding eU_{cr1} ($\sigma > 1$). In this case a white positive potential charge pattern is formed on a black zero background. When writing in the "black on white" mode, the target potential is brought to a higher equilibrium value (for example, equal to the potential of the anode or collector). The writing is effected by low-velocity electrons ($\sigma < 1$) reducing the potential of the target elements to the lowest (zero) equilibrium value. In this case a black negative potential charge pattern is produced on a white positive background.

In spite of the fact that the bistable writing does not allow information recording comprising gradations of grey (halftones), it is employed by storage tubes for computers using a binary system. The advantage of the bistable writing is in that it permits a written signal (charge pattern) to be maintained (fixed) by simple means. This makes it possible to read the written information without limiting. In actual tubes the retention time is limited by unavoidable leakage currents over the target surface and compensation of the charge by the scattered positive ions which results in gradually smoothing down the stored charge pattern.

The *nonequilibrium writing* is performed by shifting the potentials of the elements of the target from their equilibrium value acquired during the preparation for the writing or due to erasure. Usually the equilibrium value is close to the collector potential. Before writing, applied to the signal plate is a voltage other than the collector potential. When writing by a modulated beam the potentials of the elements of the target are shifted from their equilibrium value so that a charge pattern appears on the target surface. The nonequilibrium writing can be accomplished by a beam with a very low current. If that is the case, the charge brought by the electron beams is insufficient for bringing the target potential to an equilibrium value and a charge pattern is formed on the target.

The nonequilibrium writing also finds its application in storage tubes with a photosensitive target (for example, in orthicon, see

Section 12.5) in which the shift of the element potential occurs due to escape of photoelectrons from the target surface.

The induced-conductivity writing is based on electrical conductivity of thin layers of dielectric (0.5 to 1 μm) occurring when irradiated by high-speed ($\varepsilon_1 \approx 10$ keV) electrons. Before writing the target surface is brought to an equilibrium potential (for example, equal to the collector potential). The signal plate is fed with a voltage substantially differing from the equilibrium potential of the target surface. The writing is performed by a beam of high-speed electrons capable of inducing conductivity. Because of the induced conductivity, at the point of impingement of the recording beam, the target potential is shifted to the potential of the signal plate so that a charge pattern is produced on the target surface.

The induced conductivity can occur under the action of a light flux falling on a target from a semiconductor having an internal photoelectromagnetic effect. Such a target is employed, for example, in vidicons (see Section 12.8).

The signals to be written (information to be "stored") can be applied to a storage tube by modulating the voltages on different electrodes of the tube. Accordingly, the following methods of introducing information are known:

- (1) Changing the current of the writing beam by varying the voltage across the gun modulator;

- (2) Changing the rate of scanning of the beam (frequency or amplitude of the scanning voltages or currents);

- (3) Changing the accelerating voltage across the writing gun or changing the collector voltage;

- (4) Changing the voltage on the signal plate. Writing can be performed by projecting the optical image on a photosensitive target. This method of writing is used in television tubes (see Section 12.3).

In the first and second methods the charge pattern is determined by the charge brought by the writing beam to the target. In the fourth method the target charge is determined by the number of secondary electrons leaving the target and landing on the collector. In the third method the target charge depends either on the number of reflected primary electrons or on the variation of the secondary emission coefficient.

The reading can be accomplished also by several methods. Storage tubes employ capacity-discharge reading, grid-control reading and redistribution reading.

The capacity-discharge reading includes a recharge of elementary capacitors formed by the elements of the target surface and the signal plate (backplate). When writing by forming stored charge patterns the elementary capacitors are charged in accordance with the value of the signal being written. During the capacity-discharge

reading, an unmodulated reading beam discharges the elementary capacitors. In this case the potentials of all elements of the target are shifted towards the equilibrium value.

If the reading beam current is sufficient for completely shifting the potential of the charge pattern to the equilibrium value, then after a single reading operation the entire surface of the target assumes the equilibrium potential, the stored charge pattern becomes completely destroyed and further reading is impossible. At smaller beam current the charge is insufficient to recharge the elementary capacitors completely. In this case the potentials of the target elements are shifted only a portion of the way towards the equilibrium value without reaching it. After each reading (scanning) the potential of the charge pattern is shifted more and the potentials of all the elements of the target approach the equilibrium value. Since during each reading the charge pattern potential is shifted only partially, it is possible at a smaller beam current to read the information from several times to several hundreds of times (depending on the charge pattern potential and the current of the reading beam).

In the *capacity-discharge reading* discharging the elementary capacitors causes a capacitive current in the signal plate circuit. This current produces an output signal. Since the discharge of the capacitors depends on the value of the written signal, the electrons returning from the target to the collector become modulated by the written signal. Hence, the output signal can also be obtained in the collector circuit. In the capacity-discharge reading, the output signal fully corresponds to the written signal, if the stored charge pattern potential is shifted all the way to the equilibrium potential. In the process of repeated reading the output signal may be distorted particularly when transmitting gradations of grey (halftones).

During the *grid-control reading* the stored charge pattern on the surface of the target produces local electrostatic fields which can act on the electrons flying near the target. Such a "control" action of the local fields is similar to the action of a control grid of an electron valve on the electron flow between the cathode and valve plate.

In the case of grid-control reading there may be no grid used as an element of the storage tube. The local electrostatic fields near the target surface play the role of such a grid. In some types of storage tubes with grid-control reading the target is made in the form of a metal grid one side of which is coated with a layer of a dielectric material.

In this case the stored charge pattern varies the permeability ("transparency") of the target for the reading beam. In other types of storage tubes with grid-control reading the stored charge pattern (local fields) controls the secondary electrons or the reflected primary electrons emerging from the target.

In the process of grid-control reading the electrons of the reading beam do not settle on the target and do not eliminate the potential of the charge pattern. Therefore, such reading is used when it is necessary to reproduce the written information many times.

The grid-control reading can be performed by a focused beam scanned over the target surface or by flooding the target surface with a wide unfocused electron beam. The grid-control reading is also used in storage tubes having an output signal in the form of a visible image on a phosphor-coated screen. In this case the stored charge pattern modulates the electron beam impinging the screen.

Reading by redistributing charges over the target surface employed in storage tubes does not differ from that used in iconoscopes (see Section 12.4).

In addition to the operations of writing and reading, some types of storage tubes employ an operation for erasing the written information to prepare the target for writing new information. The erasure is usually accomplished so that the potential of all elements of the target is shifted to one of the possible equilibrium values regardless of the charge having been acquired by these elements.

11.3. Converter Signal-storage Tubes (Electrical-electrical)

One of the commonly used electrical-electrical storage tubes is a barrier-grid tube which is also known as radochon. The barrier-grid tubes are used in radar indicators, in system of program control, and for writing (memorizing) rapid processes with subsequent reproduction of the data.

A barrier-grid storage tube has a single electron gun producing a focused beam of fast ($\epsilon_1 \approx 1000$ eV) electrons and a dielectric target on a metallic backplate serving as a signal plate. The collector is usually made in the form of a conductive coating applied to the internal surface of the bulb. A characteristic feature of this storage tube is in that before the target there is a grid at a potential several hundred volts lower than the collector potential. In this case the secondary electrons emerging from the target surface and at sufficiently high initial velocity penetrate the grid and are trapped by the collector, while the low-velocity electrons are returned to the target practically to the point of emission. Thus the redistribution of charges over the target surface is prevented as well as their elimination of the charge pattern potential.

A schematic diagram of a barrier-grid storage tube is shown in Fig. 11.2.

Mounted in a cylindrical bulb is a target with a barrier grid. The bulb has a neck in which an electron gun is located. The cylindrical part of the bulb has an electrically conductive coating subdivi-

ded into several rings. The first ring, next to the gun, is connected to the gun anode. The second ring serving as a collector is at a potential of 400 to 500 V above the anode potential. The next ring plays the role of a guide cylinder producing a field directing the secondary electrons to the collector. Its potential is 200-250 V below that of the collector. Finally, the last ring is connected to the grid and is at a potential of -300 to -400 V with respect to the collector. In some types of tubes the collector and the last electrode connected to the grid are made in the form of short wide metallic cylinders and the conductive coating has only two parts—between the gun and collector and between the collector and grid cylinder.

Writing and reading (and erasure, if necessary) are accomplished by the unmodulated electron beam at an accelerating voltage of

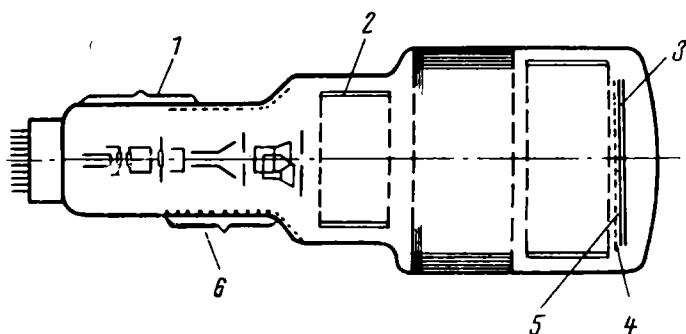


Fig. 11.2. Storage tube with barrier grid

1—electron gun; 2—collector; 3—signal plate; 4—target; 5—grid; 6—deflection system

1000 to 1500 V. Since the resolution of the tubes with a uniform target is determined largely by the diameter of the writing and reading spot, the focusing must be fairly good. The gun may be equipped with an electrostatic or magnetic focusing system. With low beam deflection angles ($12-15^\circ$), small current and comparatively high voltage (1000-1500 V) the gun with electrostatic focusing can provide a spot on the screen from 0.1 to 0.15 mm in dia. The spot of such a size provides for adequately high resolution.

The beam deflection may be electrostatic or magnetic. Since the beam deflection angles are low and the requirements to the spot size are not too rigorous, economical electrostatic deflection can be used with a success. Some barrier-grid storage tubes are equipped with a target in the form of a thin glass or mica plate (20-25 microns thick). A metallic layer forming a signal plate is deposited on one side of the dielectric by vacuum evaporation. In other types of these tubes, on the contrary, the base of the target is made of a comparatively thick metal substrate (signal plate). The dielectric (for example, magnesium fluoride) is applied to the signal plate by vacuum evaporation. An oxidized aluminum plate can also be used as a target.

A barrier grid is mounted near the target. Since the storage tubes frequently employ line (television) scanning of the raster, the grid is made in the form of a row of parallel wires by the number of the scanning lines. In this case the deflection system can be adjusted so that the electron beam will pass between the grid wires and practically will not be trapped by these wires.

Before the writing, the target surface is scanned by an unmodulated beam. In this case, since $\sigma > 1$ (the energy of the primary electrons is of 1000-1100 eV), the target surface acquires an equilibrium potential approximately equal to the grid potential.

The barrier-grid storage tubes can be used in different writing modes. The modes of equilibrium and nonequilibrium writing are most popular. In the mode of equilibrium writing the input signal is applied to the conductive backplate of the target, i.e. to the signal plate. The output information is obtained in the collector circuit. When the potential of the signal plate is modulated by the input information with simultaneous scanning of the target surface by the unmodulated beam, the potential of all elements of the target is shifted to an equilibrium value (equilibrium writing), but the charge required for this operation varies, depending on the instantaneous potential difference between the signal plate and the target surface. Thus, a charge pattern (with respect to the signal plate) is produced on the target surface.

If the supply of signals to be written is stopped but the scanning is continued, the reading beam will discharge the elementary capacitors bringing the target surface again to the equilibrium potential (the capacity-discharge reading). Since in writing the charge over the target surface is distributed in accordance with the recorded information, the current of the target and, therefore, the current of the secondary electrons emerging from the target to the collector will be modulated by the written signal and an output signal is formed in the collector circuit. As the writing is performed with a signal applied to the signal plate and the output signal is taken from the collector circuit, the polarity of the output signal is opposite to that of the input signal.

During each writing operation the target potential is shifted to the equilibrium value and there is no need for special operation of erasing the information, i.e. new writing automatically erases the earlier recorded information. In the case under consideration the operations of writing and reading are shifted in time, i.e. the written information can be stored on the target in the form of a charge pattern and then be read out when necessary.

It should be noted that the writing current of the collector varies in accordance with the potential of the signal plate, since the number of secondary electrons emerging from the target is determined by the potential of the scanning element, i.e. in the process of writing,

the output current (collector current) is modulated by the input signal, the polarity of the output signal coinciding with the polarity of the input signal in the process of writing. This feature makes it possible to use the barrier-grid storage tube as a "subtracting" tube. If the target surface is continuously scanned by an electron beam and the input information is simultaneously fed to the signal plate, the electron beam switching the target elements will read the signal written on the given element during the preceding cycle of scanning and will write the signal being applied to the signal plate at the moment of switching. Since the polarity of the output signal is different during the writing and reading operations, the resultant output signal thus obtained in the collector circuit will reproduce the difference of the values of the signals of two successive scanning cycles.

When the storage tube is used in the nonequilibrium writing mode, the input signal is fed to the gun modulator, varying the beam current. Depending on the beam current, when $\sigma > 1$, the number of secondary electrons emerging from the target will vary, i.e. a positive charge pattern will be produced on the target surface. The wider the variation of the beam current, the larger the potential of charge pattern. When the target with written information is scanned by an unmodulated beam, the elementary capacitors are discharged, i.e. in this mode we have capacity-discharge reading. In this case the collector current is modulated in accordance with the potential distribution over the target and the output signal will reproduce the recorded information. An output signal can also be obtained in the signal plate circuit.

The nonequilibrium writing allows several signals to be summed up. If several signals are fed to the modulator and are written on the same scanning line, the charge pattern will be determined by the combination of input signals.

During the reading operation the output current will reproduce the sum of all the written signals. This method of summation is used in radar indicators for improving the signal-to-noise ratio. When a radar signal is received against a background of random noise, the useful periodically repeated signals are summed up and with n cycles of writing the value of the potential charge pattern in the area of useful signal writing rises approximately n times. On the other hand, the random signals of noise will change in the potential charge pattern of the noisy background only $\sqrt{n}$ times. Thus, the signal-to-noise ratio is materially raised, the number of writing cycles being not very small.

The change-over from one scanning standard to another is necessary when relaying telecasts from a transmitter operating at a different number of scanning lines, as well as in the case of conversion of a radar picture obtained on the screen of a plan-position indicator

with circular scanning into a television (raster) picture. The change-over from one scanning standard to another can be accomplished by means of a storage tube known as *graphechon*.

The graphechon is a storage tube with a dielectric target and two electron guns (writing and reading). It uses the induced-conductivity writing and the capacity-discharge reading. The writing and reading guns may be arranged from one side of the target. In that the writing gun is located in the neck along the tube axis, while the reading gun is arranged in the other neck inclined to this axis. In this case the target is unilateral. It is of a thin layer of dielectric applied to a metallic backplate (signal plate). The guns can also

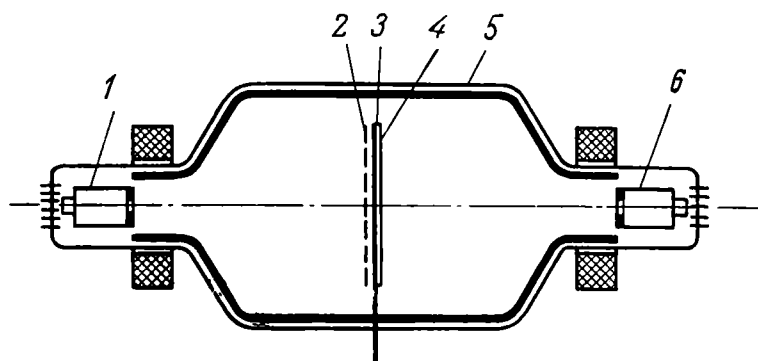


Fig. 11.3. Graphechon with bilateral target

1—writing gun; 2—grid; 3—signal plate; 4—target; 5—collector; 6—reading gun

be arranged from both sides of the target—in the two necks along the tube axis. In this case the target is bilateral. A thin aluminum layer transparent for high-speed electrons is applied to a fine-mesh grid. The aluminum film serves as the signal plate whose surface opposite to the grid is coated with dielectric forming the target.

A schematic diagram of a graphechon with a bilateral target is shown in Fig. 11.3.

The target is mounted at the centre of the wide part of the tube, the writing gun, in the left-hand neck and the reading gun, in the right-hand neck. The writing and reading functions in this tube cannot be combined since these operations are performed by electrons with substantially different energies (~ 10 keV and ~ 1 keV). Similarly, the current of the writing beam somewhat exceeds that of the reading beam. The induced conductivity is determined mainly by the energy of the electrons. At the same time, in order to obtain a good signal-to-noise ratio, it is necessary that in the process of reading the beam perceptibly discharges the elementary capacitors formed by the elements of the target surface and the signal plate. Since a graphechon can store a large charge (the potential of the charge pattern can be of several volts), a moderate beam current is necessary for effectively discharging the capacitors.

The different requirements placed on the writing and reading beams determine construction differences of the writing and reading guns. Generally, the graphecons can employ guns both with electrostatic and magnetic focusing. Since the resolution requirements are usually not too rigorous and can easily be satisfied, for a comparatively large (for example 4×6 cm) target surface, guns with electrostatic focusing are more common.

The writing and reading beams are usually deflected by means of magnetic coils. Some types of graphecons employ electrostatic deflection.

A material portion of the bulb (the cylindrical part and the neck) has an electrically conductive coating on the internal surface. The coating near the target serves as a collector for the secondary electrons emerging from the target. As a rule, the graphecon operates with an earthed collector. In this case the potentials of the cathodes of the writing and reading guns are equal to -10 and -1 kV, respectively, relative to the earth.

The thickness of the dielectric layer is determined by the necessity to gain induced conductivity at moderate energies of the electrons of the writing beam. The induced conductivity has its maximum when the depth of penetration of the electrons does not exceed the thickness of the layer. For example, with most of dielectrics used as targets of storage tubes (quartz, magnesium fluoride, calcium fluoride) the depth of penetration of the electrons having energy of 10 keV is less than 1 micron. Therefore, a value of $0.5-0.8 \mu\text{m}$ should be considered as optimum at the writing beam energy ranging from 10 to 12 keV.

It is interesting to note that the maximum charge that can be stored on the target is almost independent of the layer thickness. Indeed, the stored charge $q = CU$ (where C is the capacity of the elementary capacitor, U is the potential of the charge pattern) increases in proportion to the capacitance and the potential difference between the target surface and the signal plate. As the thickness of the dielectric decreases, the capacitance of the elementary capacitors increases but the dielectric strength, i.e. the magnitude U drops down almost to the same degree. Hence, the stored charge remains practically the same.

The retention time of the information written on the target and a possible number of reading operations depend largely on the resistance of the dielectric layer and the vacuum in the tube.

The graphecon operates as follows. In the process of reading or special preparation of the target for writing operation the target surface is subjected to scanning by an unmodulated reading beam. Since the energy of the electrons of the reading beam does not exceed 1000 eV, there is no effect of induced conductivity. At the same time, the secondary emission coefficient is larger than unity. There-

fore, in scanning by the reading beam the target potential reaches an equilibrium value approximately equal to the collector potential.

During the writing operation the signal plate is fed with a voltage (a few dozens of volts) which is negative with respect to the collector. The recording beam is modulated by the input signal fed to the modulator of the recording gun. When scanning the target by the writing beam the high-speed electrons easily pierce the thin signal plate and, having penetrated into the dielectric layer, cause induced conductivity. Due to the induced conductivity at the place where the beam strikes the target, the potential of its surface decreases, i.e. approximates the potential of the signal plate. Since the induced conductivity depends on the current of its "exciting" beam, the modulation of the writing beam current causes different variations of the potential of the target elements. A great current of the recording beam may result in levelling the potentials of the target surface and the signal plate.

Thus, during the induced-conductivity writing a stored charge pattern is formed on the target surface. The maximum potential of the charge pattern may equal the potential difference between the signal plate and collector. A great potential of charge pattern permits repeated reading operations. Without reading the information written on the graphechon target the retention period may equal several days and even months.

The reading is performed by an unmodulated reading beam which, during the scanning of the target, discharges the elementary capacitors bringing the potential of the target surface to an equilibrium value equal to the collector potential. During the discharge of the elementary capacitors capacitive currents flow in the signal plate circuit and an output signal is generated in this circuit. Due to a great potential of the charge pattern the reading beam discharges the elementary capacitors gradually. Therefore, an output signal with a satisfactory signal-to-noise ratio can remain in multiple operations used for reading the once recorded information.

Since the charge pattern potential is gradually reduced during the reading operations, no special erasing operation is necessary. If the written information must be quickly erased, the target surface is scanned by an intensive reading beam. The complete erasure is indicated by the absence of output signal modulation.

The graphechon can be used under the conditions for alternating writing and reading operations. However, it is possible to perform both of these operations simultaneously.

If that is the case, measures should be taken to prevent the occurrence of stray output signals caused by the modulation of the writing beam current. The simultaneous writing and reading are particularly convenient when the graphechon is used for conversion of a scanning standard. In this case the deflection systems of the writing and

reading sides are fed from individual generators, providing for scanning the beam over the target at different frequencies or by different laws.

The above considered storage tubes are based on capacity-discharge reading in which each act of obtaining information reduces the potential of the charge pattern, i.e. the number of readings, in principle, cannot be great. In some cases, particularly when the storage tube is used in computer systems, it must allow multiple reading without significant distortion of the output signal. In this case use is made of storage tubes with grid-control reading or with a beam holding the stored charge pattern.

One of the tubes permitting multiple reading is a storage tube with a collector-reflector. This tube has a target transparent for

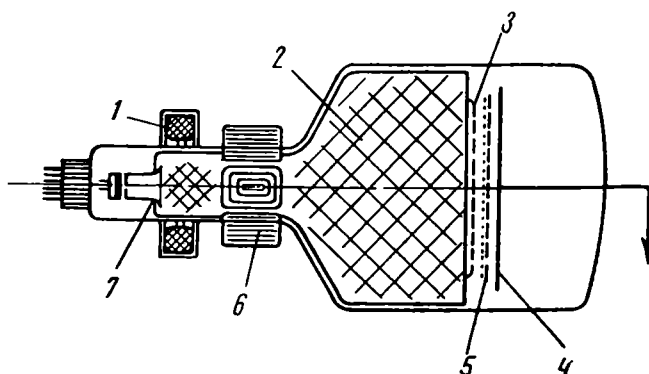


Fig. 11.4. Storage tube with reflector-collector

1—focusing coil; 2—conductive coating; 3—screen grid; 4—reflector-collector; 5—target; 6—deflection system; 7—electron gun

the electrons made in the form of a grid coated with a dielectric material on one side. The tube has a single electron gun whose beam performs the functions of writing, reading and erasing. The writing is effected by the modulation of the current of the recording beam, i.e. the input signal is fed to the modulator of the electron gun. The output signal is formed in the collector-reflector circuit. This tube employs nonequilibrium writing and grid-control reading.

The design of the storage tube with a collector-reflector is shown in Fig. 11.4.

All the components of the tube are assembled in a cylindrical glass bulb. The bulb has an adequate length in order to provide an angle of incidence of the electron beam close to 90° , i.e. to use small beam deflection angles. An electron gun is mounted in the bulb neck. A greater part of the internal surface of the bulb has an electrically conductive coating.

In the bulb neck the electrically conductive coating is a continuation of the gun anode. In the cylindrical portion of the bulb it serves as an electrostatic shield maintaining a constant potential

in the region of propagation of the electron beam. In this tube the electrically conductive coating is not a collector of the electrons emerging from the target.

The electron gun of this storage tube must produce a well focused beam, since the beam must pass through the holes of a grid-target. At the same time, the grid-target must have a fine structure to provide for high resolution. An increase in the resolution due to increasing the target size is inexpedient, since the larger the target, the greater the difference of the angle of incidence of the electrons

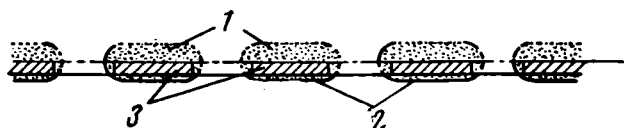


Fig. 11.5. Design of grid-target

1—dielectric; 2—aluminum film; 3—target backplate

from 90° (at a moderate length of the tube). Furthermore, making a large grid-type target of sufficient strength is difficult from the engineering point of view.

Since any defocusing of the beam during the deflection is undesirable, the deflection of the beam in this tube is effected by magnetic fields. The deflection system consists of two pair of coils often provided with steel cores with parallel-added magnetic fluxes.

A potential carrier (target) transparent for electrons is a fine-mesh grid provided with a thin dielectric coating on the side opposite to the gun. The electrons of the beam must not penetrate to the dielectric during their motion from the gun to the target. The design of the grid-target is shown schematically in Fig. 11.5.

During the operation of the tube the grid potential is always substantially low than the potential of the gun anode. Therefore, the electrons are decelerated in the zone near the target. In order to reduce the beam defocusing during the deceleration, it is expedient that this operation is effected in the immediate vicinity to the target, i.e. the length of the path of the slow electrons should be minimized. This is achieved by mounting a special screen grid having a potential equal to that of the gun anode (conductive coating) near the target from the gun side.

Mounted behind the target is a flat metal electrode, collector-reflector playing the role of an electron mirror. As a rule, all the three electrodes, the screen grid, target and collector-reflector, are assembled in a common adequately rigid metal ring which is then mounted in the tube.

The above-described tube uses the same electron gun for writing, reading and erasing. The change-over from one operating mode to another is carried out by varying the voltage on the grid-target and on the collector-reflector.

When preparing for writing (or when erasing the previously written information) the potential of the grid-target is set below the potential of the screen grid. The potential of the collector-reflector is set below the potential of the gun cathode. Under such conditions the electrons are considerably decelerated in the screen grid to target space and, having passed at a low velocity through the holes of the target grid, are reflected by the electron mirror (a zero equipotential surface) formed between the target and the collector-reflector. The reflected electrons impinge the dielectric coating of the target and charge it to an equilibrium potential value.

Since in the case of a low energy of the reflected electrons the secondary emission coefficient of the target is lower than unity, the target surface acquires a negative charge and the equilibrium potential value is zero (with respect to the gun cathode). Thus, the potential carrier prepared for writing is at a zero equilibrium potential.

The writing is accomplished by an electron beam modulated by the input signal. In writing the potential of the collector-reflector remains negative, while the grid-target potential increases to such an extent that the reflected electrons impinge the dielectric surface with an energy providing for $\sigma > 1$. Owing to a small distance between the target and the collector-reflector and the fall of the electrons on the target grid at angles close to 90° , the reflected electrons impinge the dielectric in the immediate vicinity to the place of fall of the beam on the grid. This ensures high resolution.

The secondary electrons knocked out from the dielectric during the writing operations are collected by the target grid which, when writing, plays the role of a collector of secondary electrons. Thus, in writing a positive potential charge pattern is formed on the zero equilibrium background. Since the potential of the target grid much differs (by 300-500 V) from the equilibrium value, the potential of the stored charge pattern can be sufficiently large (a few dozens of volts). As the writing operation is performed by a modulated beam, the stored charge pattern contains half-tones, i.e. gradations of gray can be recorded.

The retention period of the written signal is fairly long, as reduction of the charge pattern potential is only due to the leakage and ion discharge and may be of negligible value, provided the dielectric and vacuum are good.

The reading is effected by scanning the target surface with an unmodulated beam. In this case the target grid is set at 40-60 and the collector-reflector at 200-240 volts. The target functions as a control grid of an electron valve (grid-control reading). The collector-reflector plays the role of the anode of the electron valve; the current in the collector-reflector circuit varies with the grid potential. Thus the collector-reflector current is modulated by the

written signal, i.e. the output signal appears in the collector-reflector circuit.

Since the electrons of the unmodulated beam do not impinge the dielectric during the process of reading, this operation does not reduce the charge pattern potential. Hence the reading may be accomplished during a fairly long period of time. The service of such tubes shows that an undistorted output signal (with a good signal-to-noise ratio) can be obtained after 20-30 thousand continuous reading operations (by one reading operation is meant one period of vertical scan).

A storage tube with a collector-reflector provides only for alternating operations of writing, reading and erasing, since during the changeover from one operation to another it is necessary to change the potential of the grid-target and collector-reflector, i.e. these operations cannot be coincident in time.

The above-discussed storage tubes make it possible to memorize and reproduce information containing gradations of the value of signal, i.e. they relate to the so-called half-tone tubes. Obtaining half-tones of the gray is due to the fact that the target of such a tube can be used for forming thereon stored charge patterns at different

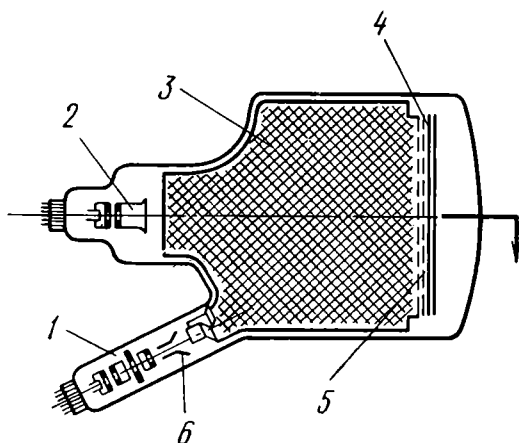


Fig. 11.6. Storage tube with "holding beam"

potentials corresponding to the distribution of the amplitudes of the signal being written. In contrast to these storage tubes, the memory tubes employed in computers usually operate under the bistable conditions, i.e. the target potential can have only two quite different values. Such conditions are very convenient, since most of the computers employ a binary system and, in order to record any information, two values of the target potential corresponding to zero (black) and unity (white) are sufficient.

The computers can also be provided with a storage tube having a barrier grid. However, the capacity-discharge reading in this tube does not allow the stored information to be read many times which is often necessary when performing mathematical operations. To this end, special tubes with a holding beam restoring the charge pattern potential after each reading cycle have been developed for the computer systems. The use of such holding of the charge pattern potential makes the number of readings practically unlimited.

A schematic diagram of a storage tube with a holding beam is shown in Fig. 11.6. The cylindrical bulb has two necks: one is arranged

along the tube axis and the other is positioned at an angle of $25\text{--}30^\circ$ to the axis. The cylindrical part has an electrically conductive circular coating 3 connected to a grid-collector 5. The necks have electrically conductive coatings connected to the gun anodes. The tube operates with an earthed collector so that all three sections of the electrically conductive coating are connected together and earthed.

The writing and reading are performed by the electron beam of a gun 1 mounted in the tilted neck. The holding beam is produced by a gun 2 mounted along the tube axis in the "straight-line" neck. In order to produce a high potential charge pattern during the writing operation which is required for correct action of the holding beam, the beam current of the recording gun must be several times the holding beam current. In order to obtain good focusing at a fairly high beam current, it is necessary to have a fairly high accelerating voltage (2.5 to 3 kV).

The storage tube with a holding beam is usually equipped with an electrostatic deflection system 6 for the writing and reading beams because of its simplicity and low energy consumption. Insufficient linearity of the deflection system and possible distortion of the raster at an inclined path of the beam cause no distortion to the output signal, as the writing and reading operations are accomplished by the same beam deflected by the same deflection system fed from a common sweep generator.

The target 4 of the storage tube consists of a thin layer of dielectric applied to a metal signal plate. Located in front of the target is a transparent (grid-type) collector of electrons.

The holding beam must have an electron energy equal to eU_{cr1} for the given material of the target. The holding can be performed by a focused beam scanned over the target surface. The same effect can be obtained by flooding the target surface with a scattered beam of slow electrons. Since in this case the gun construction is simpler and a deflection system is no longer necessary, the holding is effected through the use of a stationary defocused beam covering the entire surface of the target. Accordingly, the holding gun utilizes a simple triode circuit (cathode—modulator—anode) and is an immersion objective from the optical point of view. The anode potential of the holding gun is equal to zero, the cathode potential is close to $-U_{cr1}$, and the modulator potential is selected in compliance with the current required for reliable holding of the charge pattern potential.

The bistable writing in a storage tube with a holding beam can be either positive ("white on black") or negative ("black on white"). Before the writing, depending on the energy of the focusing beam electrons, the target potential is set to be equal to one of the possible equilibrium values. In positive writing the preparation of the

target for writing (erasing) comes to making the energy of the holding beam electrons somewhat less than eU_{cr1} . In this case $\sigma < 1$ and the target surface acquires an equilibrium potential equal to the potential of the cathode of the holding gun. In negative writing the preparations of the target consists in bringing its potential to zero (collector) potential (erasing at $\sigma > 1$).

The signal being written is applied to the modulator of the normally nonconducting recording gun in the form of a triggering pulse. At the same time a potential of 100 V (with respect to the collector) is fed to the signal plate. This voltage is negative during the positive writing and positive during the negative writing.

The electrons of the writing beam have an energy of 2 to 2.5 keV which in any case provides $\sigma > 1$. Thus, during the occurrence of the writing beam on the given element of the target its potential is brought to an equilibrium value approximately equal to the collector potential. However, after the writing the element potential changes due to the charge stored in the elementary capacitor and becomes higher than the collector potential at positive writing. In this case the charge pattern potential rises to several dozens of volts.

The reading is effected by an unmodulated beam discharging the elementary capacitors (capacity-discharge reading). When discharging the elementary capacitors a capacitive current pulse flows through the signal plate circuit. This current forms the output signal. Owing to the high potential of the charge pattern, each operation of reading does not cause complete discharge of the elementary capacitor, i.e. the charge pattern potential is maintained. Since the holding beam irradiates the target surface continuously, the reduced potential of charge pattern is permanently restored. This takes place because of the fact that between the grid-collector and the target there is a decelerating or accelerating field (depending on the polarity of the stored charge). Since the decelerating voltage of the holding gun is approximately equal to the first critical potential, in a decelerating field the electrons of the holding beam, when they approach the target, are retarded and the secondary emission coefficient becomes less than unity. In this case the reduced potential of the elements, involved in the writing operation, is brought to an equilibrium value equal to the potential of the cathode of the holding gun. In the presence of an accelerating field (positive writing) the electrons of the holding beam approaching the target are additionally accelerated thus providing for $\sigma > 1$. In that the potential of the target elements involved in writing is restored to an equilibrium value equal to the grid-collector voltage. Thus, despite the reading reducing the charge pattern potential, it is continuously restored and keeps its original value determined by the difference of the equilibrium values of the potential at $\sigma < 1$

and $\sigma > 1$ (the cathode potential of the holding gun and the grid-collector potential).

The number of readings is unlimited. Practically it is determined by the surface resistance of the dielectric layer and the degree of vacuum (the presence of ions). A good tube allows a few millions of reading operations without substantial distortion of the output signal. Like in any system with bistable writing reproduction of half-tones in this tube is impossible. Just the same, since the writing and reading are performed by one gun, these operations can be performed only in succession. From published sources tubes are known which utilize the same scheme but have three guns (writing, reading and holding) to allow the writing and reading operations to be performed simultaneously. Storage tubes with bistable writing can employ mosaic targets permitting the potential of charge pattern to be increased substantially. In addition to a signal plate and a dielectric layer the target of such a storage tube has a mosaic consisting of electrically conductive particles insulated from one another and arranged on the dielectric surface. With a higher potential of charge pattern the mosaic target has some advantages. In the case of bistable writing and a higher potential of charge target the potential difference between adjacent elements of the target may be of 100 V and more. Large surface potential gradients may cause shift of the charges over the uniform dielectric surface thus causing false signals. In the presence of an electrically conductive mosaic the potential of each mosaic particle is brought to equilibrium at the expense of its conductivity. In this case the leakage currents over the potential boundaries between mosaic particles at different potentials can be compensated for by increasing or decreasing the charge of the whole surface of the mosaic particle due to the secondary emission caused by the holding beam. The mosaic target is a sheet of homogeneous mica. One side of this sheet is coated with silver to form a signal plate. The other side carries a mosaic consisting of separate (insulated from one another) particles of berillium. The use of berillium is explained by a comparatively low (about 50-60 V) and stable value of the first critical potential, which is very significant for holding the charge pattern potential by the holding electron beam. In other respects the construction and principle of operation of the storage tube with a mosaic target do not differ from those of the above-described tubes.

11.4. Viewing Storage Tubes (Electrical-visual)

In some cases (for example, when oscillographing single-stage processes, in radar engineering) it is necessary to obtain an output information not only as a sequence of electric signals but also as a visible image on a fluorescent screen. There are also widely employ-

ed storage tubes producing output information only in the form of a visible image. The viewing storage tubes perform the same functions as oscillograph tubes, radar tubes or iconoscopes. But in contrast to conventional picture tubes these storage tubes allow the written information to be stored during a long period of time and, if necessary, to display it in the form of an image on a screen.

The viewing storage tubes can employ different methods of writing, the reading being, as a rule, performed by the grid-control method.

The storage tube with simultaneous reproduction of output information in the form of electric signals and a visible image has no principal difference from a storage tube with a collector-reflector. The only distinction is in some modification of the collector-reflector. If the collector-reflector is made in the form of a grid and a layer of phosphor is applied to the bulb face, then the storage tube will produce an output electrical signal in the collector-reflector circuit, as usually. However, due to the grid structure of the collector-reflector, some electrons will pass through the grid and form a visible image on the screen. In order to increase the screen brilliance, an additional accelerating electrode is mounted between the collector-reflector and the screen or the latter is provided with an electrically conductive coating transparent for the electrons and an adequately high (with respect to the cathode of the reading gun) positive potential is applied to this conductive film.

In storage tubes with bistable writing used in electronic computers (see Section 11.3) the output information can be obtained simultaneously in the form of electrical signals and a visible image. If in these tubes the signal plate is made semitransparent and a phosphor layer is used as a dielectric (many commercial luminophor compositions have good dielectric properties), then the written information can be observed through the signal plate in the form of an image. Since the holding beam has a high energy when approaching positive (white) elements, they will glow, i.e. we shall obtain a white image against a black background (with positive writing) or a black image against a white background (with negative writing).

The storage tubes of this group, in which the output information is obtained only as a visible image, are more common, bistable tubes being used in oscillographic technique and half-tone tubes being used in radar engineering.

Figure 11.7 illustrates a schematic diagram of viewing storage tube employed as an oscillograph tube.

The tube employs a gun 1 with electrostatic focusing system utilizing a "bipotential objective + unipotential lens" scheme. The deflection is also electrostatic (2). The target 4 is a fine-mesh grid coated with a dielectric layer on one side. The dielectric consists

either of calcium fluoride or a mixture of aluminum oxide or magnesium oxide. The screen 5 has an electrically conductive coating fed with a high voltage (up to 6-10 kV) which provides high brightness of luminescence. The writing is bistable. The signals to be written are fed either to the modulator or to the cathode of the writing gun. The accelerating voltage of the writing gun is equal to 1.5-2 kV thus ensuring $\sigma > 1$.

An interesting feature of this tube is in that it has no reading gun. A wide flow of "reading" electrons uniformly flooding the entire surface of the target is produced by the so-called viewing cathode 3 constituted by a row of parallel wires spaced at a few millimetres. The wires are located in a plane perpendicular to the

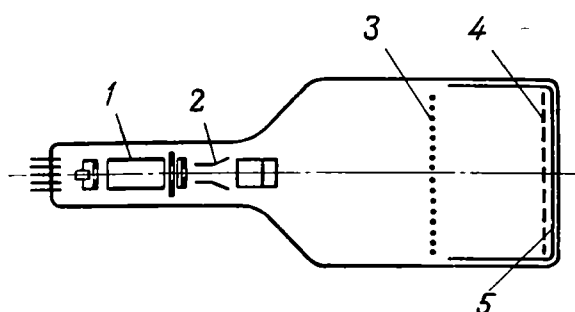


Fig. 11.7. Viewing storage tube

1—gun; 2—deflection system; 3—viewing cathode; 4—target grid; 5—screen

tube axis, between the deflection system and the target. The wires are given an oxide coating and are heated, by an electric current flowing in them, to a temperature providing perceptible emission of electrons. The electron flow produced by the viewing cathode passes through the target grid and is admitted into a strong accelerating field. In this case the electrons fly to the screen practically at right angles to its surface so that the information recorded on the target is accurately displayed on the screen.

Since the electrons of the reading beam do not bombard the dielectric (grid-control reading), the stored charge pattern is reliably kept on the target. In any case, during continuous reading a stable image can be observed on the screen for one hour and more.

Radar indicator storage tubes produce a half-tone image using the nonequilibrium writing and grid-control reading. The tubes have a writing gun (with electrostatic or magnetic focusing) located in an inclined neck shifted from the tube axis. The deflection system in most cases is magnetic. The reading gun is always located in the axis neck of the tube, since the electrons must be directed at right angles to the screen surface for correct display of the information. The reading gun utilizes a simple optical scheme: cathode + modulator + anode, i.e. is an immersion objective. The electrons of the reading gun diverge as a wide beam behind the

crossover plane uniformly flooding the entire screen surface. The resolution of the radar storage tubes is as high as 50 lines per cm, the recording speed is several thousands of km/s.

Employed in radar engineering are also storage tubes with character indication. In contrast to the digit printing tubes studied in Section 8.3, the storage tube with character indication has a target (potential carrier) storing the written information. These tubes employ equilibrium writing and grid-control reading. The path of the writing electron beam is intersected by a matrix giving the beam a cross section in the form of a certain sign. When such a matrix-modulated electron beam falls on a grid-target coated with

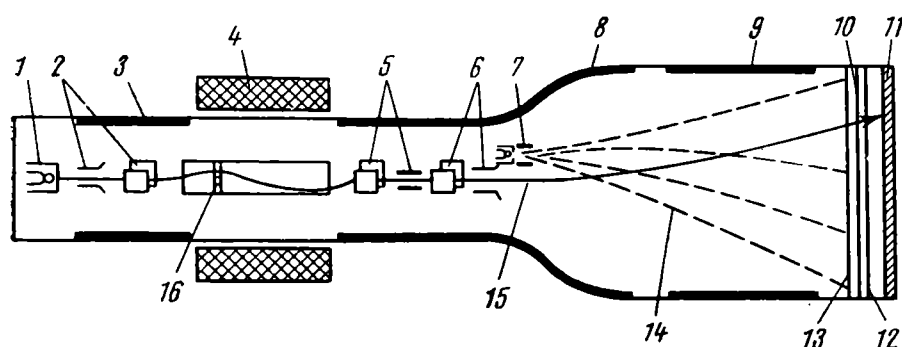


Fig. 11.8. Typotron

1—recording gun; 2—sign selection plates; 3—first anode; 4—focusing coil; 5—compensating plates; 6—address plates; 7—reading gun; 8—second anode; 9—third anode; 10—grid; 11—screen; 12—target; 13—ion deflector; 14—reading beam; 15—electron beam; 16—matrix

a layer of a dielectric material, a charge pattern in the form of the sign being written is produced on this target.

During the reading operation the reading gun beam passes through the grid-target only at the place of writing the signal and, when striking the screen, produces the written sign.

An example of a character printing storage tube is the so-called typotron (Fig. 11.8).

The neck of this tube houses an electron gun, sign selection plates directing the beam to a corresponding place on the matrix, as well as compensating and address plates. The compensating plates direct the electron beam passed through the matrix along the tube axis. The address plates direct the beam to a required region of the target. In the region behind the matrix (before the compensating plates) the beam is focused by a magnetic coil mounted on the tube neck.

Mounted in the wide portion of the bulb is a target in the form of a fine-mesh grid coated with a dielectric layer from the gun side. The bulb face is coated with a phosphor layer. The neck, adapter part and wide portion of the bulb have individual electrically conductive coatings at different potentials. One of the address plates

mounts a reading gun producing a wide non-focused electron beam flooding the entire surface of the target. During the writing operation the beam of the writing gun formed by the matrix as a sign is directed by the address plates to the selected area of the target. The energy of the electrons of the writing beam must be sufficiently large to provide for $\sigma > 1$. In this case formed on the target is a positive potential charge pattern due to the escape of secondary electrons.

During the reading the slow-velocity electrons of the reading beam do not pass through the grid-target where there is no written information. On the other hand the target sections with recorded characters are at a higher potential and the electrons of the reading beam get into an accelerating field, pass through the target and,

having been accelerated by the field behind the target, strike the screen and excite its luminescence.

A tube with photoelectron excitation of the phosphor is a modification of the viewing storage tube (Fig. 11.9).

A target is mounted in the cylindrical bulb of this tube. The bulb face opposite to the target is coated with a phosphor. The bulb has a neck arranged at an angle to the tube axis. The internal surface of the cylindrical part of the bulb and neck has an electrically conductive coating playing the

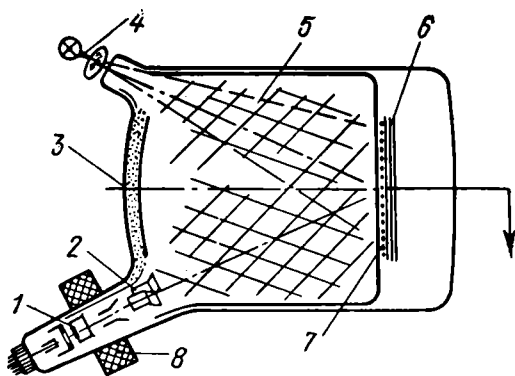


Fig. 11.9. Viewing storage tube with photoelectron excitation of luminophor 1—electron gun; 2—deflection system; 3—screen; 4—light source; 5—conductive coating; 6—target; 7—photocathode; 8—focusing coil

role of a collector. The tube has a single gun which is placed in the neck and is as writing and reading one at a time. This storage tube employs nonequilibrium writing and grid-control reading.

The gun of the storage tube is used during the writing (and erasing). The reading is accomplished by illuminating the target with an external source of light. During the process of reading the gun is turned off. At writing use is made of a fairly high accelerating voltage required for producing a negative potential charge pattern as $\sigma < 1$ in the region behind the second critical potential. In erasing the accelerating voltage is reduced to provide for $\sigma > 1$. The gun is usually based on a triode scheme (cathode-modulator-anode), a short magnetic coil put on the tube neck being used as the second lens. The beam deflection may be electrostatic or magnetic.

The target is a dielectric coating applied to the signal plate. Located on the dielectric surface are photosensitive particles (miniature photocathodes) electrically connected to each other. The

dielectric surface not covered with photocathodes serves as the potential carrier.

When preparing the storage tube for writing (or when erasing the written information), the target surface is scanned by an unmodulated beam at an adequately high current and an accelerating voltage providing $\sigma > 1$. In this case the signal plate and the associated photocathode are at a potential equal to that of the collector. In the process of erasing, since $\sigma > 1$, the target surface assumes an equilibrium potential approximately equal to the collector potential.

During the writing operation the accelerating voltage is increased to a value exceeding the second critical potential and a signal to be recorded is applied to the modulator. Since in this case we have $\sigma < 1$, the target potential is reduced and formed on the target surface is a negative potential charge pattern whose potential is approximately proportional to the current of the writing beam. Thus, the tube in question makes it possible to record half-tones. The written signal (charge pattern) in the absence of reading and erasing has a retention period of up to several days.

The reading can be accomplished by illuminating the elementary photocathodes with an external source of light. In this case the potential of the signal plate (and photocathodes) becomes negative with respect to the collector (conductive coating). The electrons emitted by the photocathodes (due to the illumination) are accelerated by the field of the collector and focused by a longitudinal uniform magnetic field built by an elongated coil mounted on the cylindrical part of the bulb. The stored charge pattern acts as a control grid of an electron valve. The electrons from the photocathodes located near the negatively charged target elements are decelerated and do not reach the screen or reach it in a small amount. Since the uniform magnetic field transfers the electron image from the target to the screen, the written signal is displayed on the screen as an image.

It is obvious that the output signal has the reverse polarity compared to the recorded signal, i.e. the maximum amplitude of the input signal corresponds to the dark spots of the image. The tube permits transmission of gradations of the gray, since in the case of nonequilibrium recording the potential of charge pattern may be of any value in the interval between the two equilibrium potentials (collector potential and second critical potential); the screen brilliance can also vary within a wide range depending on the current of the photoelectrons. The written data can be read within 10 to 15 minutes, then the image becomes deteriorated mainly due to reducing the potential of the charge pattern by positive ions.

Camera Tubes

12.1. General Principles of Telecasting

The first stage of the telecasting process is conversion of an optical image into a sequence of electrical pulses (video signals) carrying the information about the content (tonal gradation) of the picture being televised. The amplified video signal is radiated by the TV transmitting aerial. The TV signal is picked up by the aerial of a television receiver and after amplification is fed to the modulator of a kinescope on the screen of which the information carried by the video signal is reproduced as a picture identical to the original optical image. At normal operation of all the elements of the television channel the distribution of brightness on the kinescope screen corresponds to the brightness distribution over the surface of the scene being televised, i.e. we have an undistorted television picture on the kinescope screen.

The camera tube is the first element in the television channel and is used for conversion of an optical image into a video signal. Thus the camera tube performs the function opposite to that of the kinescope. The camera tube is a photoelectric cell to the cathode of which alternately projected are separate elements of the scene being televised. We may also imagine a single photoelectric cell, as a plurality of microscopic photocells (equal in number to the scanning elements of the picture) to which the optical image is projected. The signals from each photocell must be alternately switched on/off at a high speed for obtaining a required time sequence of the pulses. Generally three methods of scanning the image can be used: (1) moving the optical image with respect to a stationary photocell; (2) moving the photocell with respect to a stationary optical image; (3) electrical switching (commutation) of signals from a plurality of stationary photocells to which the optical image has been projected.

These methods of scanning can be accomplished by moving the entire optical image, the light ray "probing" the object or the photocell, as well as by using a mechanical switch (commutator) having an appropriate speed of operation. The first telecasting systems employed mechanical scanning. However, with an increase in the

television standard (increase in the number of scanning elements) practical realization of the mechanical scanning encountered still higher engineering difficulties. Therefore, at the present time electrical (electronic) scanning is used in all television systems. The speed of electronic scanning can be very high because the electron ray performing the scanning may be considered practically inertialess at frequencies up to 10-20 MHz.

Electronic scanning cannot be used for directly moving an optical image or a photoelectric cell. But if the optical image is preliminarily transformed into an electronic one by means of a photocell, it can be moved over the receiving element at a required speed. This method is practically employed in the camera tube known as a dissector. An electron ray can also be used for producing a point-type light source moving at a high speed and alternately illuminating the elements of the scene being televised. This possibility is realized in a system with a travelling beam used for transmission of motion picture films and diapositives. The systems with movable photocells have not been practically realized.

The most popular are camera tubes in which an electron ray scans an optical image projected onto a photosensitive surface or an electronic image transferred from a photocathode to a special target. The first method of scanning is practically used in camera tubes without image transfer (iconoscopes, orthicons), the other is used in image-transfer tubes (super-orthicons, super-iconoscopes). However, in the first case too, the electron ray scans essentially not an optical but an electronic image, since the optical image projected onto the photocathode is converted into an electronic image on its surface due to photoelectronic emission.

Transmitting devices in which the scanning elements successively produce electric pulses proportional to the brightness of the scanning elements of the picture being televised are known as systems of instantaneous or alternating action. Such devices include dissector and travelling-beam tubes. The tubes of the other group (iconoscope, orthicon, super-orthicon, super-iconoscope, vidicon) employ the principle of charge accumulation (see Section 12.3), hence they are called the electrostatic storage systems.

Regardless of the method of conversion and the construction, the camera tubes must meet some general requirements. First of all, they must have high signal-to-noise ratio. The camera tube itself and the other elements of a television channel may be a source of noise. Theoretical study and experimental works have shown that defects of the image caused by noise are not noticed by the viewer at a signal-to-noise ratio not less than 150. A satisfactory picture can be produced at a signal-to-noise ratio of not less than 10. This requirement, however, is not strictly obligatory. The experiments show that a high-quality TV picture can be obtained at

a lower signal-to-noise ratio. Thus, for example, we may assume conventionally that using super-orthicon tubes a perfect picture is obtained at $\psi=80-100$ (ψ stands for the signal-to-noise ratio), a good picture is obtained at $\psi=40-50$ and a satisfactory picture — at $\psi=25-30$. For tubes without secondary-emission amplifying (super-iconoscope, vidicon) a perfect picture is obtained at $\psi=25$, a good one — at $\psi=10-12$, and a satisfactory picture is produced at $\psi=7-8$. It should be noted that the presence of parasitic signals (noise) affects such important parameters of the tubes as transmission of tonal gradations, resolution, sensitivity.

The resolution of a camera tube determines the smallest size of picture details to be reproduced on the screen. At any rate, the resolving power of a camera tube expressed by a number of distinguishable lines laid on the whole image must not be less (preferably higher) than 625 lines according to the TV standard accepted in the USSR and in some other countries.

The picture quality is largely determined by the number of distinguishable tonal gradations. A human eye well adapted to average brightness can distinguish up to 100 brightness gradations. However, the contrast sensitivity of the eye defining the number of distinguishable brightness gradations depends on the average brightness of the entire image. The lower the average brightness, the lower is the contrast sensitivity of the eye.

Therefore, it is very desirable that the camera tube provide for an adequate number (at least 10) of tonal gradations at low and average brightness of the picture.

One of the main characteristics of camera tubes is the dependence of the useful signal on the illumination of the corresponding element of the photocathode receiving the luminous flux from the scene. This dependence is known as the light characteristic of the camera tube. It is obvious that the correct transmission of an image, i.e. distribution of the brightness on the screen corresponding to the distribution of the light over the surface of the scene being televised, is obtained when the light-signal-light transmission characteristic of the entire television channel is linear. The dependence of the kinescope brightness on the signal determined by the modulation characteristic of the kinescope gun is not linear and in the general case can be expressed as

$$B_s = k_2 U_{vs}^{\gamma_c} \quad (12.1)$$

where B_s is the screen brightness, U_{vs} is the voltage of the video signal fed to the kinescope modulator. In this case γ_c is greater than 1 ($\gamma_c \approx 5/2$ for an electrostatically focused gun, see Section 4.3).

Assuming that the intermediate links of the television channel such as video amplifiers found between the camera tube and the kinescope have linear characteristics ($\gamma_a = 1$), the optimum "light-

signal" characteristic of the camera tube can be expressed through the following dependence

$$i_{vs} = k_1 E^{\gamma_{ct}} \quad (12.2)$$

where i_{vs} is the current of the video signal produced by the camera tube; E is the illumination of the photocathode of the camera tube, in which case $\gamma_{ct} < 1$.

The linearity of the transmission characteristic of the whole television channel results in the equality $\gamma_{ct} + \gamma_a \gamma_c = 1$ which in a first approximation (at $\gamma_a = 1$) corresponds to $\gamma_{ct} = 0.4$.

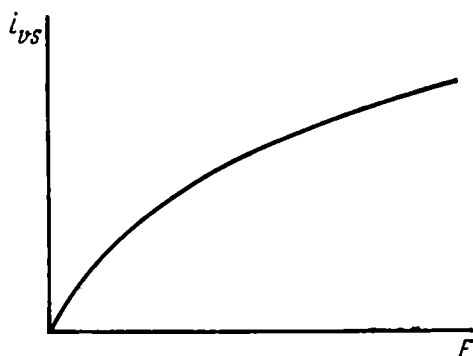


Fig. 12.1. Optimum light characteristic of camera tube

An approximate view of the optimum light characteristic of the camera tube is shown in Fig. 12.1.

It should be noted that the given characteristic is optimum also from the point of view of the possibility of conveying of a large number of tonal gradations, since with such a shape of the curve under the conditions of inadequate illumination small changes in E cause greater variations of the video signal than in the case of good illumination. Actual camera tubes have light characteristics perceptibly differing from the optimum one. More than that, these characteristics are not single-valued, since they vary with variation of the content of the picture being transmitted (distribution and size of bright and dark areas, contrast, etc.). Therefore, in a television channel, between the camera tube and the kinescope there must be inserted γ -correctors eliminating the discrepancy between the characteristics of the camera and picture tubes and the optimum relationships.

The spectral characteristic of the photocathode of a camera tube is an important characteristic of it. The picture on the screen of a black-and-white TV receiver has a constant spectral composition determined by the type of the phosphor of the iconoscope. At the same time, the scene being televised is usually not monochromatic.

It is clear that the spectral characteristic of the iconoscope must correspond to the "visibility curve" of the human eye. Only under such correspondence of the characteristics of the photocathode and the eye the brightest elements of the scene will be the brightest ones on the television picture as well. Consequently, the cathodes of camera tubes must have spectral characteristics sufficiently close to the "visibility curve" of an ordinary human eye. In some cases colour light filters are used for correction of the spectral characteristic of the photocathode. The photocathodes of colour camera tubes must have the maximum sensitivity respectively in the blue, green and red portions of the spectrum.

A camera tube employed in television must have high sensitivity. The tube sensitivity is evaluated by the minimum illumination of the photocathode at which there is still obtained a signal-to-noise ratio corresponding to the adequate display of the picture. The higher the sensitivity, the lower is the illumination of the scene whose image is satisfactorily conveyed by the given tube. The requirement for high sensitivity is particularly important for camera tubes used for televising of natural outdoor scenes. Modern highly sensitive tubes produce satisfactory pictures ($\psi = 15-20$) at illumination of the photocathode lower than 1 lx.

One of the requirements placed on camera tubes is their low persistence. The residual signal from the preceding frame must not be noticeable during the transmission of the next frame, otherwise bad distortions appear during the transmission of moving objects.

Finally, it is desirable that the camera tube reproduce the black level, i.e. the initial level from which the absolute value of the video signal is read. In fact, when transmitting a video signal from a camera tube to a kinescope through a radio engineering channel, there is preserved only an alternating component carrying the information about a change in the illumination of the photocathode elements. For correct display of a picture on the kinescope screen it is necessary to select an initial point on the modulation characteristic of the kinescope gun corresponding to the reference point of the video signal value. This initial point corresponds to the darkest (black) level of illumination of the scene (and the photocathode). Some types of camera tubes form a signal corresponding to the black level during the retrace of the sweep electron beam, and this much simplifies the problem of correct distribution of the brightness on the kinescope screen in accordance with that of the scene.

The general requirements given above, of course, are of different importance for different camera tubes. Some of these requirements may be of no significance, while the others turn to be of special importance, depending on the field of application and construction of the tube. However, the requirement for sufficient signal-to-noise ratio is of the greatest importance in all practical cases.

One can easily see that the sensitivity of modern photocathodes in alternating-action systems is insufficient for obtaining a satisfactory picture even of brightly illuminated scenes.

Estimate the signal-to-noise ratio assuming that the basic noise component is caused by the statistic fluctuations of the photocathode current. The mean value of the square of the noise current appearing due to the photocurrent fluctuation is given by

$$\bar{i}_n^2 = 2ei_p\Delta f \quad (12.3)$$

where i_p is the photocurrent, Δf is the frequency band used for the transmission of the image.

Let us calculate a possible value of the photocurrent produced by the photocurrent area corresponding to one scanning element of the image being transmitted. Assume that the scene has brightness B_0 . This scene is projected on the photocathode through a lens with an aperture angle Θ . In this case the illumination of the image on the photocathode is

$$E_0 = \pi B_0 \sin^2 \frac{\Theta}{2} \quad (12.4)$$

The luminous flux incident on the photocathode with an area s_p is

$$\Phi = E_0 s_p \quad (12.5)$$

With a TV standard of N lines and the raster side ratio of 3 to 4 the number of scanning elements is equal to

$$n = \frac{4}{3} N^2 \quad (12.6)$$

When the photocathode sensitivity is k_p the photocurrent of one scanning element is equal to

$$i_{pn} = \frac{k_p \pi B_0 \sin^2 \frac{\Theta}{2} s_p}{\frac{4}{3} N^2} \quad (12.7)$$

On a bright sunny day the illumination may be as high as 10^5 lx which corresponds to the brightness of an ideally illuminated surface (for example, snow), about $30,000$ cd/m². With the photocathode sensitivity of $k_p = 50 \mu\text{A/lm}$, an aperture angle $\Theta = 90^\circ$ (this is realizable in good lenses), the photocathode area of 0.02 m² and the number of lines $N = 600$, the value of the photocathode current of each element is given by

$$i_{pn} = \frac{50 \times 10^{-6} \times 3.14 \times 3 \times 10^4 \times 0.5 \times 10^{-2}}{\frac{4}{3} (600)^2} \approx 5.2 \times 10^{-12} \text{ A}$$

The mean value of the noise current on a frequency band of 5 MHz is equal to

$$i_{nn} = \sqrt{2ei_{pn}\Delta f} = \sqrt{2 \times 1.6 \times 10^{-19} \times 5.2 \times 10^{-12} \times 5 \times 10^6} \approx 2.8 \times 10^{-12} \text{ A}$$

The result obtained shows that even in the case of an ideally illuminated scene the signal-to-noise ratio is less than two, which is insufficient for obtaining a good picture.

Thus, the element-to-element transmission of an image with the use of conventional photoelectric cells cannot be practically realized. Consequently, the camera tubes operate with amplification of the photocurrent either in the device itself or with intensifying the image (as in the image converters) and increasing the sensitivity either due to the secondary electronic amplification, the so-called storage effect or, finally, by means of the combination of these factors.

The use of external amplifiers for increasing the photocurrent will obviously not improve the signal-to-noise ratio, since in this case the noise is amplified together with the useful signal.

12.2. Instantaneous Action Systems

An instantaneous action device, the so-called dissector (Fig. 12.2), was one of the first camera tubes.

In this device an optical image is projected on a semitransparent photocathode formed on the internal surface of a flat face of a cy-

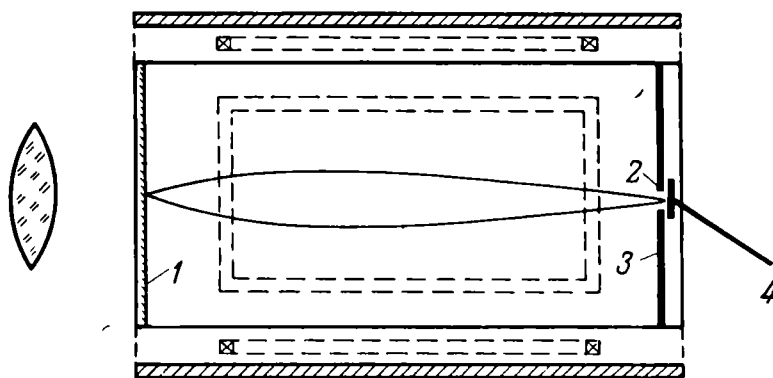


Fig. 12.2. Dissector tube

1—photocathode; 2—aperture; 3—screen; 4—collector

lindrical vacuum tube. Arranged opposite to the photocathode is a metal screen with a small aperture equal in size to one element of a television picture. The potential difference between the photocathode and the screen produces approximately uniform longitudinal

electrostatic field accelerating the electrons moving towards the screen. The entire tube is arranged in a longitudinal uniform magnetic field produced by a long solenoid located outside the tube. The magnetic field transfers the electron image from the photocathode to the screen. The electron image is moved by two pair of deflecting magnetic coils fed with saw-tooth currents at a frequency of line and frame scanning in the screen plane so that all the elements of the image are coincident with the aperture in succession, line by line. It is obvious that at any moment of time passed through the aperture is a photocurrent corresponding to the illumination of a single image element. Thus, formed in the circuit of the collector mounted behind the aperture is a succession of electric pulses, i.e. a video signal is produced.

A simple dissector can produce an adequate image (sufficiently high signal-to-noise ratio) only at high illumination of the scene (a few hundreds of thousands of luxes). Therefore, it has not gained wide recognition for transmission of natural scenes. At the same time, the dissector features high contrast and resolution (defined by the size of the aperture which can be made sufficiently small), a simple construction (due to the absence of an electron gun with an incandescant cathode), small overall dimensions and weight. The light characteristic of a dissector is practically linear. The positive properties of the dissector allowed it to be used for motion picture transmission, since in this case the required high illumination of the photocathode can be obtained by means of a high-power external source.

The development of low-noise secondary-emission multipliers made it possible to improve dissectors. In modern dissectors the photocurrent from each scanning element, having passed the aperture, is taken by an input dynode of the secondary-emission multiplier placed directly behind the screen opposite to the aperture. The secondary-emission multiplier can provide multiplication of the photoelectric current by 10^6 times. Since the intrinsic noise level of the multiplier is comparatively low, it is possible to obtain a signal-to-noise ratio of the order of a few dozens with relatively small illumination of the object. When employing secondary-emission multiplication, the dissector can successfully compete with other types of camera tubes due to its simplicity, reliability, high resolution and long life owing to the absence of an incandescent cathode.

The so-called channel electron multiplier is very promising for use in dissectors. The channel electron multiplier consists of a tube of a special electrically conductive glass having a high secondary emission coefficient. A voltage is applied to the ends of the tube, which produces a longitudinal electrostatic field.

In the tube channel the electron is accelerated by the electrostatic field and, when impinging the channel wall, knocks out a few second-

dary electrons. The secondary electrons are again accelerated by the field and knock out new secondary electrons, etc. (Fig. 12.3).

With correct selection of the geometric dimensions (length and diameter) of the tube and the accelerating voltage it is possible to obtain multiplication by a factor of 10^6 and higher. Since the

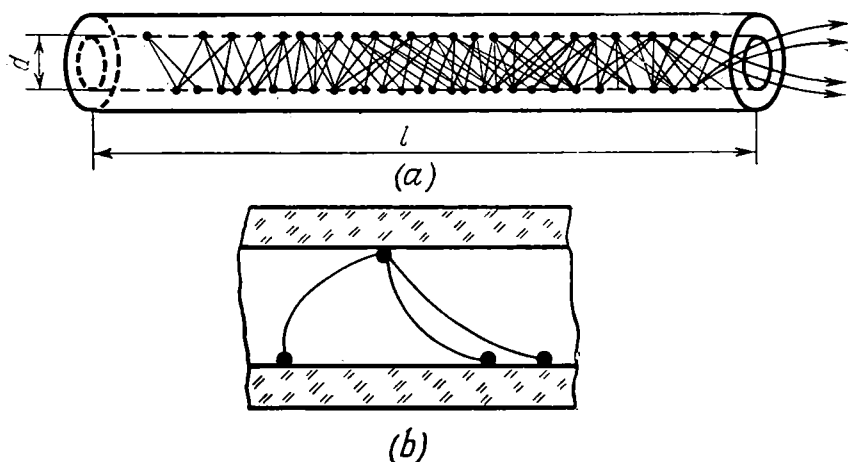


Fig. 12.3. Channel electron multiplier
(a) design; (b) electron trajectories

dissector aperture is small, it is expedient that a channel multiplier with a small diameter of the tube is located behind the aperture. In order to reduce the overall dimensions of the device, the channel multiplier tube can be coiled into a spiral.

Another alternating-action transmitting device is a scanning beam tube used for transmission of transparent objects such as

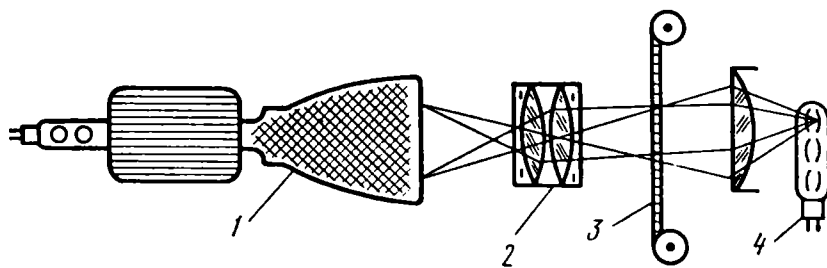


Fig. 12.4. Scanning-beam system

motion pictures and diapositives. The scanning-beam system (Fig. 12.4) consists of a kinescope 1 with a small brightly glowing screen having a high resolving power, a light-optical system 2 projecting the television raster formed on the kinescope screen to a transparent object (cinema film) 3, and a photoelectric multiplier 4 sensing the luminous flux having passed through the object.

Since when scanning the kinescope screen with an electron beam, a separate screen element corresponding to one resolution element of the image glows at a time, the light passed through the film is modulated in accordance with the distribution of the transparency of the elements of the image on the film. Thus, a video signal is formed at the multiplier output. The use of special kinescopes with magnetic focusing and deflection of the beam at accelerating voltages of 15-20 kV allows one to obtain a screen brightness of at least several thousands of nits at resolution of up to 1000 lines. The scanning-beam system provides high-quality motion picture transmission, it is comparatively simple and reliable in operation.

Thus employed in the alternating-action systems, having found practical application, is the principle of secondary-emission amplification of the photocurrent in the device itself, since only in this case it is possible to obtain a sufficiently high value of signal-to-noise ratio providing for reproduction of a high-quality picture.

12.3 Charge Storage Effect

As mentioned above, a significant disadvantage of the alternating-action transmission systems consists in the use of a low photoelectric current from a single element of the image for forming a video signal at a certain instant of time. In the charge storage systems a video signal is produced by means of an electric charge formed on a photosensitive surface during the transmission of a single frame, i.e. of all n elements of the picture thus increasing the sensitivity of the transmission system and improving the signal-to-noise ratio. The principle of charge storage employed in camera tubes does not differ from the principle of forming a stored charge pattern on the target of a memory tube discussed in Section 11.1. An optical image in TV charge storage tubes is projected onto the surface of a photosensitive target. In this case a stored charge pattern is produced on this surface due to the removal of photoelectrons. During the electron-beam scanning this pattern is transformed into a succession of electric pulses. Therefore, a camera tube using the charge storage principle serves as a memory tube with writing of information due to the photoelectron emission of the target and with reading by means of scanning the target surface with an electron beam. In charge storage tubes the electrons emitted by a photocathode receiving an optical image are used for charging the elementary capacitors formed by the elements of the photosensitive target and the signal plate. When scanned with an electron beam the charged capacitors discharge producing a succession of electric pulses. If the value of the charges stored by the elementary capacitors is proportional to the luminous fluxes incident to each element of the photocathode, it is clear that the discharge currents of the capacitors

will correctly transmit the distribution of illumination over the cathode surface.

Since the charge of the capacitors occurs during the whole interval of time while the scanning beam runs $n - 1$ elements of the image, it might be expected that the magnitude of the signal during the capacitor discharge is n times the magnitude of the signal from each element in the alternating-action systems.

Assume that the entire photocathode is exposed to a luminous flux Φ_0 . With n resolution elements the luminous flux incident on each element is equal to $\Phi_n = \Phi_0/n$. This luminous flux acting on the photocathode having sensitivity k_p will cause a photoelectric current from one element equal to $i_{pn} = K_p \Phi_0/n$. During the time T for scanning the whole raster the beam rests on each element during a time period $\tau = T/n$. If each element of the photocathode is connected to the elementary capacitor having capacitance C , then the charge stored by the capacitor in scanning all the photocathode elements is

$$q = CU = \int_0^T i dt \quad (12.8)$$

If during the transmission of one frame the illumination of the given element is constant, $i = \text{const} = i_{pn}$, then the charge stored by the elementary capacitor is equal to

$$q = i_{pn}T = \frac{k_p \Phi_0 T}{n} \quad (12.9)$$

The capacitor is discharged during the scanning of one element, i.e. $\tau = T/n$. The mean discharge current is approximately

$$i_n \approx \frac{q}{\tau} = \frac{k_p \Phi_0 T}{\tau n} = k_p \Phi_0 \quad (12.10)$$

Expressing the luminous flux through the brightness of the transmitted image, B_0 , we have

$$i_n = k_p \Phi_0 = k_p \pi B_0 \sin^2 \frac{\theta}{2} s_p \quad (12.11)$$

Comparing the obtained expression with (12.7) we may see that the discharge current of each element is increased as compared to the photoelectric current $4/3 N^2$ times, i.e. $i_n = \frac{4}{3} N^2 i_{pn}$.

In this case the sensitivity determined by the minimum illumination of the object, which produces the useful signal, is 10 times the noise signal and can rise even more than n times. This is explained by the fact that during the discharge of the capacitor, random oscillations (fluctuations) of the photoelectric current within the scanning period may either increase or decrease. Therefore, generally the fluctuations of the discharge current may be less than the fluctuations of the photoelectric current.

Thus the charge storage effect theoretically allows the sensitivity of a transmitting device to be increased more than N^2 times, where N is the number of lines.

The use of the charge storage effect made it possible to develop high-sensitive camera tubes which ensure signal-to-noise ratios higher than 10 with very low brightness of the object (of the order of a few dozens of lux). Combination of the charge storage effect with electronic amplification of an image allows the sensitivity of transmitting system to be increased several times. At the present time the charge storage tubes with electron amplification provide high-quality pictures with illumination of the photocathode of less than 1 lx which corresponds to the brightness of the scene being televised of not higher than 1 cd/m².

12.4. Iconoscope

The *iconoscope* is one of the first camera pick-up photoelectric storage tubes having found practical application. The iconoscope is a memory tube with a photosensitive target, scanning by a beam of high-speed electrons, writing and reading by means of redistribution of the charges over the target surface. The iconoscope has a comparatively simple construction, since it has no electron amplification (image transfer) and secondary-emission multiplication. The principles of operation of the iconoscope were formulated in 1931 by Soviet scientist S. Kataev; somewhat later the American scientist V. K. Zworykin made the first model of such a tube. Despite some disadvantages, the iconoscope was for a long time the most popular pick-up tube, since it provided a high-quality image at adequate illumination of the scene being televised.

A schematic diagram of the iconoscope is given in Fig. 12.5.

The iconoscope consists of a cylindrical bulb with a neck fixed at an angle of approximately 30° to the bulb axis. The bulb encloses a photosensitive target consisting of a great number of elementary photoelectric cells located on a thin dielectric plate. The back side of the dielectric plate is coated with a metal layer having a terminal. This layer serves as a signal plate. An electron gun producing a well focused beam of high-speed electrons is mounted in the neck. The electron beam is moved over the target surface by the deflection device and forms an ordinary television line-structure raster.

The electrically conductive coating of the bulb walls is a collector of the photo and secondary electrons emerging from the target. The image transmitted by optical means is projected onto the front (photosensitive) side of the target. The video signal is taken from the signal plate. The photosensitive surface of the target consists of separate metal particles electrically insulated from one another. Each particle forms an elementary photoelectric cell, i.e. the photo-

sensitive surface of the target may be regarded as a combination of independent photoelectric cells whose number exceeds the number of the scanning elements of the televised picture. Due to the discrete structure of the photosensitive layer of the target it is called a *mosaic*.

The elementary photoelectric cell of the mosaic and the signal plate separated by a dielectric form a capacitor having a capacitance of 100-300 pF per cm² of the surface. The photosensitivity of the mosaic attains a value of 10-15 μ A/lm. Since the scanning

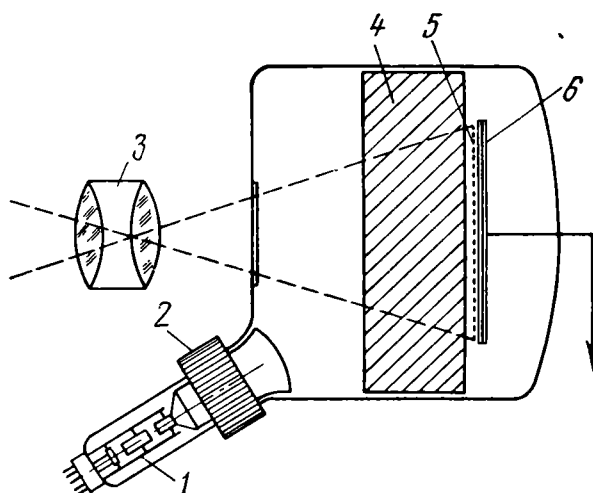


Fig. 12.5. Iconoscope

1—electron gun; 2—deflection system; 3—lens; 4—collector; 5—mosaic; 6—signal plate

is effected by a beam of high-speed electrons the secondary emission coefficient of the mosaic is higher than unity and may vary in the range of 3 to 7, depending on the type of the target.

The electron beam current is usually small within the range of 0.1 to 0.3 μ A. Since the secondary emission coefficient of the mosaic is higher than unity, one could expect that during the scanning of a *nonilluminated* surface of the target its potential assumes an equilibrium value which is somewhat higher than the collector potential.

However, due to a comparatively large distance from the target to the collector (electrically conductive coating) the field near the target surface is very weak. Consequently, a considerable portion of the secondary electrons released by the beam is returned back to the target thus reducing its potential. When scanning the non-illuminated surface of the mosaic with an electron beam the potential of any of its points does not remain constant. The mosaic element directly under the electron beam acquires a positive potential of 3-4 V with respect to the collector due to the secondary emission.

The secondary electrons returning to the mosaic reduce its potential. The current of these electrons is distributed over the entire surface of the target so that the mosaic elements, which at this moment are not subjected to the electron bombardment, acquire a potential 1-1.5 V below the collector potential. Upon reaching a certain small negative value, the potential is no longer reduced, since the secondary electrons cannot return to the negatively charged target.

Thus, when the nonilluminated mosaic surface is scanned by an electron beam, the potential of its elements varies in the range from $+(3-4)$ to $-(1-1.5)$ V with respect to the collector. However, the mean current of the secondary electrons emerging from the target remains unchanged and equal to the beam current, i.e. there is no video signal.

When an optical image is projected onto the mosaic, its elements start losing electrons and acquire a positive charge due to the photoelectric emission. Since the target surface has no strong field capable of removing all the electrons from the target, a considerable number of these electrons returns to the mosaic. Therefore, the photoelectronic emission as a whole is unsaturated. Thus the writing of an image is effected largely due to the redistribution of the charges on the mosaic surface. Owing to such redistribution a pattern of charges is produced on the target surface. This pattern corresponds to the distribution of light over the mosaic elements. Since the light image is projected continuously, the process of redistribution of charges is also continuous within the time interval between the "on/off switchings" of the given element by the electron beam. Consequently, in spite of the unsaturated photoelectric emission, the storage effect is utilized to a sufficient degree.

The reading of the image written on the target is performed by scanning its surface with the electron beam. When the electron beam falls on the target elements, their potential changes approximately the same amount as during the scanning of the dark surface of the target. Yet the number of electrons emerging from various elements of the target will be not the same due to the presence of the potential of the charge pattern produced by the optical image on the account of the photoelectric emission. Less electrons will emerge from the illuminated element than from the dark element because their elements are different potentials. Because of this, the collector current and, therefore, the capacitive current in the signal plate circuit will vary in accordance with a change in the target illumination. The modulated capacitive current flowing in the resistor in the signal plate circuit produces a voltage drop varying in proportion to the mosaic illumination and serving as the video signal. Approximate variation of the potentials of the nonilluminated and illuminated elements of the mosaic in the time interval

between the scanings of the element by the electron beam is shown in Fig. 12.6.

Due to the unsaturated state of the photoelectric current and secondary emission current a considerable number of photoelectrons and secondary electrons return to the mosaic causing the charge redistribution. All this accounts for the complexity of the quantitative theory of the iconoscope operation. We can only approximately estimate the currents taking part in producing a video signal. The photoelectrons producing the positive current i_{pn} emerge from the illuminated mosaic element which is not exposed to the electron

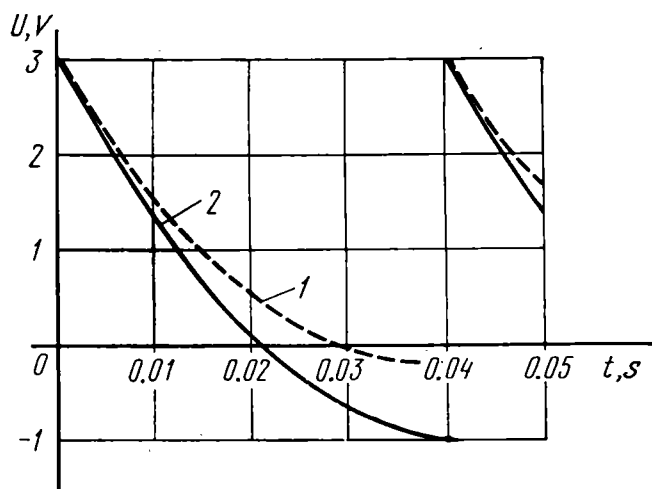


Fig. 12.6. Potential variation of illuminated mosaic element (1) and of non-illuminated element (2)

beam. At the same time, the secondary electrons from the other mosaic elements and a part of the photoelectrons return to the same element thus producing negative currents i'_{pn} and i'_{2n} .

Assume that the mosaic element in question has an area ds . Then, if the mosaic has a capacitance C_0 per sq. cm of the surface with respect to the signal plate, then a change of the element potential ds in time can be written as follows

$$\left. \frac{dU}{dt} \right|_0 = \frac{i_{pn} - i'_{2n} - i'_{pn}}{C_0 ds} \quad (12.12)$$

The mosaic element, which is not illuminated and scanned by the electron beam, does not emit electrons but receives the secondary electrons and photoelectrons returning from the adjacent illuminated and switched elements i'_{2n} and i'_{pn} . The change in the potential of this element is written as follows

$$\left. \frac{dU}{dt} \right|_{n1} = - \frac{i'_{pn} + i'_{2n}}{C_0 ds} \quad (12.13)$$

The comparison of expressions (12.12) and (12.13) shows that the illumination of the target results in a change in the potential distribution over the mosaic, i.e. it produces a stored charge pattern.

When the mosaic element is scanned by the electron beam, it receives electrons and, at the same time, emits secondary electrons. In this case the element potential rapidly changes from zero or small negative value (with respect to the collector) to a certain positive value. Let us estimate the magnitude of this change

$$\Delta U = \frac{i'_{2n} - i_b}{C_0 ds} \tau \quad (12.14)$$

where i_b is the beam current; τ is the time of residence of the beam on the given element.

Transmitting a picture with 25 frames per second at a TV standard of 600 lines, the residence time of the beam on a single scanning element is equal to

$$\tau = \frac{T}{n} = \frac{0.04}{\frac{4}{3} (600)^2} \approx 0.8 \times 10^{-7} \text{ s}$$

At a beam current of $0.2 \mu\text{A}$ and a secondary emission coefficient of the mosaic $\sigma = 5i'_{2n} = 1 \mu\text{A}$. The mean capacitance of the mosaic is equal to 100 pF/cm^2 . The area of the scanning element for a standard mosaic ($9 \times 12 \text{ cm}$) at 600-line scanning is equal to

$$ds = \frac{9 \times 12}{\frac{4}{3} (600)^2} \approx 2.2 \times 10^{-4} \text{ cm}^2$$

Then

$$dU = \frac{10^{-6} - 0.2 \times 10^{-6}}{10^{-10} \times 2.2 \times 10^{-4}} \times 0.8 \times 10^{-7} \approx 3 \text{ V}$$

Experience shows that the mosaic potential varies during the scanning operation in the range of $-(0.5-1)$ to $+(2-3) \text{ V}$, which confirms the above calculation. However, good coincidence of the calculations with the experimental data also shows that the secondary emission current of the mosaic element is apparently saturated before the scanning since the current of the electrons returning to the mosaic was not taken into account during the calculation. The saturation of the secondary emission current from the given element may be explained by the presence of scanned elements near the given element. The scanned elements are at higher potential and are capable of receiving a considerable portion of secondary electrons emerging from the element being scanned. This assumption holds also for the photoelectric current from the illuminated element of the mosaic immediately before scanning. However, the time of existence of the saturated photoelectric current is very low

as compared to the period of frame scanning. Therefore, generally the photoelectric current may be considered unsaturated. It has been found experimentally that at low illumination the average photoelectric current is 20-25 % of the saturation current. The unsaturated state of the photoelectric current increases the sensitivity of the device due to the charge storage effect.

When scanning an illuminated mosaic element a video signal is produced. The potential of the illuminated element decreases slower than that of the dark elements due to the loss of electrons during the scanning [see expressions (12.12) and (12.13)]. Thus by the instant of scanning the former is higher than the latter. The scanning beam changes the potential of the illuminated element practically by the same value as the potential of the nonilluminated element.

Thus after the scanning the illuminated element becomes more positive. Immediately after the scanning, the secondary electrons and photoelectrons from the adjacent elements rush towards this element and the current in the collector circuit is reduced. The reduction of the collector current is accompanied by a pulse of capacitive current in the signal plate circuit. The varying capacitive current flowing in the resistor in the signal plate circuit produces a voltage drop across this resistor, which serves as the video signal.

It is clear that the video signal amplitude will be the higher, the higher is the potential of the illuminated element before the scanning. However, as follows from expression (12.12), the rate of the rise of the potential of the illuminated elements is considerably reduced by the secondary electrons and photoelectrons returning to this element. Experiments have shown that 70-75 % of the stored charge is neutralized by the redistributed electrons and only 25 % takes part in the generation of the video signal. Thus, the video signal does not exceed 1/4 of a theoretically possible value.

Hence, the actual video signal is far less than the theoretical one due to the influence of two factors: unsaturated state of the photoelectric current and reduction of the charge pattern potential due to the electron redistribution. The sensitivity increase due to the charge storage is practically equal only to 3-6 % of the theoretically expected value. However, even with such a low efficiency of the iconoscope, the gain in sensitivity compared to the alternating-action systems without charge storage is (with 600-line resolution)

$$(600)^2 \frac{4}{3} 0.05 = 24,000$$

This example shows that, in spite of a low efficiency, the iconoscope is much advantageous over a system without charge storage. At low illumination levels the light characteristic of the iconoscope is close to linear. However, the conditions of collecting the photoelectrons by the collector become worse with a rise in the mosaic

illumination. Moreover, the highly illuminated mosaic elements having a higher positive potential are more intensively populated by the secondary electrons from the adjacent elements. This decreases the rise of the output signal at high illumination. Thus, the light-signal characteristic of an iconoscope is rather close to optimum (see Fig. 12.1). It can be approximated by a parabola with an exponent γ equal to $1/2$. Such a characteristic provides satisfactory tonal gradation in an adequately wide range of change of illumination.

An external view of the iconoscope is shown in Fig. 12.7. The iconoscope target consists of a mica sheet one side of which is coated with a continuous electrically conductive layer (signal plate)

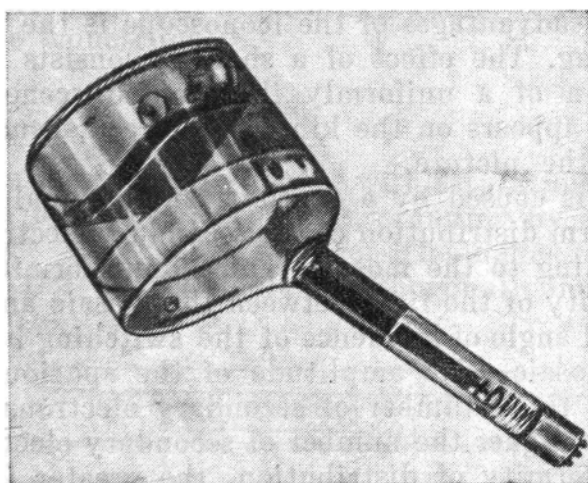


Fig. 12.7. External view of iconoscope

and the other side carrying a mosaic. The mosaic consists of separate metallic (usually silver) grains $20\text{--}30\text{ }\mu\text{m}$ in diameter coated with caesium. The size of a standard target is $9 \times 12\text{ cm}$. A comparatively large surface of the iconoscope target provides high resolution (600-800 lines) even with not-too-narrow beam. At the same time small size of the mosaic grains is useful, since the scanning electron beam covers a few hundreds of elementary photoelectric cells and the nonuniformity in the size and sensitivity of separate elements of the mosaic practically does not affect the resultant signal.

Iconoscopes are equipped with electrostatically focused guns providing a spot with a diameter of $0.1\text{--}0.15\text{ mm}$ at small currents of the beam. Two pair of magnetic coils are used for deflection of the beam. The keystoneing of the raster appearing due to the inclined path of the beam is compensated for by additionally modulating the current in the deflecting coils in such a manner that the beam deflection angle increases with a decrease in the distance to the screen (when the beam moves downwards).

The sensitivity of an operating iconoscope sometimes increases and results in a disproportionate increase in the signal from the most illuminated areas of the mosaic. This phenomenon known as line sensitivity is due to the saturation of the photoelectric current from the line preceding the line being switched. The line potential raised at the expense of the scanning effect produces a local field intensively with drawing photoelectrons from the adjacent line. If the latter has brightly illuminated elements, the saturation photoelectric current from these elements can reach a considerable value, thus raising the video signal. The presence of line sensitivity at high illumination increases the contrast of the image transmitted by the iconoscope.

One of the disadvantages of the iconoscope is the presence of the so-called shading. The effect of a shading consists in that during the transmission of a uniformly illuminated scene a shading of irregular shape appears on the kinescope screen, usually at the left upper part of the picture.

The shading is caused by a spurious video signal appearing due to the nonuniform distribution of the secondary electrons and photoelectrons returning to the mosaic. This nonuniformity results from the nonuniformity of the field between the mosaic and the collector and the unequal angle of incidence of the switching beam on various parts of the mosaic. The amplitude of the spurious video signal depends on the total number of secondary electrons released from the mosaic. The higher the number of secondary electrons, the greater the nonuniformity of distribution, the greater is the spurious video signal. A decrease in the electron beam current and the secondary emission coefficient of the mosaic contribute to reduction of the shading effect. However, a decrease in the beam current below a value of 0.15-0.1 μA is inexpedient, as the number of electrons of the beam may be insufficient for adequately changing the potential of the mosaic at the instant of scanning.

A decrease in the secondary emission coefficient can hardly be realized in practice, since the photosensitive surface with a low yield is usually a good secondary emitter. The shading effect is reduced by introducing into the television channel, an artificially generated signal of the opposite polarity.

A cathode-ray tube having a construction similar to that of an iconoscope is used for transmission of a test card, i.e. a stationary standard image for checking and testing television receivers. This tube known as a monoscope has an electron gun and deflection systems similar to those of a standard iconoscope, but instead of a mosaic, the monoscope has a plate made of a material with a high secondary emission coefficient. Such a material may, for example, be composed of oxidized aluminum having $\sigma > 10$ at the energy of the primary electrons of the order of 1 keV. The image to be trans-

mitted is printed on the plate with the aid of graphite or another substance having a low secondary emission coefficient.

When the electron beam scans the secondary emission target, the current of the secondary electrons flying to the collector (electrically conductive coating of the walls of the bulb) varies in accordance with the variation of the secondary emission coefficient. Thus, the current in the circuit of the collector of signal plate whose role is played by the non-oxidized side of the target opposite to the gun will be modulated due to the variation of the secondary emission coefficient, i.e. formed at the output of the tube is a video signal corresponding to the picture printed on the target. Since no optical image is projected onto the monoscope target, the gun of this tube is arranged on the bulb axis rather than in an inclined neck like in the iconoscope.

12.5. Orthicon

The *orthicon* is one of the first tubes with saturation photoemission provided with a photosensitive target, scanning by a beam of low-velocity electrons and charge storage. The name of this tube is determined by the perpendicular (orthogonal) incidence of the

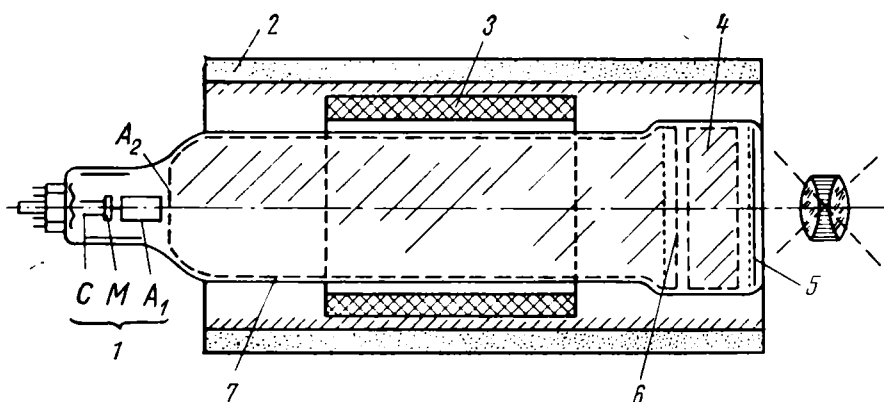


Fig. 12.8. Orthicon

1—electron gun; 2—focusing coil; 3—deflecting coils; 4—decelerating electrode; 5—target; 6—grid; 7—conductive coating

scanning beam onto the target. The orthicon is a storage tube with non-equilibrium writing and capacity-discharge reading.

A schematic diagram of the orthicon is given in Fig. 12.8. The orthicon is assembled in a cylindrical bulb of a comparatively low diameter (the bulb diameter of modern orthicons is equal to 70-80 mm). At one end the bulb has a neck for accommodation of an electron gun, at the other end it terminates with a face of polished optical glass through which the light image is projected onto the target.

Since the orthicon target is small (about 50 mm in diagonal), the required high resolution is obtained by making the diameter of the electron beam near the target of 25-30 μm or less. At the same time, the beam current must be equal to 0.3-1.0 μA for reliable recharging the target during the reading. The difficulties in obtaining a beam of a low cross section (at a small current) are explained by the fact that the orthicon employs low-velocity electrons (the accelerating voltage is only 100-200 V), and near the target surface the electron velocity drops down almost to zero.

Electrostatic focusing cannot provide the required parameters of the beam, consequently, in tubes with scanning by low-velocity electrons (an example is orthicon) the beam is subjected to additional focusing by means of a longitudinal uniform magnetic field. The presence of a longitudinal magnetic field makes it possible to provide a perpendicular path of the electrons what is important for correct reading of the information stored on the target.

Most of orthicons have magnetic deflection systems, however, some types of orthicons employ combined deflection: vertical—magnetic and horizontal—electrostatic.

Regardless of the type of the deflection system, the presence of a longitudinal focusing magnetic field makes the electron describe a complex trajectory, twice subjected to refraction in the space between the gun and the target. The electron trajectory in an orthicon is determined by the combined action of two fields: a uniform longitudinal magnetic field B_z and a transverse electrostatic field E_y or magnetic field B_x . Let us use the Cartesian system of coordinates x, y, z with an origin at the point the beam emerges from the gun and the axis OZ coinciding with the axis of the non-deflected beam. Let us discuss the deflection of the beam only in one direction (Y).

To this end, the electrostatic field must be directed along the axis OY (vertically) or the magnetic field along the axis OX (horizontally). The equations of electrons motion in this case are written in the form

$$\left. \begin{aligned} m \frac{d^2x}{dt^2} &= -e \frac{dy}{dt} B_z \\ m \frac{d^2y}{dt^2} &= e \frac{dx}{dt} B_z - eF \\ m \frac{d^2z}{dt^2} &= eE_z \end{aligned} \right\} \quad (12.15)$$

where $F = E_y$ is for the electrostatic deflection and $F = B_x v_z$ for the magnetic deflection.

Restricting ourselves to small deflection angles, we may assume the electron velocity v to be equal approximately to $v_z = \sqrt{\frac{2e\epsilon}{m}} U_a$, where U_a is the potential of the second anode (and of the electrically

conductive coating in the deflection zone). Such an assumption is quite possible since in the tubes with low-velocity electrons the deflection zone extends along the axis OZ , while the target size is small. A real deflection angle is not higher than $10\text{--}12^\circ$. Under these conditions the error due to the replacement of v by v_z does not exceed 1%.

Write the solution of system (12.15)

$$\left. \begin{aligned} \frac{dx}{dz} &= \sqrt{\frac{e}{2mU_a}} \int_0^z F(\xi) \sin \left(\int_\xi^z \sqrt{\frac{e}{2mU_a}} B_z dz \right) d\xi \\ \frac{dy}{dz} &= \sqrt{\frac{e}{2mU_a}} \int_0^z F(\xi) \cos \left(\int_\xi^z \sqrt{\frac{e}{2mU_a}} B_z dz \right) d\xi \end{aligned} \right\} \quad (12.16)$$

where ξ is a new variable obtained from the replacement

$$v_z = \frac{dz}{dt} = \sqrt{\frac{2e}{m} U_a}$$

The integration of equations (12.16) yields expressions for the electron coordinates $x(z)$ and $y(z)$ at any distance between z and the coordinate origin.

However, the analysis of equations (12.16) presents some difficulties, since $F(\xi)$ cannot be presented by a simple analytic function due to the presence of stray fields.

Some simplifications may be introduced for clear representation of the trajectory in the deflection zone. Assuming that the longitudinal focusing field is constant ($B_z = \text{const}$) and the deflecting field F is also constant in the deflection zone ($E_y = \text{const}$ or $B_x = \text{const}$), equations (12.15) are transformed into equations with constant coefficients and can easily be solved. In the absence of longitudinal electrostatic field ($E_z = 0$), which takes place in reality since in the deflection zone the potential is constant and equal to U_a , the third equation of system (12.15) yields $v_z = \text{const} = \sqrt{\frac{2e}{m} U_a}$, while the first and second equations give the projection of the trajectory onto the plane XOY . In this case the system of equations (12.15) is conveniently written in the following form

$$\left. \begin{aligned} \frac{d^2x}{dt^2} &= -\frac{e}{m} \times \frac{dy}{dt} B_z \\ \frac{d^2y}{dt^2} &= \frac{e}{m} \times \frac{dx}{dt} B_z - \frac{e}{m} F \\ \frac{d^2z}{dt^2} &= 0 \end{aligned} \right\} \quad (12.17)$$

Denoting $\frac{e}{m} B_z = \omega$ and $\frac{e}{m} F = A$ (it is obvious that $\omega = \text{const}$ and $A = \text{const}$ under the given assumptions), we obtain a system

of equations

$$\left. \begin{aligned} \frac{dv_x}{dt} &= \omega v_y \\ \frac{dv_y}{dt} &= \omega v_x - A \end{aligned} \right\} \quad (12.18)$$

where $v_x = dx/dt$ and $v_y = dy/dt$ are the velocity components along the axes OX and OZ .

The solution of the system of equations (12.18) is presented in the form of equations of cycloid

$$\begin{aligned} x &= \frac{A}{\omega^2} (\omega t - \sin \omega t) \\ y &= \frac{A}{\omega^2} (1 - \cos \omega t) \end{aligned} \quad (12.19)$$

The time t in these equations is determined for any value of z as $t = z/v_z = \frac{z}{\sqrt{\frac{2e}{m} U_a}}$. Due to the presence of the constant velocity

along the axis OZ the cycloid moves in parallel to itself in this direc-

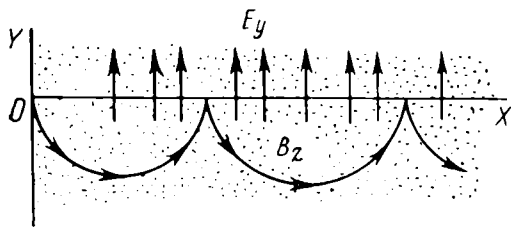


Fig. 12.9. Projection of electron trajectory on plane XOY

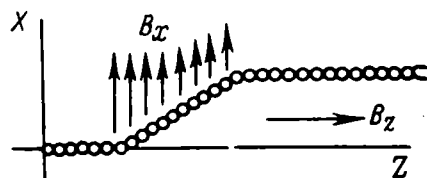


Fig. 12.10. Electron trajectory in beam deflection space

tion. Thus, the electron trajectory in the deflection zone is a complex space curve having a curvature and torsion. The projection of the electron trajectory on the plane XOY is shown in Fig. 12.9.

Equations (12.19) are obtained, assuming that the deflecting electrostatic field ($E_y = F$) is directed along the axis OY or the deflecting magnetic field ($B_x = \frac{F}{v_z}$) is directed along the axis OX .

From equations (12.19) it follows that the electron oscillating in the direction of the axis OY is continuously shifted in the direction of the axis OX , i.e. the deflection takes place in the direction at right angle to the deflecting electrostatic field or in the direction coinciding with the direction of the deflecting magnetic field. This result shows that in the tubes with scanning by low-velocity electrons in the presence of a uniform longitudinal focusing magnetic field the deflection systems deflect the electron in directions perpendicular to the deflection of electrons in ordinary tubes

The example of the tubes with a beam of low-velocity electrons shows that in a longitudinal magnetic field the electron is as if "guided" by the magnetic line of force and twists about the latter along spiral paths. Just the same, in the presence of a transverse deflecting magnetic field the electron moves near the resultant vector of the magnetic field produced during the summation of the longitudinal focusing magnetic field and the transverse deflecting magnetic field (Fig. 12.10).

The ability of the magnetic field to "guide" electrons approximately along the lines of force is employed in tubes with scanning by low-velocity electrons near the target. As the deflecting fields practically go to zero near the target, the magnetic field turns to be at right angles to the target surface. This field again curves the paths of the electrons and forces the electrons to come to the target approximately normal to its surface. Of course, the "twisting" action of the magnetic field somewhat disturbs the condition of square incidence of the beam to the target, however, by selecting the electron velocity (accelerating voltage) and the value of the longitudinal magnetic field, one can make the angle of incidence sufficiently close to 90° .

The deflected electron beam is decelerated before the target by a field of a special decelerating electrode usually made in the form of a narrow ring-shaped electrically conductive coating applied to the cylindrical portion of the bulb directly in front of the target. The retarding electrode has a separate lead and is 20-50 V positive with respect to the gun cathode.

In the orthicon the projection of an optical image onto the target from the gun side is practically difficult due to the incidence of the electron beam in the direction normal to the target surface. For this reason such a tube employs a target which receives the light image from the side opposite to that flooded by the electrons. The target backplate is made of a thin (about 0.1 mm) glass sheet. The signal plate is formed by a transparent electrically conductive tin layer. A photosensitive mosaic is applied to the opposite surface (from the side of the electron gun).

When the dark surface of the target is scanned by a beam of low-velocity electrons, its potential starts reducing, since the secondary emission coefficient of the target is less than unity for the energy of the primary electrons, $\varepsilon_1 < eU_{cr1}$. It is clear that an equilibrium will be obtained when the target reaches the cathode potential (zero). Strictly speaking, the target potential can be equal to 0.1-0.2 V relative to the cathode due to the presence of initial velocities of the electrons.

Under these conditions the electrons are reflected from the target and are collected by the conductive coating, since even a low positive potential of the conductive coating produces

an accelerating field near the entire surface of the target directed from the latter.

When an optical image is projected onto the target, its surface starts emitting photoelectrons. The presence of an accelerating field makes the photoemission saturated, and all the electrons move towards the conductive coating or the second anode of the gun. The emission of photoelectrons makes the target elements charged positively, i.e. the equilibrium (zero) potential of the target becomes distorted (nonequilibrium writing). Since the number of emitted photoelectrons is proportional to the luminous flux incident to the target, the charge stored by the target elements will reproduce the brightness distribution of the scene being televised. Thus a stored charge pattern is produced on the target surface, the highest positive potential corresponding to the brightest areas of the image.

When the electron beam scans the illuminated surface of the target, the electrons neutralize the stored positive charge, the elementary capacitors discharge and a video signal is formed in the circuit of the signal plate. Thus, the electron beam recharges the target elements bringing their potential to an equilibrium value (capacity-discharge reading).

It is clear that, in order to bring the target potential to the equilibrium value, the number of electrons per given target element must be equal to the number of electrons leaving the target. The excessive electrons of the beam will not reach the target and will be entrapped by the conductive coating.

Since the photoemission in the orthicon is saturated, there is no effect of charge redistribution which reduces the potential of the charge pattern on the mosaic. As a result the orthicon sensitivity becomes much higher (up to 20 times) than the sensitivity of an iconoscope.

The light characteristic of the orthicon is linear ($\gamma = 1$) which is a certain disadvantage. At the same time, with the gun beam cut-off, the output signal is produced only by the photoelectrons, therefore, it is equal to the signal formed when scanning a non-illuminated element of the mosaic. Thus the orthicon provides automatic reproduction of the black level.

Normal operation of the orthicon is possible only when the switching beam completely discharges the target (brings it to an equilibrium state) within one period of scanning. With high brightness of the scene being televised the loss of photoelectrons and, therefore, an increase in the potential of separate target elements may be so high that the charge brought by the scanning beam is insufficient for bringing the potential of these elements to an equilibrium (zero) state. In this case the potential of bright elements starts rising. If it overpasses the critical potential, then the secondary emission coefficient becomes higher than unity and the scanning effect will

lead to a still higher increase in the potential due to the secondary emission. A decrease in the illumination does not result in setting the equilibrium potential, a persistent white spot independent of the character of the scene being televised will be superimposed on the image. The white spot tends to spread over the entire image so that the operation of the orthicon becomes unstable at high brightness values of the scene being televised.

The white spot can be prevented by increasing the current of the scanning beam. However, an accelerating voltage of 100-200 V and a current of a few tenths of one microampere hinder the focusing. Hence, in order to provide a required resolution, low beam current should be used.

In spite of higher sensitivity as compared to the iconoscope, the orthicon has not gained general recognition mainly due to its lower resolving power and unstable operation (possibility of white spot) at intensive illumination of the scene.

The orthicon was used as a base for developing an image tube with secondary electron multiplication known as a superorthicon overpassing the orthicon by all parameters (except for simplicity in adjustment). This tube has gained the widest application in the electron optics

12.6. Super-iconoscope

Insufficient sensitivity of the iconoscope has resulted in some modification of this generally good tube. The most important improvement of this tube was the introduction of an image conversion stage proposed in 1938 by Soviet scientists P. Timofeev and P. Shmakov. Since that time this tube referred to as *image iconoscope* or *super-iconoscope* was considerably improved and is presently employed mainly for transmission from TV studios. The basic advantages of the image iconoscope are its high resolution and sufficiently high sensitivity exceeding that of an ordinary iconoscope approximately 10 times.

The high resolution of the image iconoscope is explained by the high resolving power of a solid (not mosaic) photocathode and comparatively large size of the target. A gain in the sensitivity compared with an iconoscope is obtained by using a more sensitive solid photocathode, saturated photoelectric current and intensifying the image during its transfer from the photocathode to the target.

The image iconoscope is a television-camera storage tube with image transfer, scanning by high-speed electrons, writing and reading, and redistribution of the charges over the target surface. A schematic diagram of the image iconoscope is shown in Fig. 12.11.

The bulb of the image iconoscope consists of two cylinders with different diameters. The wider cylinder comprises the target, the

narrower cylinder has a flat face with a photocathode applied to its inner surface. At the joint of the wide and narrow cylinders, a neck accommodating an electron gun is fixed. The inner surface of the neck, narrow cylinder and a portion of the surface of the wide cylinder have an electrically conductive coating. The conductive coating of the neck serves as the second anode of the gun, the coating of the narrow cylinder serves as the anode accelerating the photoelectrons. The coating of the portion of the wide cylinder is the collector of secondary electrons leaving the target.

Since the part of the bulb comprising the photocathode is narrower, the neck with the gun can be arranged at a smaller angle

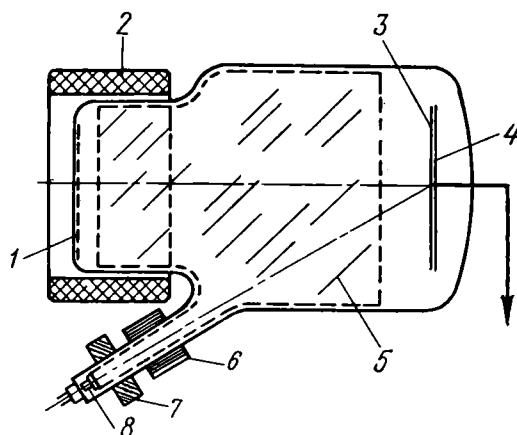


Fig. 12.11. Image iconoscope

1—photocathode; 2—electron image focus coil; 3—target; 4—signal plate; 5—collector; 6—deflection system; 7—scanning beam focusing coil; 8—electron gun

to the bulb axis to improve the scanning conditions. The angle between the gun axis and the normal to the target surface is equal to $20\text{--}25^\circ$.

Super-iconoscopes employ guns with magnetic focusing based on a simple triode system. The anode voltage of the gun is equal to 1000 to 1200 V, the beam current is of 0.1 to 0.3 μA . The beam is deflected by a magnetic field produced by two pairs of coils mounted on the tube neck. To reduce the stray fields and to prevent the influence of the deflecting fields on the image section, the deflection system is enclosed into a ferromagnetic screen.

The photocathode of the super-iconoscope is formed on the flat face of the narrow cylinder of the bulb. Since the image section allows electron enlargement of the image by 2-4 times, the photocathode area can be lower than the target area. The small size of the photocathode makes it possible to use short-focus wide-aperture lenses providing a required illumination of the photocathode at comparatively low brightness of the scene to be televised. At the same time, the quality of an optical image on a small surface can

be very high. The high quality of the optical image and the high resolution of the solid photocathode permit one to obtain a resolving power of the device not lower than in the case of an iconoscope in spite of the fact that the area of the photocathode of the image iconoscope is much less than that of the iconoscope mosaic.

In addition to the high resolution, the continuous photocathode has some other advantages compared with the mosaic. The sensitivity of a continuous photocathode can be 2-3 times that of the mosaic. Since the photocathode is separated from the target, it can be given a desirable spectral characteristic required for correct transmission of a colour image. The photocathode, which is not integral to the target, can be made by using modern technological processes providing high quality of the cathode (higher sensitivity, more uniform surface, lower "fatigue", etc.).

In order to provide high sensitivity of the tube, the photoelectric current must be saturated, for which purpose an electrostatic field is built up at the cathode surface for accelerating the electrons. This field is produced by the electrically conductive coating of the cylindrical portion of the bulb which is positive with respect to the cathode. With a relatively large diameter of the tube envelope, the field near the cathode is nonuniform. The electrons at the cathode edges are subjected to considerable acceleration, while the field in the centre of the cathode is much weaker. This nonuniformity of the field may be a cause of distortion of the image being transferred by the magnetic field. Therefore, in some types of image iconoscopes a flat fine-mesh grid is mounted near the cathode. In spite of the fact that the grid collects some of the photoelectrons, the image quality is improved due to the high uniformity of the field attracting the photoelectrons. To prevent the grid image from being passed to the target and included into the video signal, the grid is placed at such a distance from the photocathode that the elementary electron beams focused by the longitudinal magnetic field have antinodes.

The electron image is transferred from the photocathode to the target. The image can be transferred both by an electrostatic and magnetic field. A uniform electrostatic field obviously cannot transfer the image with the required resolution intact. The electrostatic systems similar to those used in image converters have not gained wide application due to perceptible aberrations occurring when use is made of wide beams (large targets) and a concave cathode interfering with the projection of the optical image. Therefore, the image in image iconoscopes is usually transferred by means of a magnetic field. In earlier constructions of these tubes use was made of a uniform field transferring the image in a scale 1 in 1. However, further improvements of image iconoscopes have resulted in development of tubes with a nonuniform magnetic field in the

image section allowing the target to receive an electron image of the photocathode enlarged 2-3 times compared with its original size. Some enlargement of the image is expedient, since in this case it is possible to obtain higher resolution. Furthermore, when scanning a large surface by an electron beam, we may use a light spot of larger diameter.

The image section of the tube employs a nonuniform longitudinal magnetic field whose intensity decreases from the cathode to the target. For the principal trajectories of the electrons the decreasing field is a diverging lens, while for the adjacent trajectories it serves as a focusing lens (see Section 1.5). Thus, matching the nodes of adjacent trajectories with the target surface by selecting the ratios of the electric field intensity to the magnetic induction, one can obtain an enlarged electron image of the photocathode on the target. Since the principal trajectories are "twisted" by the magnetic field, the entire image is turned through a certain angle. The image iconoscope target must not have photosensitivity because the functions of conversion of the optical image into the electron one and the storage of charge between the photocathode and the target are separated. At the same time, the secondary emission coefficient of the target must be sufficiently high at the primary electrons energy of 1000-1200 eV, since the target is scanned by the high-speed electrons. To maintain the stored charge pattern intact, the transverse conductivity of the target surface must be as low as practicable. To provide high resolution, the target surface must be structureless. Finally, to provide for the possibility to store an adequate charge, it is desired that the target capacitance be not less than 100-200 pF/cm².

These requirements are met by a thin dielectric sheet having an electrically conductive coating on one side (signal plate). The image iconoscope target is practically a metallic plate coated with a thin layer of non-conductive oxide (for example MgO) or another dielectric having a high secondary emission coefficient. There is also used a mica sheet one side of which is coated with magnesium oxide and the other with a silver coating forming a signal plate. The image is transferred from the photocathode to the target at an electron amplification factor of 3 to 5 (due to the $\sigma > 1$ conduction).

However, like in the iconoscope, the formation of an adequate stored charge pattern in the image iconoscope is hindered by the redistribution of secondary electrons reducing the charge pattern potential. Thus, in the image iconoscope the current of the electrons emerging from the target is not saturated. However, in this tube a greater part of secondary electrons reaches the collector due to the higher initial velocities of the secondary electrons as compared with the velocity of the photoelectrons. Some types of image icono-

scopes are provided with a metal frame mounted near the target surface and having an individual terminal. By controlling the voltage applied to this frame (within the range ± 20 V with respect to the collector), one can vary the electrostatic field near the target, thus, selecting optimum conditions for forming an electron image and more uniform distribution of the secondary electrons over the target surface which is important for reducing the spurious signal (shading).

Since the scanning in the image iconoscope is accomplished by a beam of high-speed electrons, in scanning the non-illuminated surface of the target the element under scanning acquires a potential a few volts higher than the collector potential. But, like in the iconoscope, the potential of the scanned element starts decreasing due to the action of the redistributing secondary electrons returning to the target. In general, the technique of writing and reading the image in the image iconoscope differs but little from that used in the iconoscope. The only distinction is in that during the writing operation the stored charge pattern (image pattern) is produced by the secondary electrons rather than the photoelectrons as the case is in the iconoscope. However, due to the higher sensitivity of the continuous photocathode, electron intensification of the image in the image section and better selection of secondary electrons by the collector the output signal of the image iconoscope is 8-10 times that of the iconoscope at the same illumination of.

The light characteristic of the image iconoscope is similar to that of the iconoscope, i.e. it is linear at low levels of illumination of the photocathode (approximately up to 10 lx). Then the output signal starts increasing at a lower rate than the illumination, since the reduction of the image pattern potential by the redistributed secondary electrons is more noticeable at intensive illumination.

A disadvantage of the image iconoscope is in the presence of shading and distortions associated with the transfer of the image from the photocathode to the target.

12.7. Image Orthicon

At present time the image orthicon is the most widely used camera tube employed both for studio and natural telecasting. The image orthicon surpasses the other types of camera tubes by many parameters. This tube uses charge storage, image intensification due to its transfer from the photocathode to the target and secondary-electron amplification of the output signal. A characteristic feature of the image orthicon is the use of a two-sided target one side of which is used for writing (charge storage) and the other for reading. Like the case is with the orthicon tube, this tube employs nonequilibrium writing and capacity-discharge reading.

The following factors allow the sensitivity of the image orthicon to be materially increased: a continuous highly sensitive photocathode, a saturated photoelectric current, amplification due to the image transfer, efficient collecting of secondary electrons by the grid (at low illumination), capacity-discharge reading by a beam of low-velocity electrons ($\sigma < 1$), and secondary-electron amplification of the signal. Owing to these factors the sensitivity of a modern image orthicon is hundreds and thousands of times higher than the sensitivity of an iconoscope and in some cases can exceed the sensitivity of the human eye.

The construction of the image orthicon is shown in Fig. 12.12. All components of the image orthicon are assembled in a long cylindrical bulb of a comparatively low diameter. At the side of the

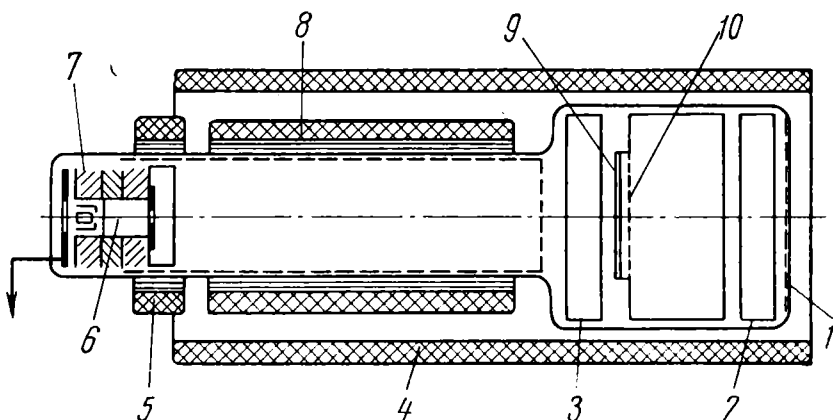


Fig. 12.12. Image orthicon

1—photocathode; 2—accelerating ring; 3—decelerating ring; 4—focusing coil; 5—alignment coil; 6—electron gun; 7—secondary electron multiplier; 8—deflecting yokes; 9—target; 10—screen

target the bulb is widened. The narrow part of the bulb is terminated with a flat face, a base with terminals used for connection of the gun electrodes and the secondary-electron multiplier to the external circuit. The wide portion of the bulb terminates with a flat face of optical glass whose inner surface is coated with a continuous semitransparent layer—translucent photocathode. In the adapter portion between the wide and narrow cylinders there are additional terminals used for connection of the external circuit to the photocathode, the accelerating ring, the target screen, and the decelerating ring. The greater part of the inner surface of the narrow cylinder has a conductive coating at the potential of the gun anode. The image orthicon gun must produce a beam having a current of 1-2 μA forming a spot on the target not higher than 30-40 microns in diameter. The gun usually employs a simple triode scheme.

The anode diaphragm of the gun has an aperture 30-50 μm in diameter. Behind the diaphragm the beam is shaped by means of a longitudinal magnetic field into a ray with almost parallel electron trajectories. Thus, the diameter of the beam is determined largely by the diameter of the anode aperture. The outer surface of the anode diaphragm serves as a first emitter of the secondary-electron multiplier.

The electron beam is deflected by two pairs of elongated coils producing approximately uniform transverse fields. In addition, mounted outside the tube in the region of the gun anode are alignment coils producing short transverse magnetic fields. By controlling the current in these coils the beam can be deflected within a small range to provide the best conditions for the passage of electrons through the anode diaphragm. This current control also acts on the electrons returning from the target to ensure their best collecting by the first emitter of the multiplier, i.e. the anode diaphragm of the gun.

The two-sided target is composed of a very thin plate (4-5 microns) made of special glass with an increased electrical conductivity. Owing to the low thickness of the plate its transverse resistance is low and the stored charge pattern formed on one side of the target transfers to the other side due to the transverse conductivity. At the same time the surface (longitudinal) resistance of the target is sufficiently large and there is no perceptible "leakage" of the charge over the target surface during the transmission of one image pattern. Thus the stored charge pattern formed in writing on the target surface facing the photocathode will appear on the reading-beam side facing the gun. In this case the transverse conductivity of the target provides a required potential distribution at both sides of the target while the high transverse resistance prevents rapid reduction of the stored charge pattern.

A fine-mesh screen is mounted in the immediate vicinity to the target surface for collecting the secondary electrons from the target at the writing side. The target surface and the neighbouring screen form a capacitor storing an electrical charge when writing.

The image orthicon may employ different types of photocathodes having high sensitivity. The most widely used are oxygen-silver-caesium, antimony-caesium, bismuth-oxygen-silver-caesium and multialkali photocathodes.

The image is transferred from the photocathode to the target by a longitudinal magnetic field. This field is a continuation of the focusing field forming the reading electron beam at the opposite side of the target. In order to obtain high sensitivity, all the electrons emitted by the photocathode must reach the target, i.e. the photoemission must be saturated. A saturated photoelectric current is generated by the accelerating electrostatic field produced by the

screen and cylinder of the target with respect to which the photocathode is at a negative potential of -250 to -500 V.

The image section of the image orthicon must provide undistorted transfer of the image from the photocathode to the target. Moreover, the presence of a screen close to the target surface dictates that the electrons are incident to the target surface at right angles. Two fields act in the image section. These are a practically uniform longitudinal magnetic field and not quite uniform electrostatic field accelerating the photoelectrons.

Strictly speaking, the undistorted transfer of the image is possible only under the condition that both fields are uniform and coincide in direction. The nonuniformity of the electrostatic field in actual tubes always causes some distortion of the image. In order to reduce these distortions, mounted between the photocathode and the target cylinder is a ring (electrode) ordinarily referred to as accelerating. This electrode makes the electrostatic field almost uniform. The potential of the accelerating ring is a midpoint between the potentials of the photocathode and the target screen. By controlling the potential of the accelerating ring, it is possible to transfer the image at minimum distortions.

The magnetic field of actual image orthicons cannot be made sufficiently uniform in the transfer section. The field intensity near the target is higher than that at the photocathode because the edge of the coil is closer to the photocathode. The photocathode cannot be inserted into the coil because in this case the coil edge would interfere with the arrangement of the lens. It should be noted that short-focus high-speed lenses must be located close to the surface of the photocathode in order to greatly reduce the image of the scene being televised.

A decrease in the field intensity at the photocathode results in that the image is transferred to the target with size reduction ($M=0.85-0.9$). In this case, of course, the image is also slightly turned ($3-5^\circ$).

Thus the electron trajectories in the image section of an image orthicon tube are similar to those in the image iconoscope. The difference lies in the fact that in the image iconoscope we have an enlarged image due the reduction of the magnetic field, while in the image orthicon it is reduced due to an increase in the magnetic induction.

The reduction of the image is disadvantageous, as the resolution worsens at the same diameter of the reading beam. Furthermore, the smaller the working surface of the target, the lower is the screen-target capacitance, i.e. the lower the stored charge.

An increase in the capacitance due to reducing the gap between the target and the screen is effective only to distances of 30-20 microns (with the same size of screen meshes). Further reduction

of this distance would increase the capacitance only if the grid meshes are reduced proportionally. Both these operations are associated with engineering difficulties. Therefore, the capacitance may be increased by increasing the working surface of the target (using an image orthicon tube with an enlarged target). The increase in the target size is limited by the necessity to have small deflection angles of the reading beam. In the image orthicon with an enlarged target the linear magnification in the image section is equal to about 1.5. In this case the target capacitance is increased almost 3 times and this enables the signal-to-noise ratio to be improved at poor illumination of the scene. The magnetic field in the image section of image orthicon with an enlarged target, like that in an image iconoscope, has its maximum intensity near the cathode. However, the field does go to zero at the target but becomes uniform and sufficient in magnitude for focusing the reading beam.

The quality of the image formed by an image orthicon is largely determined by the image section. The electron image on the target must be so neat that the resolution is determined by the diameter of the reading beam rather than by the quality of the image on the target. A blurred image on the target may be the result of nonuniform electrostatic and magnetic fields in the image section and also of aberrations in the electron-optical system. The disturbances to the axial symmetry of the fields appearing due to bad workmanship when assembling the system target—screen—cylinder—accelerating ring have the greatest effect on the resolution.

Geometric aberrations in the image section are as a rule insignificant. This is explained by the fact that the electrostatic and magnetic fields in the image section are practically uniform and, according to the design and experimental data, the dispersion circles due to the aberration are less than the reading beam diameter and do not limit the resolving power of the device.

However, considerable chromatic aberration takes place in the image section due to differences in the initial velocities of photoelectrons emerging from the cathode. With a television standard of 600 lines and more the chromatic aberration reduces the resolution to a considerable degree. Since it is impossible to reduce the scatter of the initial velocities, the chromatic aberration can be reduced only at the expense of increasing the magnetic induction and accelerating voltage in the image section. It is inexpedient to increase the magnetic field along the entire tube by increasing the current in the coil or the number of turns, since in this case the voltage of all the electrodes and the amplitude of the current in the deflecting coils must be increased correspondingly in order to maintain the required focus. Therefore, sometimes it is expedient to intensify the magnetic field in the image section. To this end an additional coil is used in the region between the photocathode and the target.

For the same purpose, the potential difference between the cathode and the screen can be increased. However, an increase in the potential above 500 V may cause spurious emission of electrons from the photocathode surface, thus reducing its efficiency.

The image orthicon sensitivity can also be increased at the expense of secondary-emission amplification of the signal. The use of a secondary-electron multiplier in the very camera tube is advantageous over the amplification of the signal by means of an individual amplifier with electron valves.

First, electron multipliers have less intrinsic noise than amplifiers. This allows for a satisfactory signal-to-noise ratio at low magnitudes of the input signals. Owing to the fact that in the image orthicon the maximum output current corresponds to the darker elements of the image, the use of valve amplifiers instead of a multiplier would cause considerable noise at low intensity of illumination. Second, the connecting wire between the camera tube and the amplifier causes additional induction noise. When the signal is amplified by a multiplier, this kind of noise can be considerably reduced (the signal-to-noise ratio is improved). Finally, the secondary-electron multiplier has a linear characteristic within a very wide range of variation of the input signal and serves as a wide-band amplifier suitable for transmission of an undistorted image at the highest television standards.

The secondary-electron multiplier of an image orthicon consists of a few (up to seven) rings encircling the anode cylinder of the gun. The rings have radial slots with bent edges ("louvres"). The anode diaphragm of the gun serves as a first emitter of the multiplier, the rings form a second, third, etc. multiplication stages. For better direction of the electrons to the subsequent emitters, mounted between the second and third, third and fourth, etc. stages are grids at the potential of the next emitter (i.e. a grid between the second and third emitters is connected to the third emitter, etc.).

The voltages across the multiplier stages are increased from stage to stage so that the voltage across the last emitter with respect to the gun cathode may be higher than 1.5 kV. Mounted before the last ring-emitter (without slots) is a secondary-electron collector made as a grid at a potential 100-200 V higher than the potential of the last emitter. No electrodes having a positive (with respect to the multiplier) potential are provided near the secondary-electron multiplier. Therefore, the electrons returning from the target are effectively collected by the multiplier, thus producing an output current (video signal).

The first modifications of image orthicons employed silver-caesium or antimony-caesium secondary emitters. At the present time use is made of alloyed emitters such as silver-magnesium, copper-berillium, aluminium-magnesium. These emitters are more stable

in operation, less sensitive to overloads and are more convenient from the engineering point of view.

When the non-illuminated target surface is scanned by a beam of low-velocity electrons, it is gradually charged by the landing electrons ($\sigma < 1$) assuming an equilibrium potential approximately equal to the gun cathode potential (zero). When an optical image is passed to the photocathode, the electron image appearing due to the photoelectric emission is transferred by the magnetic field to the target. The photoelectric emission is saturated, i.e. all electrons emitted by the photocathode land on the target. The electron energy at an accelerating voltage of 300-400 V is sufficient to make $\sigma > 1$. Thus, in the image section the image is amplified σ times. The emission of secondary electrons results in a positive potential charge pattern on the target surface. Since the projection of the optical image and the transfer of the electron image to the target are continuous processes, the stored charge pattern is formed during the transmission of one frame, i.e. the effect of charge storing is realized to a full extent. Due to the transverse conductivity of the target the potential of its reading-beam side facing the gun varies either. Therefore, we have nonequilibrium writing.

When the target is scanned by a beam of low-velocity electrons ($\sigma < 1$), the potential of its surface is reduced due to the compensation for the positive charge by the electrons of the scanning beam. Since the reading beam changes the potential stored on the target, the reading in the super-orthicon is of the capacity-discharge type.

When the photocathode is not illuminated, i.e. in the absence of the scene to be televised, there is no charge pattern on the target. When the dark surface of the target is scanned by a beam of low-velocity electrons, its potential tends to the equilibrium value equal to the gun cathode potential (zero). In this case the electrons of the reading beam will be reflected by the target surface and will return to the gun. The longitudinal magnetic field "guides" the reflected electrons to the anode diaphragm of the gun serving as the first secondary emitter of the electron multiplier. The electrostatic field of the second emitter directs the secondary electrons from the gun diaphragm to the secondary emitter. The multiplier cylinder also takes place in the formation of this field. From the second emitter the electrons are directed to the third emitter, etc., from the last emitter—to the multiplier collector (anode). The amplification factor of the multiplier may be as high as a few thousands.

When the target has a stored charge (image) pattern corresponding to the distribution of illumination over the photocathode surface, some of the electrons of the reading beam are used to compensate for the positive charge stored on the target. Owing to this fact the current of the electrons returning from the target is reduced. It is clear that the higher the positive charge stored on the given

element of the target, the lower is the current flowing from this element to the multiplier input.

Thus, the maximum output current corresponds to the non-illuminated elements of the target and the minimum current flow, to the brightest elements. Hence, the polarity of the video signal of the image orthicon is the reverse of that used in the above-discussed tubes.

In a simple manner, the formation of a video signal in an image orthicon may be presented as follows. Assume that the photocathode current is saturated, all the secondary electrons from the target are collected by the screen, i.e. the secondary electron current is also saturated, the reading beam current is sufficient for full compensa-

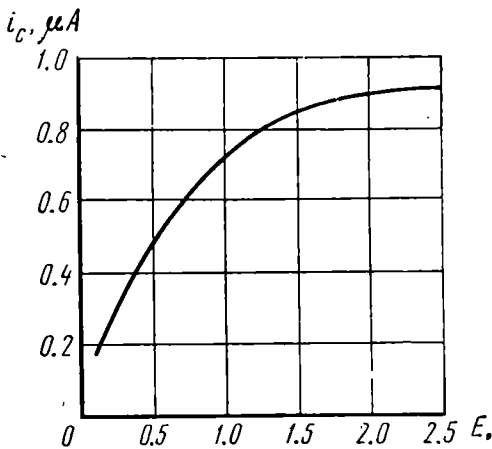


Fig. 12.13. Light characteristic of image orthicon

tion for the charge stored on any element of the target, while the characteristic of the secondary-electron multiplier is linear within the whole possible range of variation of the input current. It is obvious that in the case of an ideal image orthicon the video signal value must be proportional to the luminous flux incident to the photocathode. In other words, the light-signal characteristic must be linear within the whole possible range of variation of the brightness of the transmitted image.

The ideal conditions, however, can exist only at very low intensity of illumination. Only the first condition—photoelectric current saturation—is fulfilled at comparatively high illumination of the photocathode due to a considerable potential difference between the photocathode and the target screen. At the same time, the field between the target surface and the screen is very low and can change its polarity with storage of positive charge on the target surface. Thus, there is no sense in speaking of the saturation of the secondary-electron current from the target at moderate illumination. In the same manner, the reading beam may not “succeed” to completely discharge the target elements at a high value of the stored charge. The result of these factors is in that, as the illumination of the photocathode increases, the proportionality between the illumination and the value of the video signal becomes disturbed, the increase in the amplitude of the output signal lags behind the rise of the illumination, i.e. the light-signal characteristic becomes nonlinear.

An approximate view of a real light characteristic of an image orthicon is shown in Fig. 12.13.

As shown in the drawing, the light characteristic is close to the linear one only at very low illumination of the photocathode. The steepness is reduced with a rise in the illumination. Thus the actual light-signal characteristic of the image orthicon is close to optimum. Owing to the high steepness at low illumination it is possible to transmit an adequate number of tonal gradations. At the same time, at high illumination the light characteristic is mildly sloping and we may assume that a further increase in the illumination (behind the "knee" of the light characteristic) will make the formation of a video signal impossible.

However, the experience has shown that even when the illumination of the photocathode considerably exceeds the value corresponding to the "knee" of the light characteristic, the image orthicon tube reproduces the tonal gradations satisfactorily. Moreover, when operating behind the "knee" of the light characteristic, the signal-to-noise ratio even increases and the image quality is improved. To explain this the theory of the video signal formation by an image orthicon must be made more precise.

So far we did not take into account the possibility of redistribution of the secondary electrons emerging from the target among the unequally illuminated elements of its surface. Yet this process plays an important role in the storage of the charge and formation of the image pattern on the target surface. As indicated above, the field collecting the secondary electrons from the target surface is very small, even when the photocathode is not illuminated. At high illumination this field may become retarding. At the same time, the potential difference between the neighbouring light and dark elements of the target surface may turn to be sufficient for producing local fields contributing to the transfer of the secondary electrons from one target element to another. Moreover, the presence of an interelement potential difference causes a charge leakage due to the non-zero surface conductivity of the target. The allowance for the redistribution of the secondary electrons makes it possible to explain the feasibility of formation of a charge pattern on the target at the photocathode illumination corresponding to the region behind the "knee" of the light characteristic. Indeed, white element of the target will always lose more secondary electrons than a black one. However, while the secondary electrons are collected by the screen at low illumination, a perceptible number of the secondary electrons goes from the white elements to the "black" ones at higher illumination thus adding to the potential of the charge pattern.

During the recent years the problem of formation of a charge pattern on the image orthicon target with due account for the redis-

tribution of the secondary electrons and the charge leakage was studied by N. Galinsky. In accordance with the theoretical and experimental studies of N. Galinsky, the formation of a video signal by an image orthicon tube may be presented as follows. Assume that a test pattern ("mira") made of alternating light and dark strips is projected onto the photocathode of the image orthicon, the illumination of the light strips being perceptible higher than the illumination corresponding to the light characteristic "knee". Under these conditions we may assume that all the secondary electrons emerging from the target surface are returned back to the target, since the field in the screen-target space is too weak. Furthermore at the high transparency of the screen (up to 70-80% in modern image orthicon tubes) the electrons from the target moving towards the screen pass through the latter and are returned back to the target by the field of the image section. Let us also assume that the scattering range of the secondary electrons is substantially greater than the width of the strips of the electron image on the target. Generally, the secondary electrons fly both from white to black and vice versa but the number of electrons emerging from the white is always greater than the number of electrons emerging from the black. Therefore, the potential of the white elements increases with resultant potential difference between the white and black elements. This potential difference, however, cannot be infinitely high, as the rise of the potential of the white elements is associated with an increase in the retarding field returning the secondary electrons emerging from the white elements back to these elements. A dynamic equilibrium is set in which case we may approximately assume that half the secondary electrons emerging from each element is transferred to the neighbouring parts of the target, while the other half returns to the same element. Supposing that the amount of electrons flying from one target element to another is proportional to the potential difference U , we can set up equations for the currents of the white (i_w) and black (i_b) elements of the target

$$\left. \begin{aligned} i_w &= -i_{pw} + \frac{1}{2} i_{pw} \sigma (1 - \alpha U) - \frac{1}{2} i_{pb} \sigma (1 - \alpha U) \\ i_b &= -i_{pb} + \frac{1}{2} i_{pb} \sigma (1 - \alpha U) - \frac{1}{2} i_{pw} \sigma (1 - \alpha U) \end{aligned} \right\} \quad (12.20)$$

where i_{pw} and i_{pb} are the photoelectric currents of the white and black elements of the photocathode, respectively; α is the proportionality coefficient.

The coefficient α can be determined assuming that, when reaching the ultimate (maximum) value of the interelement potential difference U_{in} , the current difference of the white and black elements becomes equal to $2i_b$

$$(i_w - i_b)_{U=U_{in}} = 2i_{pb} (\sigma - 1) \quad (12.24)$$

Finding the value of α , we obtain

$$i_w - i_b = (i_{pw} - i_{pb})(\sigma - 1) - (i_{pw} + i_{pb})(\sigma - 1) \frac{U}{U_{in}} \quad (12.22)$$

Since the target features surface conductivity, the leakage current should be taken into account

$$i_l = \frac{U}{R} \quad (12.23)$$

where R is the interelement surface resistance.

Introducing the concept of interelement capacitance C , we may set up an equation for the electron exchange between the target elements:

$$C \frac{dU}{dt} = i_w - i_b - i_l \quad (12.24)$$

Expressing the photoelectric current through the illumination E and the sensitivity k_p of the photocathode, we may write the solution of equation (12.24) in the form

$$U = U_{in} \frac{E_w - E_b}{E_w + E_b + \frac{a}{R}} \left[1 - e^{-\frac{1}{aC} (E_w + E_b + \frac{a}{R}) t} \right] \quad (12.25)$$

where

$$a = \frac{U_{in}}{k_p s_p (\sigma - 1)} \quad (12.26)$$

(s_p is the photocathode area).

Equation (12.25) makes it possible to find the signal current i_s ,

$$[i_s = \frac{U_{in} C}{\tau} \frac{E_w - E_b}{E_w + E_b + \frac{a}{R}} \left[1 - e^{-\frac{1}{aC} (E_w + E_b + \frac{a}{R}) T} \right]] \quad (12.27)$$

where τ is the time for scanning of one element; T is the period of the frame scanning.

Expressions (12.25) and (12.27) make it possible to obtain quantitative relationships between the photocathode illumination, the charge pattern potential and the signal current. Experiments have shown that the design values are in satisfactory agreement with the experimental data and this confirms the validity of the above assumptions about the significant role of the redistribution of secondary electrons in the process of formation of a stored charge pattern on the target surface of the image orthicon. Furthermore, the redistribution of the secondary electrons reduces the noise level, since any change in the number of secondary electrons flying from element to element with respect to the equilibrium potential value immediately causes local fields which restore this equilibrium value.

Thus, the statement formulated by N. Galinsky clearly explains the processes of the video signal formation by an image orthicon including these processes occurring at high illumination (behind the "knee" of the light characteristic) as well as the presence of the higher signal-to-noise ratio than that expected on the basis of the generally accepted ideas.

Due to its high sensitivity, adequate resolution, good signal-to-noise ratio, near-optimum characteristic the image orthicon is

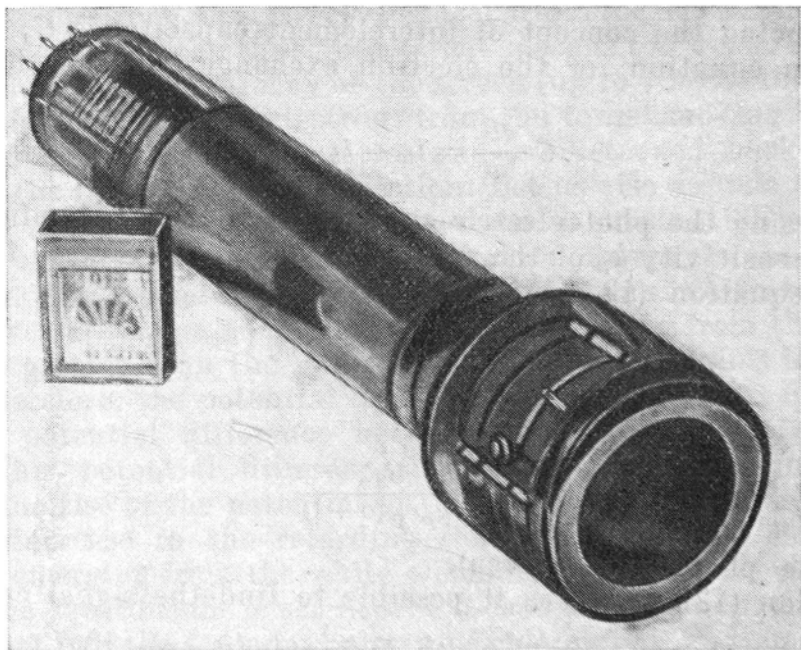


Fig. 12.14. External view of image orthicon

presently most popular camera tube widely used both in monochrome and colour telecasting either from a studio or outdoors. The external view of a modern image orthicon tube is shown in Fig. 12.14.

All the same, the image orthicon has some disadvantages. These are first of all an increase in the signal-to-noise ratio at poor illumination of the scene being televised, occurrence of a dark frame encircling the bright details of the image, and a decrease in the resolving power when transmitting objects with pronounced tonal gradations. Furthermore, the image orthicon is a complex tube requiring careful adjustment and control for obtaining a good picture.

A relatively high noise level at poor illumination of the televised scene is due to the fact that in the image orthicon the dark elements of the image correspond to the highest current of the electrons returning to the gun. It is well known that the mean-square value of the

noise current is proportional to the signal current (see Section 12.1). The noise increases with the current of the electrons returning to the multiplier. At the same time, the useful signal with a poorly illuminated photocathode is very low, since in this case the potential of the charge pattern on the target is not high. Thus, the noise rises up with a decrease in the useful signal, i.e. the signal-to-noise ratio drops down. The use of a multiplier is not effective because the noise appears in the reading beam, when the electrons are reflected from the target, and on the first emitter (anode diaphragm). The next in sequence multiplication stages amplify the noise together with the useful signal. As a result, the signal-to-noise ratio of the output signal becomes unsatisfactory at illuminations when the tube sensitivity is still adequate.

The appearance of a dark frame around the bright elements of the image is explained by the redistribution of the secondary electrons over the target surface. If the transmitted object has some brightly illuminated spots, the number of electrons on the corresponding spots of the image may be high. In this case some of secondary electrons will return to the target, thus reducing the potential around the place corresponding to the brightest element of the image. Since the reduction of the target potential is equivalent to the transmission of the darker elements of the image, the bright spot of the image is surrounded by a dark frame. The same effect of redistribution of the secondary electrons leads to the so-called emphasis of the bright elements of the image. Finally the reduction of the resolving power during the transmission of objects with high tonal gradation is explained by the leakage of the charge over the target surface. The charge leakage is particularly noticeable when transmitting objects having small bright elements against a poorly illuminated background. The large charge stored on image sections corresponding to the brightly illuminated elements of the scene leaks over the target surface during the transmission of a single frame. Thus the boundary of these elements becomes blurred. Therefore, the resolving power of the tube transmitting the objects with high tonal gradation becomes lower than during the transmission of uniformly illuminated objects.

In addition, some image orthicons are characterized by nonuniformity of the background, i.e. during the transmission of a uniformly illuminated picture some places on the image appear brighter or, on the contrary, darker. The background nonuniformity is caused mainly by a different value of the secondary emission coefficient of different places of the first emitter of the multiplier. Since the deflecting fields act also on the electrons returning from the target, the point of incidence of the electrons to the first emitter is displaced continuously in accordance with the scanning of the reading beam over the target surface. This disadvantage can

be reduced by making the anode diaphragm (first emitter of the multiplier) from a highly uniform material and by carefully cleaning its surface from foreign matter influencing the secondary emission coefficient. The background is levelled by mounting a fine-mesh screen in the image section (before the target). This screen reduces the scatter of the electrons returning from the target.

As mentioned above, a high noise level at low illumination is one of the disadvantages of the image orthicon. It is clear that the change in the polarity of the output signal of the image camera tube and secondary-electron multiplication of the signal would result in significant improvement of the signal-to-noise ratio at low illumination, since in this case the low illumination corresponds to a low current of the electrons returning from the target. The electron current returning from the target can be regarded as consisting of two parts: the electrons elastically reflected from the target and those scattered due to the interaction of the reading beam with the target surface. The higher the potential of the target elements (the brighter the elements), the greater number of the reading beam electrons will travel to the target and the greater will be the scattering of the electrons. Thus the number of scattered electrons is approximately proportional to the target illumination, and the flow of scattered electrons can be used for forming an output signal.

The above principle underlies the operation of the image camera tube with charge storage and secondary-electron amplification of the signal known as an image isocon. The electrode system of the image section, the target, the gun and deflection systems of the image isocon are identical to the respective elements of the typical image orthicon. The only difference is in the construction of the secondary-electron multiplier. The first dynode of the multiplier is located so that it receives only the scattered electrons, while the elastically reflected electrons return along the path practically coinciding with the trajectory of the reading beam and are collected by the gun anode. In this tube the anode diaphragm of the gun is not the first emitter of the multiplier.

The scattered electrons collected by the input electrode of the multiplier produce an output signal. Since in the case of low illumination most of the reading beam electrons are reflected by the target, the signal current is small and the signal-to-noise ratio is higher than the case is with the image orthicon. The tests of experimental models of image isocons have shown that on the dark portions of the image the signal-to-noise ratio is much higher than in an image orthicon. On the bright portions it is approximately the same for both tubes. In spite of better parameters at low illumination levels, the image isocons have found limited application mainly due to their complexity.

12.8. Vidicon

The camera tubes discussed in the preceding sections employ photoemission for conversion of an optical image into an electron image, the sensitivity of the photocathodes of these tubes being not higher than $40\text{--}50 \mu\text{A}/\text{lm}$. Therefore, special measures should be taken for increasing the sensitivity. These are electron intensification or secondary-electron multiplication. At the same time, available commercially are high-sensitivity photoelectric cells with internal photoelectric effect (photoconductors) whose sensitivity is tens and hundreds of times the sensitivity of the photoelectric cells with external photoelectric effect. Wide application of

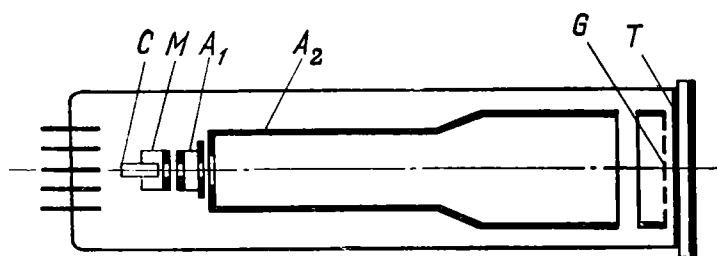


Fig. 12.15. Vidicon

photoconductors in camera tubes has been hindered by high persistence of the devices with internal photoelectric effect. The development of highly sensitive and comparatively low-persistence semiconductor photoconductors made it possible to produce a simple camera tube with a photoconductive charge storage device which is called the *vidicon*.

The vidicon is a camera tube with a semiconductor photoconductive target, charge storage and capacity-discharge reading. The construction of the vidicon is schematically shown in Fig. 12.15.

An electron gun is mounted in a cylindrical tube. The gun produces a beam $20\text{--}30 \mu\text{m}$ in diameter at a current of $0.3\text{--}0.6 \mu\text{A}$. In many types of vidicons the additional focusing of the electron beam is performed by a uniform longitudinal magnetic field produced by a long coil put on the tube. The presence of a longitudinal magnetic field also assists in normal incidence of the beam to the target (like in the orthicon). The beam is deflected by two pairs of deflecting coils arranged outside the tube. Also known are vidicons with purely electrostatic focusing of the beam and electrostatic deflecting plates. In any case the vidicon requires a well formed beam of a small diameter but at a small current. Such beams are produced rather simply by guns with small diaphragms in the crossover plane (see Section 3.6).

The bulb face opposite to the gun end is made of a flat polished sheet glass. A transparent signal plate and a photoconductive target

are formed on this face. The signal plate is composed either of a tin oxide coating applied to the glass or of a thin transparent film of metal such as gold or platinum. The photoconductive layer forming the vidicon target is applied to the signal plate by vacuum evaporation. Antimony trisulphide or another semiconductor having photosensitive properties (examples are selenium and lead sulphide) is used as the target material. Selection of the target material is dictated by the necessity to have a definite spectral characteristic, adequate sensitivity and low persistence. Unfortunately, the requirements for high sensitivity and low persistence are often incompatible, the most sensitive photoconductors have the highest persistence. The photosensitive layer must have an adequate dark resistivity—at least of 10^{11} to 10^{12} ohm·cm; otherwise there is a possibility of reducing the potential of charge pattern (leveling potentials of the neighbouring elements) and decreasing the resolving power. A fine-mesh screen leveling the electrostatic field is mounted before the target from the gun side. This screen also performs the function of a collector for the secondary electrons returning from the target.

The vidicon can operate in two modes: (1) the target is scanned by low-velocity electrons; (2) the target is scanned by high-velocity electrons. In the former case the potential of the signal plate exceeds the gun cathode potential by dozens of volts. In the absence of the scanning beam (the gun is cut-off) the target surface assumes the signal plate potential due to the conductance of the photolayer. When the surface of the dark target is scanned by a beam of low-velocity electrons ($\sigma < 1$) its potential tends to an equilibrium value approximately equal to the gun cathode potential. In the interval between the scannings the potential of the target elements increases, but slightly, since the dark resistance of the photoconductive layer is high. Thus, the electron beam, which brings the potential of the target surface to the equilibrium value, charges the elementary capacitors, i.e. a potential difference is produced between the signal plate and the target surface. In the interval between the scannings each elementary capacitor is discharged, but very slightly, as the discharge current is very low due to the high transverse resistance of the photoconductive layer. As the dark resistance is the same for all the elements of the target, these elements produce no output signal in the signal plate circuit because their potentials change identically during the scanning of the surface of the dark target.

When an optical image is projected through the signal plate, the target resistance is changed according to the distribution of illumination. The higher the illumination of the target element, the lower is the transverse resistance of the photoconductive layer at this area. Consequently, in the interval between the scannings the potential of the bright target elements increases much higher than the potential of the dark elements. Thus, a positive potential

charge pattern is produced on the scanned surface of the target. The potential of this charge pattern may be fairly high due to the perceptible difference in the transverse resistance of the dark and bright elements of the target. When scanning the surface of the illuminated target, the potential of its elements is brought by the electron beam to the equilibrium value equal to the cathode potential. However, since the potentials of the target elements are different (in accordance with the illumination distribution), the potentials of the target elements change differently thus producing a video signal in the signal plate circuit.

When the scanning is performed by high-speed electrons, the signal plate potential is dozens of volts lower than the potential of the secondary-electron collector. When the surface of the dark target is scanned by high-speed electrons, ($\sigma > 1$), its potential is brought to the equilibrium value which is approximately equal to the potential of the screen performing the role of the secondary-electron collector. In the case of illuminating the target by changing the transverse conductance of the photoconductive layer a stored charge pattern is formed on the target surface which is converted into a video signal in the signal plate circuit when the elements are scanned by the electron beam.

Modern vidicons make it possible to obtain a satisfactory image (with a signal-to-noise ratio of >10) at an illumination level of only a few luxes, i.e. their sensitivity is close to that of the image orthicons. The light-signal characteristic of a vidicon is largely determined by the dependence of the conductance of the photoconductive layer on the illumination. Since for most of the practically used photoconductors this dependence is roughly described by the ratio $\frac{1}{R} = kE^n$, where $n < 1$, the light characteristic of the vidicon may be close to optimum.

The resolution of vidicons having 26 mm in diameter (with an image size of 9×12 mm) is equal to 450-600 lines. When the tube diameter is increased to 40 mm (the image 25 mm in diagonal) the resolution is increased to 1000-1200 lines. The persistence of vidicons, in spite of new types of photoconductors, is still rather high and the lower the illumination of the target, the higher is the persistence. Thus, for example, with one of the Soviet-made vidicons operated at illumination of 1000 lx, the residual signal does not exceed 10% after 40 ms (in the second frame) while at illumination of 10 lx the signal persistence in the second frame rises up to 40%.

The vidicons, like other devices with photoconductors have disadvantages in that the resistance of the photoconductive materials involved depends upon temperature. A change in the transverse resistance of the target caused by a temperature variation results

in a change in the amplitude of the output signal at a constant voltage of the signal plate. The amplitude of the output signal with variations of the target temperature can be maintained only by changing the voltage of the signal plate, what is not always possible. Therefore, during the operation of the vidicon the target temperature must be kept constant.

The simple construction, high sensitivity, adequate resolution, near-optimum light characteristic make the vidicon a very convenient and promising camera tube, particularly for portable TV

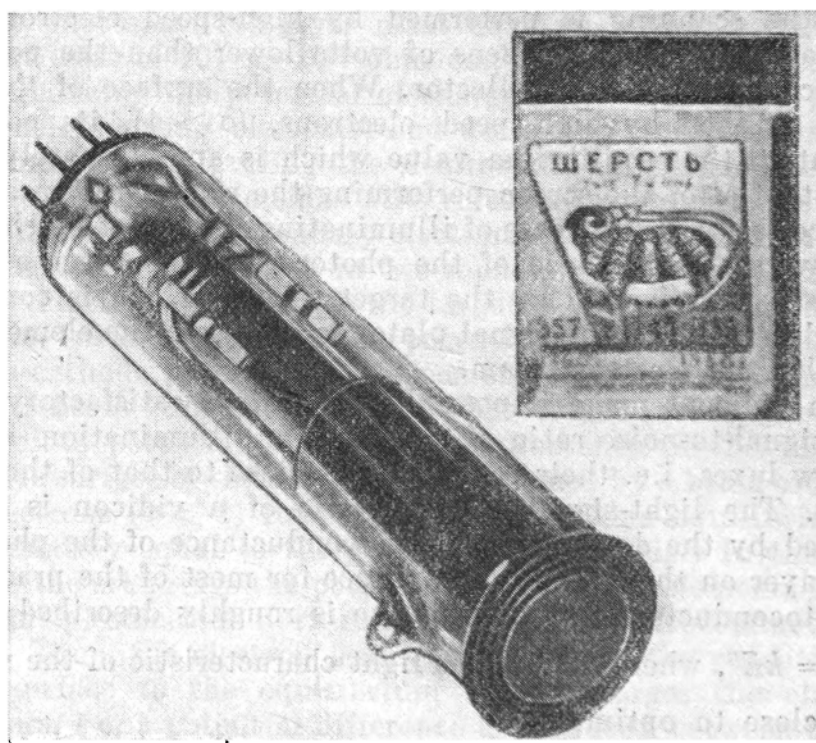


Fig. 12.16. External view of vidicon

broadcasting stations. Vidicons are also widely used in industrial television systems. An external view of the vidicon is shown in Fig. 12.16.

Close to the vidicon in principle of operation is a camera tube called the *ibicon*. This tube employs the phenomenon of the induced conductivity in the dielectric layer (see Section 11.2). The *ibicon* target is composed of a dielectric layer applied to a thin metal (aluminium) signal plate transparent for high-velocity electrons. The photocathode image is transferred to the target by an electrostatic or magnetic field (like in an image converter). The potential difference used in the image section for the acceleration of electrons is adequately high (20-25 kV). If that is the case the electrons emer-

ging from the photocathode acquire an energy sufficient to pass the thin signal plate and induce conductivity in the layer of the target dielectric. Thus, an image pattern corresponding to the distribution of illumination on the photocathode is formed on the target reading beam side opposite to the photocathode. When the target surface is scanned by an electron beam, its potential is brought to an equilibrium value equal to the potential of the gun cathode or secondary-electron collector (depending on the energy of the beam electrons). In this case a video signal is formed in the signal plate circuit. Owing to the amplification of the image during its transfer from the photocathode to the target and its multiplication due to the induced conductivity, the vidicon sensitivity can be higher than that of the image orthicon.

There are described several modifications of camera tubes with a dielectric target employing the phenomenon of induced conductivity, yet so far these tubes have not gained wide application.

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